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Statistical Test for the Mathematical Theory of Democracy

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Wirtschafts- und Sozialwissenschaftliches Institut
in der Hans-Böckler-Stiftung, Düsseldorf

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Abstract

The problem formulation is as follows. Given m policy issues like 'Introduce nation-wide minimal wage' (Yes/No), 'Privatize railways' (Yes/No), etc., and positions of n parties on these issues as well as that of majority of the population revealed, say, by public polls. The questions to answer is: if one of parties (a coalition of a few parties) represents the majority opinions on $m - i$ out of m issues, with i being relatively small, should it be regarded as a manifestation of the party (coalition) representative capacity or just as a possible coincidence?

We test the null-hypothesis that the given observation can result from a coincidence by chance. For this purpose, consider a Bernoulli $(m \times n)$-matrix whose elements are independent Bernoulli random variables taking equiprobable values 0 and 1 corresponding to random (mis)representation of m majority opinions by n parties. The m rows are associated with m issues, and n columns represent the match of positions of n parties to the prevailing public opinion. For a coalition, it is assumed that its positions are determined by a majority of coalition members (parties), that is, by the vectorial sum of associated columns. For instance, a three-party coalition is 100%-representative if the sum of the three associated columns is the m-vector with all elements $\geq 3/2$ (majority of three votes).

To test the null-hypothesis for a single party, we find the probability of occurrence of a column with i or fewer zeros. If this probability is low, e.g. ≤ 0.05, a coincidence by chance looks hardly probable, and the actual observation is interpreted as a demonstration of the party's representativeness (with the statistical significance 5%). To test the null-hypothesis for a coalition of k parties, we find the probability of occurrence of k columns whose sum is the m-vector with i or fewer elements being $< k/2$. Again, if this probability is small then a coincidence is regarded unlikely and the coalition is considered representative, otherwise the question about its representativeness remains open.

The paper provides solutions for coalitions with up to three parties. It is done by meta-modeling with geometric, algebraic, and properly probabilistic meta-models. Each meta-model builds a series of models with computational formulas for particular sizes of the Bernoulli matrix. These formulas are too complex to be derived 'manually' and have no visible regularity, so the meta-modeling approach is essential. The probabilities, which cannot be found due to computer limitations, are estimated from the known ones by five interpolation techniques.

Finally, the statistical significance of representativeness of five major German parties and their coalitions is estimated.

Keywords: Mathematical theory of democracy, statistical test, parties, coalitions, representativeness, Bernoulli matrices, sums of random vectors.

JEL classification: C12—Hypothesis Testing: General, C44—Statistical Decision Theory, C63—Computational Techniques; Simulation Modeling, D71—Social Choice; Clubs; Committees; Associations, D72—Political Processes: Rent-Seeking, Lobbying, Elections, Legislatures, and Voting Behavior.

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Contents

1 Introduction

1.1 Representativeness as a statistical phenomenon

The mathematical theory of democracy, among other things, provides methods to evaluate single representatives (candidates for president, political parties) and representative bodies (parliament, cabinet of ministers) regarding their capacity to express opinions of the population (Tangian 2008a, 2012). The evaluation is based on comparing the position of representatives on selected policy issues with the public opinion revealed in public polls, referenda, or plebiscites. However, any conclusion based on a data sample has a limited reliability. Therefore, the statistical significance of evaluation has to be estimated.

To illustrate what we are going to study, suppose that five political parties define their position on six issues like 'Introduce nation-wide minimal wage', 'Privatize railways', etc., and, according to public polls, one party perfectly represents the public opinion, matching the majority opinion on all the issues. Regarding this outcome, a conclusion about the party's high representativeness is made. Nevertheless, the following questions emerge:

1. Does the outcome observed really indicate at the party's representative capacity, or it may be just a coincidence by chance? In other words, can a similar performance be expected on other policy issues, not yet considered or arising in future?
2. Are six policy issues sufficient to make any conclusion about the party's representative capacity? Or their number should be increased, say, to 10?
3. What can be said if the match of party positions to the public opinion is imperfect, for instance, is restricted to five out of six issues? Does the conclusion about the party's representativeness remain valid?

The same questions can be asked about party coalitions, whose positions on each issue are determined by a majority vote within the coalition.

The usual approach to this type of problems is developing a statistical test. Assuming that the parties meet the public opinion randomly, the probability of the actual outcome is found. If it is small then a coincidence by chance looks improbable and the actual observation is interpreted as a manifestation of the party's representative capacity. If the probability is not small then the outcome looks possible and no conclusion on the party capacity is made.

The match of party positions to public opinion can be represented by a table, in our example of six issues versus five parties, with 1s standing for match and 0s for no match. If the match is assumed random, the table elements turn into independent Bernoulli random variables, taking values 0 and 1 with equal probabilities 1/2.

If an alone-standing party is considered then the table consists of a single column. The match on six out of six issues has the probability $(1/2)^6 = 1/64 < 0.02$. It is sufficiently small, so a coincidence is unlikely. Hence, the party is regarded representative.

For five parties, the situation is different. The probability that one party out of five expresses the majority opinion on all the six issues is $1-\left[1-(1/2)^6\right]^5 \approx 0.08$. It is not small enough to say that the actual outcome is little probable, so the party's representative capacity is in question. However, if the perfect match is observed for seven out of seven issues then the probability $1-\left[1-(1/2)^7\right]^5 < 0.04$ is sufficiently small, arguing for the party's representativeness.

Making conclusions about coalitions is similar, but random coincidences are more frequent than for single parties. In our example of five parties, the occurrence of a three-party coalition which represents the public opinion on six out of six issues has the probability of about 0.10. Hence, the perfect coalition performance observed is not much promising for the future. The common probability threshold 0.05 (= 5%-significance level) can be surpassed with as many as eight hits out of eight. If the match is imperfect then the sample of issues should be extended further. For three-party coalitions, a single mismatch on $i = 1$ issue must be outbalanced by at least $m - i = 10$ hits, otherwise the 5%-significance is not attained.

Computing the probabilities required to statistically 'prove' the representative capacity of coalitions is not easy, and just this task is the subject of this paper.

1.2 Problem formulation

Perfect column pairs and column triplets A *Bernoulli* $(m \times n)$*-matrix* $B = \{b_{ij}\}$ is a matrix whose elements b_{ij} are independent Bernoulli random variables, taking values 0 and 1 with equal probabilities 1/2. A k-tuple of its columns is called *perfect* if

$$\text{Its sum along rows is a column } m\text{-vector with all } m \text{ elements being } \geq k/2 \ . \tag{1}$$

Label every k-tuple of columns of Bernoulli matrix with the set of corresponding column numbers $J = [j_1, \ldots, j_k]$. Order these labels J and use them as scalar indices of column k-tuples.

By A_J denote the event that the J-th k-tuple is perfect. We are interested in the probability of union of these events, meaning that there occurs at least one perfect k-tuple of columns:

$$\mathsf{P}\left(\bigcup A_J\right) = ? \tag{2}$$

A table with random 0–1 codes of match of party positions to public opinion is nothing else but a Bernoulli matrix. Here, m rows are associated with m issues, and n columns are associated with n parties. If the majority opinion on the i-th issue is represented by the j-th party then the matrix element $b_{ij} = 1$, otherwise $b_{ij} = 0$.

A perfect k-tuple of columns corresponds to a coalition of k parties whose internal majority ($\geq k/2$ parties) shares the prevailing public opinion on every issue. The probability (2) characterizes the occurrence of such coalitions by chance and is needed to statistically test the representative capacity of coalitions with 100%-representativeness observed. That is, it is addressed to answer Questions 1–2.

***i*-imperfect column pairs and column triplets** To study Question 3 about imperfect match of party positions to public opinion, weaken the condition (1). If it is violated in i or fewer rows, the k-tuple of columns is called *i-imperfect*, that is,

$$\text{Its sum along rows is a column } m\text{-vector with at least } m - i \text{ elements being } \geq k/2 \ .$$

Obviously, perfect k-tuples of columns are 0-imperfect. A i-imperfect k-tuple of columns corresponds to a coalition which represents the majority opinion incompletely, failing to do it on i or fewer issues. The events A_J and the probability (2) are respectively redefined for i-imperfect k-tuples of columns.

Existing literature Besides the mathematical theory of democracy, the problem of estimating the probabilities mentioned arises in genetics, logistics, and some other applications like traffic control or finances (Coffman and Luecker 1991, Garey, Graham and Johnson 1976, Tangian

2007, 2008b). Random matrices are considered in numerous publications; for a survey see Eaton (2007), Edelman and Rao (2005), Kendrick (1981), and Mehta (2004). In particular, there are papers focused on sums of random vectors and their approximations; see Acosta (1992), Barbe and Broniatowsky (2005), Bolthausen (1987), Coffman and Lueker (1991), and Stanke (2003).

These publications study trends in large random matrices or in large sums of random vectors rather than propose solutions for small and medium-sized practical applications where asymptotic properties are not salient. The given paper attempts to fill in this gap by developing approaches to the problem for column pairs and column triplets in small and medium-sized Bernoulli matrices, that is, for coalitions with two or three parties if the total number of parties and the number of reference policy issues are rather limited.

Meta-modeling approach For Bernoulli matrices, three ways to find the probability (2) are developed. One method is geometric, another algebraic, and the third properly probabilistic. In theory, each of these methods solves the problem, but in practice every method has its computational limits. The geometric solution is computationally appropriate for Bernoulli matrices with a few columns, the algebraic — for Bernoulli matrices with a few rows, and the probabilistic — for Bernoulli matrices with twice more rows than columns. Therefore, the united computational solution is combined from the three methods. There are still non-computable probabilities, and their approximations are estimated from the known probabilities by five interpolation techniques.

The general approach is based on meta-modelling. Each meta-model builds a series of models with computational formulas for particular sizes of the Bernoulli matrix. These formulas are too complex to be derived 'manually' and have no visible regularity, so the meta-modeling approach is essential.

The complexity and lack of regularities may evoke suspects in the model errors. The doubts are resolved by equal output from different methods. As we shall see, the probabilities computed by alternative methods, say, geometric and algebraic, coincide with the precision better than $\epsilon = 2^{-25}$.

1.3 Structure of the paper

Chapter 2, 'Geometric method', is primarily targeted at Bernoulli matrices with up to eight columns. The (im)perfect outcomes are represented by multi-dimensional constructs based on the Sierpinski triangle (2012). Its repetitive structure enables to obtain systems of recursive equations for counting elements of these constructs and thereby to compute the probability (2). Depending on the width of Bernoulli matrix, the recursive systems are of variable dimension. Thus, perfect outcomes of Bernoulli matrices with 8 columns are counted with a system of almost 3,000 recursive equations, and it is shown that this system cannot be made simpler. For counting imperfect outcomes, the number of equations in certain cases attains 1.5 million.[1]

Chapter 3, 'Algebraic method', is aimed at Bernoulli matrices with up to seven rows. The corresponding meta-model constructs disjoint events with (im)perfect column pairs and triplets, and then the outcomes are counted as sums of multinomial coefficients expressed by exponential polynomials. In our application, some polynomials with a few dozens of terms require constructing millions of events.

[1]It is still much better than running through all the outcomes. For instance, the 'small' example at the end of the paper with five parties and 32 issues is modeled by a Bernoulli matrix with $5 \times 32 = 160$ elements. The total number of outcomes 2^{160} has the order of the number of atoms on the Earth. Out model can deal with matrices of size 60×60 and even larger.

In Chapter 4, ‘Probabilistic method’, the probabilities of *perfect* outcomes are found with the Inclusion-Exclusion formula (Feller 1968, p. 99, Helms 1997, p. 42, 55–57, Inclusion-Exclusion Principle 2012). The meta-model constructs its successive sums, but the complexity of the task rapidly increases, so that complete models are implemented for Bernoulli matrices with up to six columns. Alternatively, the first six sums of the Inclusion-Exclusion formula provide accurate approximations for Bernoulli matrices whose rows are at least twice more numerous than columns.

Chapter 5, ‘Interpolation methods’ deals with the probabilities which cannot be calculated in reasonable time by any of the three methods. Five approaches are developed, basing on interpolation between the probabilities already found. The probabilities which cannot be approximated by the Inclusion-Exclusion formula are ‘restored’ by cubic splines, that is, piecewise-polynomial functions of third degree (Stoer and Bulirsch 2002, pp. 93–106; Stoer and Bulirsch 2007, pp. 112–148); see also Spline (mathematics) (2012) and Spline Interpolation (2012). Two methods use binomial and beta distribution functions to interpolate on the degree of imperfectness i. The interpolation on the horizontal size n of Bernoulli matrix is made with exponential functions. Finally, probabilities of imperfect outcomes are derived from that of perfect outcomes obtained with the Inclusion-Exclusion formula.

Chapter 6, ‘Example: Evaluation of German parties and coalitions’, applies the probabilities found to test statistical hypotheses about the representative capacity of German parties and their coalitions at the time of German Bundestag elections 2009.

Chapter 7, ‘Conclusions: What can be improved’, discusses possible continuations of the study.

Annex ‘Probability tables’, contains tables with probabilities of perfect and imperfect column pairs and triplets in Bernoulli matrices with up to 48 rows and 16 columns. The computer program accompanying this study is written in the MATLAB 2011b computer environment. It outputs the LaTeX template of the given paper, that is, a document with the title page and all figures and tables, including their captions and labels. The final paper is made by inserting the author’s text into this template. The computer project’s volume is about 200 pages of code and by this reason it is not included in this paper but is available on request.

As for the computer performance, the time required by the meta-model to derive particular formulas rises up to several days, and the total time to derive all computational formulas for this paper is about one month, constraining further advancements. The computations with the pre-computed formulas, although still not feasible ’manually’, take a few seconds. Throughout the paper, the computation time is given for the MATLAB 2011b implementation running a PC with Intel(R) Core(TM)i7-2700K CPU 3.50GHz, 16GB RAM, under the WINDOWS 7 Professional Version 2009 Service Pack 1 64-bit operation system.

2 Geometric method

2.1 Pairs of m-columns and Sierpinski triangles

To study outcomes of Bernoulli $(m \times n)$-matrices construct tables T_m with all types of binary m-columns indexed in the ascending lexicographic order (as if reading the columns from bottom as binary numbers); see Tables 1–4. Since each column element takes values 0 or 1, the total number of m-column types is 2^m.Note that columns are indexed starting from 1 rather than from 0. It is done to meet the standard indexing in multinomial coefficients considered in the next chapter.

Table 1: Types of 1-columns

Column index	
1	2
0	1

Table 2: Types of 2-columns

Column index			
1	2	3	4
0	1	0	1
0	0	1	1

Table 3: Types of 3-columns

Column index							
1	2	3	4	5	6	7	8
0	1	0	1	0	1	0	1
0	0	1	1	0	0	1	1
0	0	0	0	1	1	1	1

Table 4: Types of 4-columns

Column index															
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1

Proposition 1 (Recursive construction of tables T_m) *Tables T_m of types of binary m-columns can be constructed recursively:*

$$\begin{aligned} T_1 &= (0\ 1) \\ T_2 &= \begin{pmatrix} T_1 & & T_1 & \\ 0 & 0 & 1 & 1 \end{pmatrix} \\ \dots & \\ T_{m+1} &= \begin{pmatrix} T_m & T_m \\ \underbrace{0\dots 0}_{2^m} & \underbrace{1\dots 1}_{2^m} \end{pmatrix} . \end{aligned}$$

Let us show that index pairs of perfect column pairs constitute the *Sierpinski triangle* (2012) described by W.Sierpinski in 1915, and index pairs of imperfect column pairs constitute the gen-

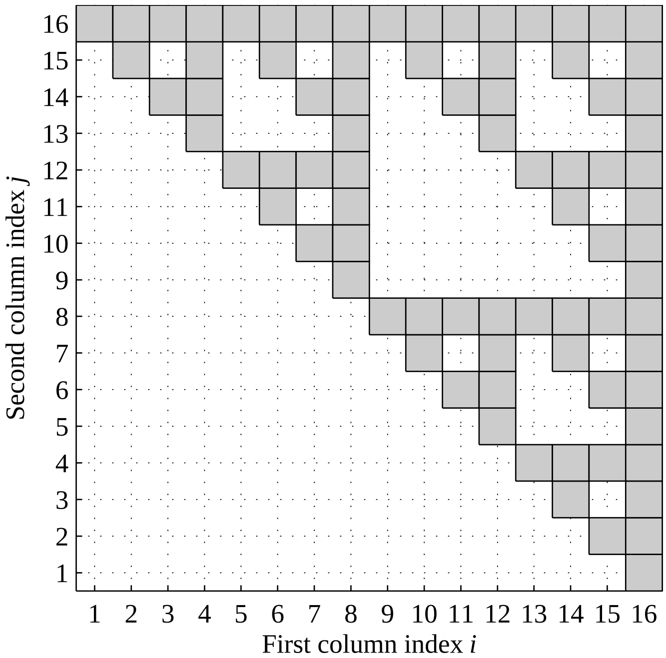


Figure 1: Perfect pairs of 4-columns (Sierpinski triangle)

eralized Sierpinski triangle defined below. Recall that the Sierpinski triangle can be constructed in several ways, in particular, recursively. The construction is initialized with the top-right element in Figure 1. At each step, the current configuration is triplicated by adding its two instances to the existing one. One instance is added at the left-hand side, and one instance is added below. The bottom-left area is left empty.

Generalize the notion of Sierpinski triangle to *Sierpinski triangle of type* i. Define the Sierpinski triangle of type 0 to be the empty set. For $i > 0$ the construction is initialized with the top-right element in Figures 2 and 3. At each step the current configuration is triplicated by adding two instances of the current configuration to the existing one. One instance is added on the left-hand side and one instance below. Besides, the bottom-left area is filled with the Sierpinski triangle of type $i-1$. As one can see, the original Sierpinski triangle is of type 1.

Proposition 2 (Imperfect column pairs as generalized Sierpinski triangles) *The index pairs of* i*-imperfect column pairs constitute the Sierpinski triangle of type* $i+1$.

PROOF. The square of index pairs of m-columns consists of four sub-squares whose coordinate axes are half-axes of the large square. Denote by 0 the half-axis with indices $\leq 2^{m-1}$ of m-columns ending with 0, and by 1 the half-axis with indices $> 2^{m-1}$ of m-columns ending with 1. The (01), (10), and (11) sub-squares correspond to pairs of m-columns with no simultaneous 0s at the bottom. Therefore, these column pairs are i-imperfect if and only if their upper $(m-1)$-segments make i-imperfect pairs, i.e. these three sub-squares have the same structure of i-imperfect outcomes. The (00) sub-square corresponds to pairs of m-columns with 0s at the bottom. Consequently, these column pairs are i-imperfect if and only if their upper $(m-1)$-segments constitute $(i-1)$-imperfect pairs. This is exactly the way the Sierpinski triangle of type $i+1$ is constructed. ■

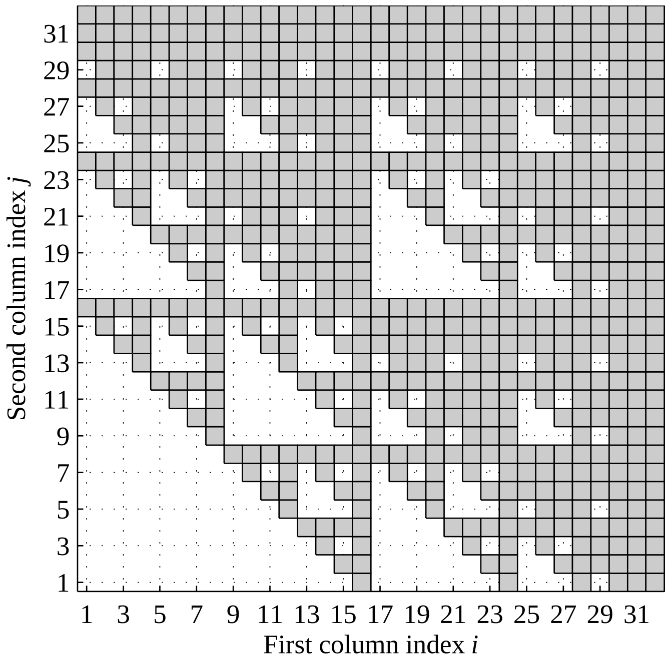


Figure 2: 1-imperfect pairs of 5-columns (Sierpinski triangle of type 2)

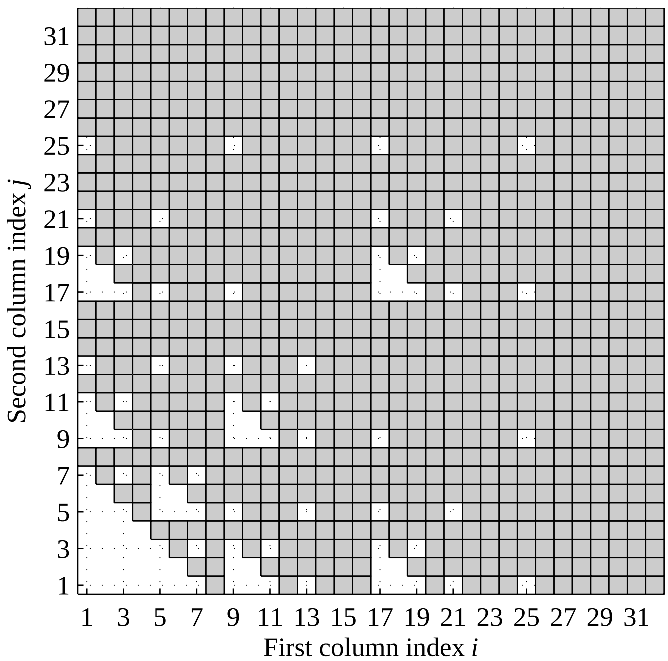


Figure 3: 2-imperfect pairs of 5-columns (Sierpinski triangle of type 3)

2.2 Recursive equations for Bernoulli two-column matrices

It is easy to show that perfect/imperfect outcomes of two-column Bernoulli matrices are binomially distributed. However, the binomial approach cannot be extended to more complex cases, and we immediately start to introduce the general method.

By definition of generalized Sierpinski triangle and Proposition 2, the transition from m-columns to $(m+1)$-columns results in triplicating the number $\overset{i+1}{x}_m$ of i-imperfect outcomes (labeled by the type $i+1$ of the corresponding Sierpinski triangle) and adding the amount $\overset{i}{x}_m$ of $(i-1)$-imperfect outcomes:

$$\begin{array}{rcllll} \overset{1}{x}_{m+1} & = & 3\,\overset{1}{x}_m & & & \text{(for Sierpinski triangle of type 1)} \\ \overset{2}{x}_{m+1} & = & \overset{1}{x}_m & +\ 3\,\overset{2}{x}_m & & \text{(for Sierpinski triangle of type 2)} \\ \overset{3}{x}_{m+1} & = & & \overset{2}{x}_m & +\ 3\,\overset{3}{x}_m & \text{(for Sierpinski triangle of type 3)} \\ \multicolumn{6}{c}{\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots} \end{array} \tag{3}$$

initialized with $\overset{1}{x}_0 = \ldots = \overset{i+1}{x}_0 = 1$. In vector-matrix notation we have

$$\mathbf{x}_{m+1} = \mathbf{A}\mathbf{x}_m = \mathbf{A}^{m+1}\mathbf{1}\ ,$$

where

$$\mathbf{x}_0 = \mathbf{1} = \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix}, \quad \mathbf{x}_m = \begin{pmatrix} \overset{1}{x}_m \\ \vdots \\ \overset{i+1}{x}_m \end{pmatrix}, \quad \text{and} \quad \mathbf{A} = \begin{pmatrix} 3 & 0 & 0 & \ldots & 0 \\ 1 & 3 & 0 & \ddots & \vdots \\ 0 & 1 & 3 & \ddots & 0 \\ \vdots & & \ddots & \ddots & 0 \\ 0 & \ldots & 0 & 1 & 3 \end{pmatrix}.$$

Since the square of outcomes of Bernoulli $(m \times 2)$-matrix has $(2^m)^2 = 4^m$ elements,

$$\begin{array}{rcl} \mathsf{P}(\text{Outcome of Bernoulli } (m \times 2)\text{-matrix is a perfect column pair}) & = & \dfrac{\overset{1}{x}_m}{4^m} = \dfrac{3^m}{4^m} \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots & & \\ \mathsf{P}(\text{Outcome of Bernoulli } (m \times 2)\text{-matrix is a } i\text{-imperfect column pair}) & = & \dfrac{\overset{i+1}{x}_m}{4^m}\ . \end{array}$$

2.3 Multigraph-based recursive equations

Identify outcomes of Bernoulli three-column $(m \times 3)$-matrix with triplets of indices of m-columns which constitute a cube with $(2^m)^3 = 8^m$ elements. A *perfect outcome* is a triplet of indices with two indices making a perfect pair, that is, belonging to the Sierpinski triangle. Therefore, perfect outcomes build 2^m-long bars over the Sierpinski triangles located in all the three cube's coordinate faces (bordered by coordinate axes) — the union of three intersecting Sierpinski cylinders; see Figure 4. For i-imperfect outcomes, the cylinder bases are obviously Sierpinski triangles of type $i+1$.

We say that a cube's coordinate face is of type j, or j-*face*, if its outcomes of interest constitute the Sierpinski triangle of type j. The number of outcomes of interest in every cube depends on j-types of its faces (as cylinder bases) and their reciprocal position. All of these are characterized

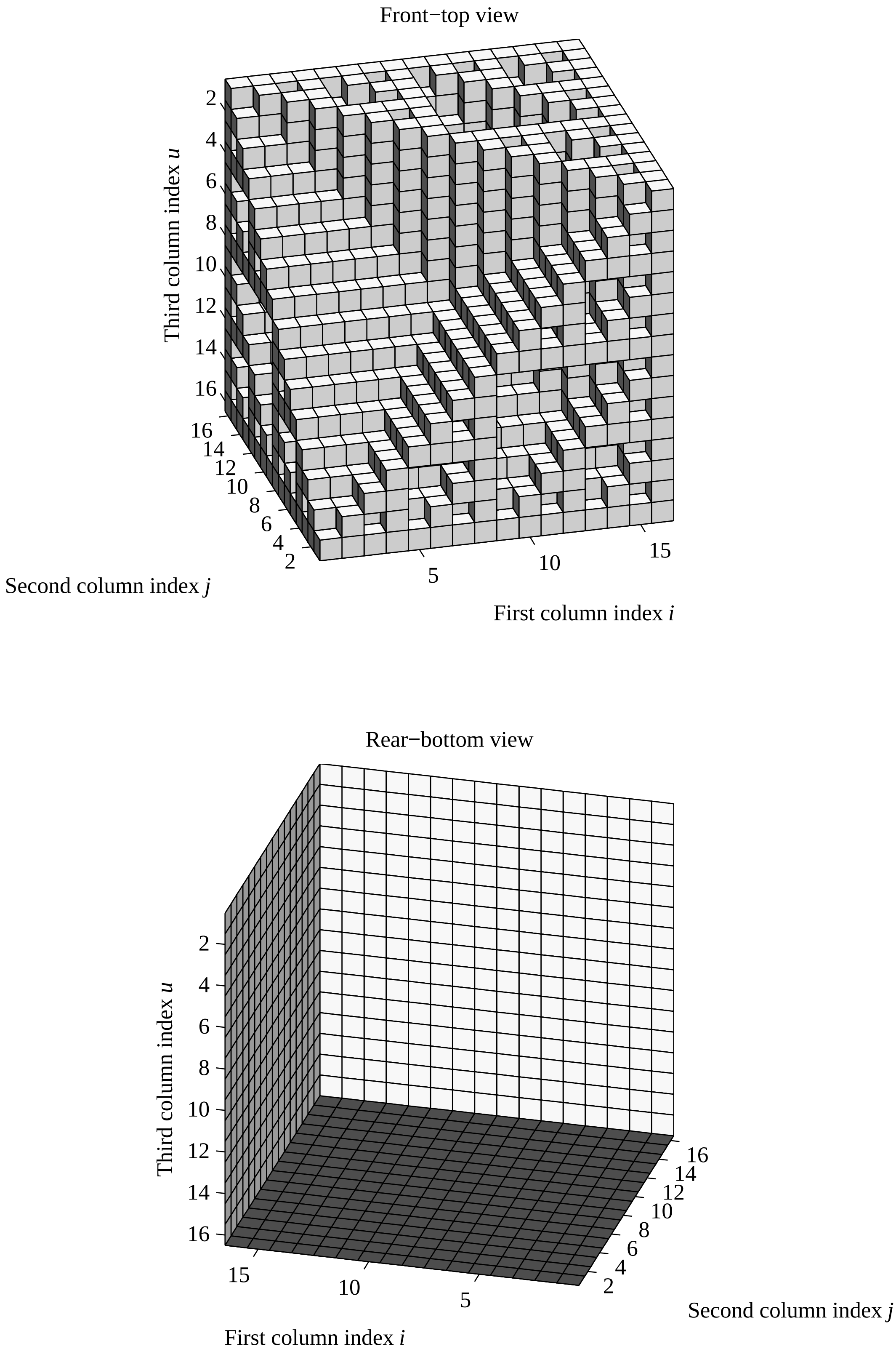


Figure 4: Triplets of 4-columns with a perfect column pair (union of three Sierpinski cylinders)

by a *multigraph* whose vertices stand for the coordinate axes and j-multiple edges for the cube's j-faces.

Recall that a multigraph with n vertices is represented by its *adjacency* $(n \times n)$*-matrix*

$$\mathbf{G} = \{e_{xy}\}, \quad e_{xy} = \begin{cases} j & \text{if } j \text{ edges connect vertices } x \text{ and } y \ (x \text{ and } y \text{ are } j\text{-adjacent}) \\ 0 & \text{otherwise} \end{cases} .$$

In our case

$$e_{xy} = \begin{cases} j & \text{if the } xy \text{ coordinate face of the cube is of type } j \\ 0 & \text{otherwise} \end{cases} .$$

For example, the 3-multigraph  shows three coordinate axes by three vertices, one 2-face by the double edge, one 1-face by the single edge, and one empty 0-face by the missing edge. Its adjacency matrix is

$$\mathbf{G} = \begin{pmatrix} 0 & 2 & 0 \\ 2 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} .$$

Since the adjacency matrix is symmetric, it is completely characterized by the vector obtained from concatenating the columns of its bottom-triangular section; for our example it is $\begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$.

For a certain permutation of the graph vertices, this vector with $\frac{n(n-1)}{2}$ elements attains its lexicographic maximum g which is called the *graph invariant* (Kuramochi and Karypsis 2007, pp. 119–121). In our example, after having permutated vertices 1 and 2, we obtain the graph invariant

$$g = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} .$$[2]

Two graphs with equal invariants are equal to within a permutation of vertices, so that the cubes with equal graph invariants have the same number of (im)perfect outcomes.

Derive recursive equations for 1-imperfect outcomes of Bernoulli $[(m+1) \times 3]$-matrix. In the notation of proof of Proposition 2, the cube of size $(2^{m+1})^3$ falls into eight sub-cubes of size $(2^m)^3$ with different 0–1 combinations of half-axes of the large cube. Depending on the 0–1 half-axis combination, every sub-cube has its own characteristic multigraph which determines its number of 1-imperfect outcomes. Therefore, we use the graph invariants to classify the cubes and to label unknowns in recursive equations as in (3).

In our geometric representation, the large cube $(2^{m+1})^3$ of 1-imperfect outcomes has three coordinate faces with the Sierpinski triangle of type 2. The cube's characteristic multigraph  has the adjacency matrix and the graph invariant, respectively,

$$\mathbf{G} = \begin{pmatrix} 0 & 2 & 2 \\ 2 & 0 & 2 \\ 2 & 2 & 0 \end{pmatrix} \quad \text{and} \quad g = \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix} .$$

Correspondingly, denote the number of 1-imperfect outcomes in the large cube by $\overset{222}{x}_{m+1}$.

[2] For i-imperfect outcomes, the elements of g adopt values $0, \ldots, i+1$. Then g can be regarded as a $(i+2)$-cimal number, in our example $(210)_3 = 2 \cdot 3^2 + 2 \cdot 3^1 + 0 \cdot 3^0 = 24$.

Note that if a face of the large cube has type j then the parallel sub-cube's faces (01), (10) and (11) inherit the same j-type. The sub-cube's face (00) has the reduced type $j-1$ (but not below 0 which means the empty face). All of these mean that the characteristic multigraph of a sub-cube is obtained from the multigraph of the large cube by reducing by 1 the degree of the edges which denote the (00)-faces of the sub-cube.

To perform these reductions, define *reduction matrices* $\mathbf{R}_c$. These matrices are of the same size as the adjacency matrix $\mathbf{G}$ and have 1s at the position of the edges whose degree must be reduced, and 0s otherwise. (Recall that the reduction stops as $e_{xy}=0$ is attained, so that $0-1=0$.) In our 3D case, we have $2^3=8$ sub-cubes with different 0–1 half-axis combinations and 8 reduction matrices with some of them being equal:

1 (000)-sub-cube with all the three (00)-faces. Here, the reduction matrix, the resulting adjacency matrix of this sub-cube and the graph invariant are, respectively,

$$\mathbf{R}_{000}=\begin{pmatrix}0&1&1\\1&0&1\\1&1&0\end{pmatrix},\quad \mathbf{G}_1=\mathbf{G}-\mathbf{R}_{000}=\begin{pmatrix}0&1&1\\1&0&1\\1&1&0\end{pmatrix},\quad g_1=\begin{pmatrix}1\\1\\1\end{pmatrix}.$$

Label the unknown number of 1-imperfect outcomes in this sub-cube by its graph invariant g_1, that is, denote it by $\overset{111}{x}_m$.

1 (001)-sub-cube. Its only (00)-face is between the first two axes, whence,

$$\mathbf{R}_{001}=\begin{pmatrix}0&1&0\\1&0&0\\0&0&0\end{pmatrix},\quad \mathbf{G}_2=\mathbf{G}-\mathbf{R}_{001}=\begin{pmatrix}0&1&2\\1&0&2\\2&2&0\end{pmatrix},\quad g_2=\begin{pmatrix}2\\2\\1\end{pmatrix}.$$

The number of 1-imperfect outcomes in this sub-cube denote by $\overset{221}{x}_m$.

1 (010)-sub-cube. Its only (00)-face is between the first and third axes, whence,

$$\mathbf{R}_{010}=\begin{pmatrix}0&0&1\\0&0&0\\1&0&0\end{pmatrix},\quad \mathbf{G}_3=\mathbf{G}-\mathbf{R}_{010}=\begin{pmatrix}0&2&1\\2&0&2\\1&2&0\end{pmatrix},\quad g_3=\begin{pmatrix}2\\2\\1\end{pmatrix}.$$

The number of 1-imperfect outcomes in this sub-cube denote by $\overset{221}{x}_m$.

1 (100)-sub-cube. Its only (00)-face is between the second and third axes, whence,

$$\mathbf{R}_{100}=\begin{pmatrix}0&0&0\\0&0&1\\0&1&0\end{pmatrix},\quad \mathbf{G}_4=\mathbf{G}-\mathbf{R}_{100}=\begin{pmatrix}0&2&2\\2&0&1\\2&1&0\end{pmatrix},\quad g_4=\begin{pmatrix}2\\2\\1\end{pmatrix}.$$

The number of 1-imperfect outcomes in this sub-cube denote by $\overset{221}{x}_m$.

4 sub-cubes with no (00)-face, that is, (111), (011), (101), and (110) sub-cubes. Since they have no (00)-faces, the corresponding reduction matrix is $\mathbf{0}$, and the resulting adjacency matrix of each sub-cube and its invariant are, respectively,

$$\mathbf{R}_{111}=\begin{pmatrix}0&0&0\\0&0&0\\0&0&0\end{pmatrix},\quad \mathbf{G}_5=\mathbf{G}-\mathbf{R}_{111}=\begin{pmatrix}0&2&2\\2&0&2\\2&2&0\end{pmatrix},\quad g_5=\begin{pmatrix}2\\2\\2\end{pmatrix}.$$

The number of 1-imperfect outcomes in each sub-cube denote by $\overset{222}{x}_m$.

Since the number of 1-imperfect outcomes in the large cube is equal to the sum of that in the eight sub-cubes, we obtain the following recursive equation

$$\overset{222}{x}_{m+1} = \overset{111}{x}_{m} + 3\overset{221}{x}_{m} + 4\overset{222}{x}_{m} \ . \tag{4}$$

This equation has two new unknowns, $\overset{111}{x}_{m}$ and $\overset{221}{x}_{m}$. Derive a recursive equation for the 'superior' unknown $\overset{221}{x}_{m+1}$ by decomposing the $(2^{m+1})^3$ cube with the graph invariant g_{221}. Apply the procedure just described to one of relevant graphs, say, $\mathbf{G}_4 = \begin{pmatrix} 0 & 2 & 2 \\ 2 & 0 & 1 \\ 2 & 1 & 0 \end{pmatrix}$, subtracting the reduction matrices $\mathbf{R}_{000}, \ldots, \mathbf{R}_{111}$ (with the latter zero-matrix repeated four times). Decompose the large cube into eight sub-cubes:

1 (000)-sub-cube with the characteristic graph and its invariant

$$\mathbf{R}_{000} = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}, \quad \mathbf{G}_6 = \mathbf{G}_4 - \mathbf{R}_{000} = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad g_6 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \ .$$

The number of i-imperfect outcomes in this sub-cube denote by $\overset{110}{x}_{m}$.

1 (001)-sub-cube with the reduction matrix, characteristic graph and its invariant

$$\mathbf{R}_{001} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \mathbf{G}_7 = \mathbf{G}_4 - \mathbf{R}_{001} = \begin{pmatrix} 0 & 1 & 2 \\ 1 & 0 & 1 \\ 2 & 1 & 0 \end{pmatrix}, \quad g_7 = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} \ .$$

The number of 1-imperfect outcomes in these sub-cube denote by $\overset{211}{x}_{m}$.

1 (010)-sub-cube with the reduction matrix, characteristic graph and its invariant

$$\mathbf{R}_{010} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad \mathbf{G}_8 = \mathbf{G}_4 - \mathbf{R}_{010} = \begin{pmatrix} 0 & 2 & 1 \\ 2 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}, \quad g_8 = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} \ .$$

The number of 1-imperfect outcomes in these sub-cube denote by $\overset{211}{x}_{m}$.

1 (100)-sub-cube with the reduction matrix, characteristic graph and its invariant

$$\mathbf{R}_{100} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad \mathbf{G}_9 = \mathbf{G}_4 - \mathbf{R}_{100} = \begin{pmatrix} 0 & 2 & 2 \\ 2 & 0 & 0 \\ 2 & 0 & 0 \end{pmatrix}, \quad g_9 = \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix} \ .$$

The number of 1-imperfect outcomes in these sub-cube denote by $\overset{220}{x}_{m}$.

4 sub-cubes with no (00)-face. Subtracting $\mathbf{R}_{111} = \mathbf{0}$ from $\mathbf{G}_4$ we obtain the same characteristic graph $\mathbf{G}_4$ with its graph invariant $g_4 = (2, 2, 1)'$. The number of 1-imperfect outcomes in each of these sub-cubes is $\overset{221}{x}_{m}$.

Thereby we obtain the next recursive equation

$$\overset{221}{x}_{m+1} \;=\; \overset{110}{x}_{m} + 2\,\overset{211}{x}_{m} + \overset{220}{x}_{m} + 4\,\overset{221}{x}_{m} \;. \tag{5}$$

Apply this decomposition procedure to $\overset{111}{x}_{m}$ from (4), and to $\overset{110}{x}_{m}$, $\overset{211}{x}_{m}$ and $\overset{220}{x}_{m}$ from (5), then to the next encountering unknowns, as long as the labeling graph invariants are not zero vectors. Finally obtain the following full system of recursive equations ordered by the graph invariants, that is, with (4) and (5) at the bottom:

$$\begin{aligned}
\overset{100}{x}_{m+1} &= 6\,\overset{100}{x}_{m}\\
\overset{110}{x}_{m+1} &= 2\,\overset{100}{x}_{m} + 5\,\overset{110}{x}_{m}\\
\overset{111}{x}_{m+1} &= 3\,\overset{110}{x}_{m} + 4\,\overset{111}{x}_{m}\\
\overset{200}{x}_{m+1} &= 2\,\overset{100}{x}_{m} + 6\,\overset{200}{x}_{m}\\
\overset{210}{x}_{m+1} &= \overset{100}{x}_{m} + \overset{110}{x}_{m} + \overset{200}{x}_{m} + 5\,\overset{210}{x}_{m}\\
\overset{211}{x}_{m+1} &= \overset{100}{x}_{m} + \overset{111}{x}_{m} + 2\,\overset{210}{x}_{m} + 4\,\overset{211}{x}_{m}\\
\overset{220}{x}_{m+1} &= \overset{110}{x}_{m} + 2\,\overset{210}{x}_{m} + 5\,\overset{220}{x}_{m}\\
\overset{221}{x}_{m+1} &= \overset{110}{x}_{m} + 2\,\overset{211}{x}_{m} + \overset{220}{x}_{m} + 4\,\overset{221}{x}_{m}\\
\overset{222}{x}_{m+1} &= \overset{111}{x}_{m} + 3\,\overset{221}{x}_{m} + 4\,\overset{222}{x}_{m}
\end{aligned}$$

and initialized, as previously, with $\overset{100}{x}_{0} = \ldots = \overset{222}{x}_{0} = 1$. In vector-matrix notation the system looks as follows

$$\mathbf{x}_{m+1} = \mathbf{A}\mathbf{x}_{m} = \mathbf{A}^{m+1}\mathbf{1}\;,$$

where

$$\mathbf{x}_0 = \mathbf{1} = \begin{pmatrix} 1\\ \vdots\\ 1\end{pmatrix}, \quad \mathbf{x}_m = \begin{pmatrix} \overset{100}{x}_{m}\\ \overset{110}{x}_{m}\\ \overset{111}{x}_{m}\\ \vdots\\ \overset{222}{x}_{m}\end{pmatrix}, \quad \text{and} \quad \mathbf{A} = \begin{pmatrix}
6&0&0&0&0&0&0&0&0\\
2&5&0&0&0&0&0&0&0\\
0&3&4&0&0&0&0&0&0\\
2&0&0&6&0&0&0&0&0\\
1&1&0&1&5&0&0&0&0\\
1&0&1&0&2&4&0&0&0\\
0&1&0&0&2&0&5&0&0\\
0&1&0&0&0&2&1&4&0\\
0&0&1&0&0&0&0&3&4
\end{pmatrix}.$$

Besides 1-imperfect outcomes $\overset{222}{x}_{m}$, this system also enables to count perfect outcomes $\overset{111}{x}_{m}$ (the invariant 111 characterizes the cube with all coordinate faces of type 1, that is, consisting of perfect outcomes). The corresponding equations 3 and 9 for counting outcomes $\overset{jjj}{x}_{m}$ for different degree of imperfectness $j-1$ are called *key equations* of the recursive system.

Since the cube of outcomes of Bernoulli $(m \times 3)$-matrix has $(2^m)^3 = 8^m$ elements,

$$\begin{aligned}
\mathsf{P}(\text{Bernoulli } (m\times 3)\text{-matrix has a perfect column pair}) &= \frac{\overset{111}{x}_{m}}{8^m}\\
\mathsf{P}(\text{Bernoulli } (m\times 3)\text{-matrix has a 1-imperfect column pair}) &= \frac{\overset{222}{x}_{m}}{8^m}\;.
\end{aligned}$$

2.4 Perfect and imperfect column pairs in the general case

Generalize the procedure described to Bernoulli matrices with $n > 3$ columns. The algorithm below is traced in Table 5.

1. (Preparing reduction matrices) Generate 2^n reduction matrices $\mathbf{R}_c$ for all 0–1 combinations of n half-axes. Since $n+1$ combinations have no two 0s, their reduction matrices are $\mathbf{0}$. The repeated matrix means several instances of similar sub-cubes.

2. (Initializing the list of records which back up recursive equations) The first record contains two parts

   (a) *Root elements*, including the adjacency matrix $\mathbf{G}$ for the cube with all faces of type $i+1$, and its graph invariant $g = (i+1, \dots, i+1)'$.

   (b) *List of sub-elements.* For every reduction matrix $\mathbf{R}_c$ find the adjacency matrix $\mathbf{G}_c = \mathbf{G} - \mathbf{R}_c$ (with reduction of matrix elements stopping at 0), and its graph invariant g_c. Count the number of instances $a_{g\,g_1}, a_{g\,g_2}, \dots,$ of each graph invariant $g_1, g_2, \dots,$ and exemplify each encountering invariant g_c with one of related matrices $\mathbf{G}_c$.

   This record backs up the first recursive equation $\overset{g}{x}_{m+1} = a_{g\,g_1} \overset{g_1}{x}_m + a_{g\,g_2} \overset{g_2}{x}_m + \dots$.

3. (Appending new records to the list) Do the following

   (a) In the list of sub-elements of the current record (resulting from the decomposition of a large cube into sub-cubes), consider every graph invariant g_c. If g_c is not among the existing root elements, append a new record to the list of records, containing (a) the invariant g_c with the exemplifying matrix $\mathbf{G}_c$ as the new root elements, and (b) the list of sub-elements for the new root computed like in Item 2b.

   Every new record backs up a new recursive equation.

   (b) After having processed the list of sub-elements of the current record, move to the next record and restart Item 3a. If there is no next record (the reduction procedure is exhausted) then stop.

This procedure is implemented in a computer program. The recursive systems derived are described in Table 6, and the probabilities computed with these recursive systems are displayed in Table 7. Some more probabilities are given in the Appendix.

Comparing the second column of Table 6 with the penultimate one, where the records are not interrupted by omission points, note that all elements of vector $\mathbf{x}_{m+1} = \mathbf{A}\mathbf{x}_m$ become different after a few successive m-iterations. That is, the sub-cubes with different multigraphs have different number of outcomes of interest. This means that there is little chance to reduce the recursive system and/or to make its derivation simpler. For instance, classifying sub-cubes by the number of faces of different types without taking into account their reciprocal position reflected in multigraphs is not possible.

Table 6 shows a rapid growth of computational complexity as n and i increase. For instance, the recursive system for $n = 4$ and $i = 16$ has about 1.5 million equations with about 12 terms each. Their computer derivation takes 18 hours. The recursive system for $n = 8$ and $i = 0$ has over 12 thousand equations with about 100 terms each, and their computer derivation takes 10 hours.

Table 5: Iterative construction of recursion equations

Record	Root elements		Sub-elements	Recursive equation
Initialization	g	$\mathbf{G}$	$\mathbf{G}_1 = \mathbf{G} - \mathbf{R}_1 \quad g_1$ $\mathbf{G}_2 = \mathbf{G} - \mathbf{R}_2 \quad g_2$ .... Classify by g_c $\Downarrow$ *List of sub-elements* $a_{g\,g_1}$ instances of g_1; e.g. $\mathbf{G}_1$ $a_{g\,g_2}$ instances of g_2; e.g. $\mathbf{G}_2$	$\overset{g}{x}_{m+1} = a_{g\,g_1} \overset{g_1}{x}_m + a_{g\,g_2} \overset{g_2}{x}_m + \dots$
1	g_1	$\mathbf{G}_1$	$\mathbf{G}_3 = \mathbf{G}_1 - \mathbf{R}_1 \quad g_3$ $\mathbf{G}_4 = \mathbf{G}_1 - \mathbf{R}_2 \quad g_4$ .... Classify by g_c $\Downarrow$ *List of sub-elements* $a_{g_1\,g_3}$ instances of g_3; e.g. $\mathbf{G}_3$ $a_{g_1\,g_4}$ instances of g_4; e.g. $\mathbf{G}_4$	$\overset{g_1}{x}_{m+1} = a_{g_1 g_3} \overset{g_3}{x}_m + a_{g_1 g_4} \overset{g_4}{x}_m + \dots$
2	g_2	$\mathbf{G}_2$	$\mathbf{G}_5 = \mathbf{G}_2 - \mathbf{R}_1 \quad g_5$ $\mathbf{G}_6 = \mathbf{G}_2 - \mathbf{R}_2 \quad g_6$ .... Classify by g_c $\Downarrow$ *List of sub-elements* $a_{g_2\,g_5}$ instances of g_5; e.g. $\mathbf{G}_5$ $a_{g_2\,g_6}$ instances of g_6; e.g. $\mathbf{G}_6$	$\overset{g_2}{x}_{m+1} = a_{g_2 g_5} \overset{g_5}{x}_m + a_{g_2 g_6} \overset{g_6}{x}_m + \dots$
....	....	....	....	....

Table 6: Description of recursive systems for counting perfect ($i = 0$) and i-imperfect outcomes of Bernoulli ($m \times n$)-matrices for column pairs ($k = 2$)

k n i	Matrix **A** of recursive system. Frames show the square sub-matrices related to the first key equations	Size of matrix **A** and vector $\mathbf{x}_m$	Average number of $\neq 0$ elements of **A** per row, also in %	Key equations	Number of different elements in vector $\mathbf{x}_m$ after successive iterations	Computer time to derive matrix **A**
2 2 48	$\begin{pmatrix} 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 1 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 0 & 1 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 0 & 0 & 1 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 0 & 0 & 0 & 1 & 3 & 0 & 0 & 0 & 0 & 0 & \dots \\ 0 & 0 & 0 & 0 & 1 & 3 & 0 & 0 & 0 & 0 & \dots \\ 0 & 0 & 0 & 0 & 0 & 1 & 3 & 0 & 0 & 0 & \dots \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 3 & 0 & 0 & \dots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 3 & 0 & \dots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 3 & \dots \\ \dots & & & & & & & & & & \ddots \end{pmatrix}$	49	2.0 4.04%	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 27 28 29 29 30 31 31 32 33 33 34 34 35 36 36 37 37 38 38 39 40 40 41 ⋮	0.01 s

k n i	Matrix $\mathbf{A}$ of recursive system. Frames show the square sub-matrices related to the first key equations	Size of matrix $\mathbf{A}$ and vector $\mathbf{x}_m$	Average number of $\neq 0$ elements of $\mathbf{A}$ per row, also in %	Key equations	Number of different elements in vector $\mathbf{x}_m$ after successive iterations	Computer time to derive matrix $\mathbf{A}$
2 3 48	$\begin{pmatrix} 6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 2 & 5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 0 & 3 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 2 & 0 & 0 & 6 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 1 & 1 & 0 & 1 & 5 & 0 & 0 & 0 & 0 & 0 & \dots \\ 1 & 0 & 1 & 0 & 2 & 4 & 0 & 0 & 0 & 0 & \dots \\ 0 & 1 & 0 & 0 & 2 & 0 & 5 & 0 & 0 & 0 & \dots \\ 0 & 1 & 0 & 0 & 0 & 2 & 1 & 4 & 0 & 0 & \dots \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 3 & 4 & 0 & \dots \\ 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 6 & \dots \\ \dots & & & & & & & & & & \ddots \end{pmatrix}$	22,099	4.8 0.02%	3 9 19 34 55 83 119 164 219 285 363 454 559 679 815 968 1,139 1,329 1,539 1,770 2,023 2,299 2,599 2,924 3,275 3,653 4,059 4,494 4,959 5,455 5,983 6,544 7,139 7,769 8,435 9,138 9,879 10,659 11,479 12,340 13,243 14,189 15,179 16,214 17,295 18,423 19,599 20,824 22,099	1 3 7 13 22 34 50 70 94 125 161 203 252 307 372 444 525 615 713 827 963 1,092 1,236 1,381 1,544 1,687 1,859 2,050 2,209 2,385 2,557 2,726 2,904 3,041 3,237 3,415 3,597 3,779 3,959 4,158 4,366 4,574 4,795 5,009 5,232 5,437 5,633 5,871 6,085 6,331 ⋮	13.07 s

k n i	Matrix **A** of recursive system. Frames show the square sub-matrices related to the first key equations	Size of matrix **A** and vector $\mathbf{x}_m$	Average number of $\neq 0$ elements of **A** per row, also in %	Key equations	Number of different elements in vector $\mathbf{x}_m$ after successive iterations	Computer time to derive matrix **A**
2 4 0	$\begin{pmatrix} 12 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 6 & 9 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 4 & 0 & 10 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 1 & 4 & 8 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & 4 & 7 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 6 & 0 & 0 & 8 & 0 & 0 & 0 & 0 \\ 3 & 0 & 3 & 0 & 0 & 0 & 9 & 0 & 0 & 0 \\ 1 & 0 & 2 & 2 & 0 & 1 & 2 & 7 & 0 & 0 \\ 0 & 0 & 2 & 0 & 1 & 0 & 2 & 4 & 6 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 & 6 & 5 \end{pmatrix}$	10	3.1 31.00%	10	1 3 10	0.01 s
2 4 1	$\begin{pmatrix} 12 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 6 & 9 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 4 & 0 & 10 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 2 & 1 & 4 & 8 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 0 & 0 & 4 & 4 & 7 & 0 & 0 & 0 & 0 & 0 & \dots \\ 0 & 0 & 6 & 0 & 0 & 8 & 0 & 0 & 0 & 0 & \dots \\ 3 & 0 & 3 & 0 & 0 & 0 & 9 & 0 & 0 & 0 & \dots \\ 1 & 0 & 2 & 2 & 0 & 1 & 2 & 7 & 0 & 0 & \dots \\ 0 & 0 & 2 & 0 & 1 & 0 & 2 & 4 & 6 & 0 & \dots \\ 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 & 6 & 5 & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \ddots \end{pmatrix}$	65	6.0 9.28%	10 65	1 4 17 41 65	0.03 s
2 4 2	$\begin{pmatrix} 12 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 6 & 9 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 4 & 0 & 10 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 2 & 1 & 4 & 8 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 0 & 0 & 4 & 4 & 7 & 0 & 0 & 0 & 0 & 0 & \dots \\ 0 & 0 & 6 & 0 & 0 & 8 & 0 & 0 & 0 & 0 & \dots \\ 3 & 0 & 3 & 0 & 0 & 0 & 9 & 0 & 0 & 0 & \dots \\ 1 & 0 & 2 & 2 & 0 & 1 & 2 & 7 & 0 & 0 & \dots \\ 0 & 0 & 2 & 0 & 1 & 0 & 2 & 4 & 6 & 0 & \dots \\ 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 & 6 & 5 & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \ddots \end{pmatrix}$	275	7.7 2.79%	10 65 275	1 4 18 53 133 217 275	0.11 s
2 4 3	$\begin{pmatrix} 12 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 6 & 9 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 4 & 0 & 10 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 2 & 1 & 4 & 8 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 0 & 0 & 4 & 4 & 7 & 0 & 0 & 0 & 0 & 0 & \dots \\ 0 & 0 & 6 & 0 & 0 & 8 & 0 & 0 & 0 & 0 & \dots \\ 3 & 0 & 3 & 0 & 0 & 0 & 9 & 0 & 0 & 0 & \dots \\ 1 & 0 & 2 & 2 & 0 & 1 & 2 & 7 & 0 & 0 & \dots \\ 0 & 0 & 2 & 0 & 1 & 0 & 2 & 4 & 6 & 0 & \dots \\ 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 & 6 & 5 & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \ddots \end{pmatrix}$	899	8.7 0.97%	10 65 275 899	1 4 18 54 154 332 583 784 899	0.39 s

k n i	Matrix **A** of recursive system. Frames show the square sub-matrices related to the first key equations	Size of matrix **A** and vector $\mathbf{x}_m$	Average number of $\neq 0$ elements of **A** per row, also in %	Key equations	Number of different elements in vector $\mathbf{x}_m$ after successive iterations	Computer time to derive matrix **A**
2 4 4	$\begin{pmatrix} 12 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 6 & 9 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 4 & 0 & 10 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 2 & 1 & 4 & 8 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 0 & 0 & 4 & 4 & 7 & 0 & 0 & 0 & 0 & 0 & \dots \\ 0 & 0 & 6 & 0 & 0 & 8 & 0 & 0 & 0 & 0 & \dots \\ 3 & 0 & 3 & 0 & 0 & 0 & 9 & 0 & 0 & 0 & \dots \\ 1 & 0 & 2 & 2 & 0 & 1 & 2 & 7 & 0 & 0 & \dots \\ 0 & 0 & 2 & 0 & 1 & 0 & 2 & 4 & 6 & 0 & \dots \\ 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 & 6 & 5 & \dots \\ \dots & & & & & & & & & & \ddots \end{pmatrix}$	2,450	9.4 0.39%	10 65 275 899 2,450	1 4 18 54 155 362 780 1,309 1,856 2,249 2,450	1.20 s
2 4 5	$\begin{pmatrix} 12 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 6 & 9 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 4 & 0 & 10 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 2 & 1 & 4 & 8 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 0 & 0 & 4 & 4 & 7 & 0 & 0 & 0 & 0 & 0 & \dots \\ 0 & 0 & 6 & 0 & 0 & 8 & 0 & 0 & 0 & 0 & \dots \\ 3 & 0 & 3 & 0 & 0 & 0 & 9 & 0 & 0 & 0 & \dots \\ 1 & 0 & 2 & 2 & 0 & 1 & 2 & 7 & 0 & 0 & \dots \\ 0 & 0 & 2 & 0 & 1 & 0 & 2 & 4 & 6 & 0 & \dots \\ 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 & 6 & 5 & \dots \\ \dots & & & & & & & & & & \ddots \end{pmatrix}$	5,830	9.9 0.17%	10 65 275 899 2,450 5,830	1 4 18 54 155 363 828 1,599 2,672 3,811 4,830 ⋮	3.43 s
2 4 6	$\begin{pmatrix} 12 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 6 & 9 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 4 & 0 & 10 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 2 & 1 & 4 & 8 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 0 & 0 & 4 & 4 & 7 & 0 & 0 & 0 & 0 & 0 & \dots \\ 0 & 0 & 6 & 0 & 0 & 8 & 0 & 0 & 0 & 0 & \dots \\ 3 & 0 & 3 & 0 & 0 & 0 & 9 & 0 & 0 & 0 & \dots \\ 1 & 0 & 2 & 2 & 0 & 1 & 2 & 7 & 0 & 0 & \dots \\ 0 & 0 & 2 & 0 & 1 & 0 & 2 & 4 & 6 & 0 & \dots \\ 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 & 6 & 5 & \dots \\ \dots & & & & & & & & & & \ddots \end{pmatrix}$	12,495	10.3 0.08%	10 65 275 899 2,450 5,830 12,495	1 4 18 54 155 363 829 1,664 3,085 4,994 7,171 ⋮	9.72 s
2 4 7	$\begin{pmatrix} 12 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 6 & 9 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 4 & 0 & 10 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 2 & 1 & 4 & 8 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 0 & 0 & 4 & 4 & 7 & 0 & 0 & 0 & 0 & 0 & \dots \\ 0 & 0 & 6 & 0 & 0 & 8 & 0 & 0 & 0 & 0 & \dots \\ 3 & 0 & 3 & 0 & 0 & 0 & 9 & 0 & 0 & 0 & \dots \\ 1 & 0 & 2 & 2 & 0 & 1 & 2 & 7 & 0 & 0 & \dots \\ 0 & 0 & 2 & 0 & 1 & 0 & 2 & 4 & 6 & 0 & \dots \\ 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 & 6 & 5 & \dots \\ \dots & & & & & & & & & & \ddots \end{pmatrix}$	24,650	10.6 0.04%	10 65 275 899 2,450 5,830 12,495 24,650	1 4 18 54 155 363 829 1,665 3,180 5,561 8,807 ⋮	28.39 s

k n i	Matrix **A** of recursive system. Frames show the square sub-matrices related to the first key equations	Size of matrix **A** and vector $\mathbf{x}_m$	Average number of $\neq 0$ elements of **A** per row, also in %	Key equations	Number of different elements in vector $\mathbf{x}_m$ after successive iterations	Computer time to derive matrix **A**
2 4 8	$\begin{pmatrix} 12 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 6 & 9 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 4 & 0 & 10 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 2 & 1 & 4 & 8 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 0 & 0 & 4 & 4 & 7 & 0 & 0 & 0 & 0 & 0 & \dots \\ 0 & 0 & 6 & 0 & 0 & 8 & 0 & 0 & 0 & 0 & \dots \\ 3 & 0 & 3 & 0 & 0 & 0 & 9 & 0 & 0 & 0 & \dots \\ 1 & 0 & 2 & 2 & 0 & 1 & 2 & 7 & 0 & 0 & \dots \\ 0 & 0 & 2 & 0 & 1 & 0 & 2 & 4 & 6 & 0 & \dots \\ 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 & 6 & 5 & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \ddots \end{pmatrix}$	45,474	10.8 0.02%	10 65 275 899 2,450 5,830 12,495 24,650 45,474	1 4 18 54 155 363 829 1,665 3,181 5,683 9,553 ⋮	1 m 17 s
2 4 10	$\begin{pmatrix} 12 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 6 & 9 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 4 & 0 & 10 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 2 & 1 & 4 & 8 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 0 & 0 & 4 & 4 & 7 & 0 & 0 & 0 & 0 & 0 & \dots \\ 0 & 0 & 6 & 0 & 0 & 8 & 0 & 0 & 0 & 0 & \dots \\ 3 & 0 & 3 & 0 & 0 & 0 & 9 & 0 & 0 & 0 & \dots \\ 1 & 0 & 2 & 2 & 0 & 1 & 2 & 7 & 0 & 0 & \dots \\ 0 & 0 & 2 & 0 & 1 & 0 & 2 & 4 & 6 & 0 & \dots \\ 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 & 6 & 5 & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \ddots \end{pmatrix}$	132,275	11.0 0.01%	10 65 275 899 2,450 5,830 12,495 24,650 45,474 79,375 132,275	1 4 18 54 155 363 829 1,665 3,181 5,684 9,715 15,878 ⋮	7 m 47 s
2 4 12	$\begin{pmatrix} 12 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 6 & 9 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 4 & 0 & 10 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 2 & 1 & 4 & 8 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 0 & 0 & 4 & 4 & 7 & 0 & 0 & 0 & 0 & 0 & \dots \\ 0 & 0 & 6 & 0 & 0 & 8 & 0 & 0 & 0 & 0 & \dots \\ 3 & 0 & 3 & 0 & 0 & 0 & 9 & 0 & 0 & 0 & \dots \\ 1 & 0 & 2 & 2 & 0 & 1 & 2 & 7 & 0 & 0 & \dots \\ 0 & 0 & 2 & 0 & 1 & 0 & 2 & 4 & 6 & 0 & \dots \\ 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 & 6 & 5 & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \ddots \end{pmatrix}$	328,250	11.2 0.00%	10 65 275 899 2,450 5,830 12,495 24,650 45,474 79,375 132,275 211,925 328,250	1 4 18 54 155 363 829 1,665 3,181 5,684 9,715 15,879 25,105 38,407 ⋮	46 m 30 s

k n i	Matrix **A** of recursive system. Frames show the square sub-matrices related to the first key equations	Size of matrix **A** and vector $\mathbf{x}_m$	Average number of $\neq 0$ elements of **A** per row, also in %	Key equations	Number of different elements in vector $\mathbf{x}_m$ after successive iterations	Computer time to derive matrix **A**
2 4 14	$\begin{pmatrix} 12 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 6 & 9 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 4 & 0 & 10 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 2 & 1 & 4 & 8 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 0 & 0 & 4 & 4 & 7 & 0 & 0 & 0 & 0 & 0 & \dots \\ 0 & 0 & 6 & 0 & 0 & 8 & 0 & 0 & 0 & 0 & \dots \\ 3 & 0 & 3 & 0 & 0 & 0 & 9 & 0 & 0 & 0 & \dots \\ 1 & 0 & 2 & 2 & 0 & 1 & 2 & 7 & 0 & 0 & \dots \\ 0 & 0 & 2 & 0 & 1 & 0 & 2 & 4 & 6 & 0 & \dots \\ 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 & 6 & 5 & \dots \\ \dots & & & & & & & & & & \ddots \end{pmatrix}$	723,775	11.4 0.00%	10 65 275 899 2,450 5,830 12,495 24,650 45,474 79,375 132,275 211,925 328,250 493,724 723,775	1 4 18 54 155 363 829 1,665 3,181 5,684 9,715 15,879 25,105 38,408 62,339 94,240 ⋮	4 h 15 m
2 4 16	$\begin{pmatrix} 12 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 6 & 9 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 4 & 0 & 10 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 2 & 1 & 4 & 8 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 0 & 0 & 4 & 4 & 7 & 0 & 0 & 0 & 0 & 0 & \dots \\ 0 & 0 & 6 & 0 & 0 & 8 & 0 & 0 & 0 & 0 & \dots \\ 3 & 0 & 3 & 0 & 0 & 0 & 9 & 0 & 0 & 0 & \dots \\ 1 & 0 & 2 & 2 & 0 & 1 & 2 & 7 & 0 & 0 & \dots \\ 0 & 0 & 2 & 0 & 1 & 0 & 2 & 4 & 6 & 0 & \dots \\ 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 & 6 & 5 & \dots \\ \dots & & & & & & & & & & \ddots \end{pmatrix}$	1,456,730	11.5 0.00%	10 65 275 899 2,450 5,830 12,495 24,650 45,474 79,375 132,275 211,925 328,250 493,724 723,775 1,037,220 1,456,730	1 4 18 54 155 363 829 1,665 3,181 5,684 9,715 15,879 25,105 38,408 62,339 94,240 136,028 190,462 ⋮	18 h 42 m
2 5 0	$\begin{pmatrix} 24 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 12 & 18 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 8 & 0 & 20 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 5 & 6 & 5 & 15 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 4 & 2 & 8 & 0 & 16 & 0 & 0 & 0 & 0 & 0 & \dots \\ 2 & 1 & 5 & 4 & 6 & 13 & 0 & 0 & 0 & 0 & \dots \\ 0 & 0 & 5 & 0 & 5 & 10 & 11 & 0 & 0 & 0 & \dots \\ 0 & 0 & 8 & 0 & 8 & 0 & 0 & 14 & 0 & 0 & \dots \\ 0 & 0 & 12 & 0 & 0 & 0 & 0 & 0 & 16 & 0 & \dots \\ 3 & 0 & 3 & 9 & 0 & 0 & 0 & 0 & 4 & 12 & \dots \\ \dots & & & & & & & & & & \ddots \end{pmatrix}$	33	6.5 19.83%	33	1 4 29 33	0.04 s

k n i	Matrix $\mathbf{A}$ of recursive system. Frames show the square sub-matrices related to the first key equations	Size of matrix $\mathbf{A}$ and vector $\mathbf{x}_m$	Average number of $\neq 0$ elements of $\mathbf{A}$ per row, also in %	Key equations	Number of different elements in vector $\mathbf{x}_m$ after successive iterations	Computer time to derive matrix $\mathbf{A}$
2 5 1	$\begin{pmatrix} 24 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 12 & 18 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 8 & 0 & 20 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 5 & 6 & 5 & 15 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 4 & 2 & 8 & 0 & 16 & 0 & 0 & 0 & 0 & 0 & \dots \\ 2 & 1 & 5 & 4 & 6 & 13 & 0 & 0 & 0 & 0 & \dots \\ 0 & 0 & 5 & 0 & 5 & 10 & 11 & 0 & 0 & 0 & \dots \\ 0 & 0 & 8 & 0 & 8 & 0 & 0 & 14 & 0 & 0 & \dots \\ 0 & 0 & 12 & 0 & 0 & 0 & 0 & 0 & 16 & 0 & \dots \\ 3 & 0 & 3 & 9 & 0 & 0 & 0 & 0 & 4 & 12 & \dots \\ \dots & & & & & & & & & & \ddots \end{pmatrix}$	791	15.5 1.96%	33 791	1 5 44 232 756 791	0.86 s
2 5 2	$\begin{pmatrix} 24 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 12 & 18 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 8 & 0 & 20 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 5 & 6 & 5 & 15 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 4 & 2 & 8 & 0 & 16 & 0 & 0 & 0 & 0 & 0 & \dots \\ 2 & 1 & 5 & 4 & 6 & 13 & 0 & 0 & 0 & 0 & \dots \\ 0 & 0 & 5 & 0 & 5 & 10 & 11 & 0 & 0 & 0 & \dots \\ 0 & 0 & 8 & 0 & 8 & 0 & 0 & 14 & 0 & 0 & \dots \\ 0 & 0 & 12 & 0 & 0 & 0 & 0 & 0 & 16 & 0 & \dots \\ 3 & 0 & 3 & 9 & 0 & 0 & 0 & 0 & 4 & 12 & \dots \\ \dots & & & & & & & & & & \ddots \end{pmatrix}$	10,687	20.1 0.19%	33 791 10, 687	1 5 45 264 1,624 5,517 10,624 10,685 10,687	20.66 s
2 5 3	$\begin{pmatrix} 24 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 12 & 18 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 8 & 0 & 20 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 5 & 6 & 5 & 15 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 4 & 2 & 8 & 0 & 16 & 0 & 0 & 0 & 0 & 0 & \dots \\ 2 & 1 & 5 & 4 & 6 & 13 & 0 & 0 & 0 & 0 & \dots \\ 0 & 0 & 5 & 0 & 5 & 10 & 11 & 0 & 0 & 0 & \dots \\ 0 & 0 & 8 & 0 & 8 & 0 & 0 & 14 & 0 & 0 & \dots \\ 0 & 0 & 12 & 0 & 0 & 0 & 0 & 0 & 16 & 0 & \dots \\ 3 & 0 & 3 & 9 & 0 & 0 & 0 & 0 & 4 & 12 & \dots \\ \dots & & & & & & & & & & \ddots \end{pmatrix}$	90,004	22.5 0.02%	33 791 10, 687 90, 004	1 5 45 265 1,719 7,727 27,854 61,462 89,872 90,003 90,004	13 m 47 s
2 5 4	$\begin{pmatrix} 24 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 12 & 18 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 8 & 0 & 20 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 5 & 6 & 5 & 15 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 4 & 2 & 8 & 0 & 16 & 0 & 0 & 0 & 0 & 0 & \dots \\ 2 & 1 & 5 & 4 & 6 & 13 & 0 & 0 & 0 & 0 & \dots \\ 0 & 0 & 5 & 0 & 5 & 10 & 11 & 0 & 0 & 0 & \dots \\ 0 & 0 & 8 & 0 & 8 & 0 & 0 & 14 & 0 & 0 & \dots \\ 0 & 0 & 12 & 0 & 0 & 0 & 0 & 0 & 16 & 0 & \dots \\ 3 & 0 & 3 & 9 & 0 & 0 & 0 & 0 & 4 & 12 & \dots \\ \dots & & & & & & & & & & \ddots \end{pmatrix}$	533,357	23.7 0.00%	33 791 10, 687 90, 004 533, 357	1 5 45 265 1,720 7,894 33,934 106,389 246,670 417,929 533,079 ⋮	8 h 32 m

k n i	Matrix **A** of recursive system. Frames show the square sub-matrices related to the first key equations	Size of matrix **A** and vector $\mathbf{x}_m$	Average number of $\neq 0$ elements of **A** per row, also in %	Key equations	Number of different elements in vector $\mathbf{x}_m$ after successive iterations	Computer time to derive matrix **A**
2 6 0	$\begin{pmatrix} 48 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 24 & 36 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 9 & 27 & 27 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 16 & 0 & 0 & 40 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 10 & 12 & 0 & 10 & 30 & 0 & 0 & 0 & 0 & 0 & \dots \\ 4 & 4 & 0 & 10 & 20 & 25 & 0 & 0 & 0 & 0 & \dots \\ 8 & 4 & 0 & 16 & 0 & 0 & 32 & 0 & 0 & 0 & \dots \\ 5 & 7 & 3 & 4 & 12 & 0 & 8 & 24 & 0 & 0 & \dots \\ 4 & 2 & 0 & 10 & 8 & 0 & 12 & 0 & 26 & 0 & \dots \\ 2 & 1 & 0 & 6 & 6 & 4 & 7 & 6 & 10 & 21 & \dots \\ \dots & & & & & & & & & & \ddots \end{pmatrix}$	155	14.8 9.55%	155	1 5 91 153 155	0.85 s
2 6 1	$\begin{pmatrix} 48 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 24 & 36 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 9 & 27 & 27 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 16 & 0 & 0 & 40 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 10 & 12 & 0 & 10 & 30 & 0 & 0 & 0 & 0 & 0 & \dots \\ 4 & 4 & 0 & 10 & 20 & 25 & 0 & 0 & 0 & 0 & \dots \\ 8 & 4 & 0 & 16 & 0 & 0 & 32 & 0 & 0 & 0 & \dots \\ 5 & 7 & 3 & 4 & 12 & 0 & 8 & 24 & 0 & 0 & \dots \\ 4 & 2 & 0 & 10 & 8 & 0 & 12 & 0 & 26 & 0 & \dots \\ 2 & 1 & 0 & 6 & 6 & 4 & 7 & 6 & 10 & 21 & \dots \\ \dots & & & & & & & & & & \ddots \end{pmatrix}$	25,505	39.1 0.15%	155 25, 505	1 6 125 1,796 21,191 25,296 25,499 25,505	10 m 54 s
2 7 0	$\begin{pmatrix} 96 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 48 & 72 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 18 & 54 & 54 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 32 & 0 & 0 & 80 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 20 & 24 & 0 & 20 & 60 & 0 & 0 & 0 & 0 & 0 & \dots \\ 8 & 21 & 18 & 5 & 30 & 45 & 0 & 0 & 0 & 0 & \dots \\ 8 & 8 & 0 & 20 & 40 & 0 & 50 & 0 & 0 & 0 & \dots \\ 16 & 8 & 0 & 32 & 0 & 0 & 0 & 64 & 0 & 0 & \dots \\ 10 & 14 & 6 & 8 & 24 & 0 & 0 & 16 & 48 & 0 & \dots \\ 4 & 5 & 2 & 9 & 18 & 5 & 20 & 8 & 16 & 40 & \dots \\ \dots & & & & & & & & & & \ddots \end{pmatrix}$	1,043	37.7 3.61%	1, 043	1 6 276 1,012 1,043	1 m 34 s
2 8 0	$\begin{pmatrix} 192 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 96 & 144 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 36 & 108 & 108 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 12 & 54 & 108 & 81 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 64 & 0 & 0 & 0 & 160 & 0 & 0 & 0 & 0 & 0 & \dots \\ 40 & 48 & 0 & 0 & 40 & 120 & 0 & 0 & 0 & 0 & \dots \\ 16 & 42 & 36 & 0 & 10 & 60 & 90 & 0 & 0 & 0 & \dots \\ 16 & 16 & 0 & 0 & 40 & 80 & 0 & 100 & 0 & 0 & \dots \\ 7 & 16 & 12 & 0 & 10 & 50 & 60 & 25 & 75 & 0 & \dots \\ 32 & 16 & 0 & 0 & 64 & 0 & 0 & 0 & 0 & 128 & \dots \\ \dots & & & & & & & & & & \ddots \end{pmatrix}$	12,345	98.5 0.80%	12, 345	1 7 791 11,381 12,345	10 h 28 m

Table 7: Probabilities of perfect ($i = 0$) and i-imperfect column pairs ($k = 2$) in Bernoulli ($m \times n$)-matrices computed by the Geometric method (with recursive systems)

k m i	Width n of Bernoulli ($m \times n$)-matrix						
	2	3	4	5	6	7	8
2 1 0	.75000000	.87500000	.93750000	.96875000	.98437500	.99218750	.99609375
2 2 0	.56250000	.76562500	.87890625	.93847656	.96899414	.98443604	.99220276
1	.93750000	.98437500	.99609375	.99902344	.99975586		
2 3 0	.42187500	.65820313	.80932617	.89724731	.94584274	.97187853	.98554820
1	.84375000	.95703125	.98876953	.99713135	.99927521		
2	.98437500	.99804688	.99975586	.99996948			
2 4 0	.31640625	.55395508	.72706604	.83994007	.90894467	.94937097	.97234511
1	.73828125	.91528320	.97444153	.99255085	.99786454		
2	.94921875	.99291992	.99906921	.99988079			
3	.99609375	.99975586	.99998474	.99999905			
2 5 0	.23730469	.45706177	.63682079	.76744083	.85591626	.91303641	.94862327
1	.63281250	.85913086	.94991684	.98295021	.99435489		
2	.89648438	.98211670	.99709511	.99953550			
3	.98437500	.99890137	.99992752	.99999535			
4	.99902344	.99996948	.99999905	.99999997			
2 6 0	.17797852	.37074661	.54485947	.68415703	.78781240	.86117292	.91116768
1	.53393555	.79124832	.91349691	.96588820	.98702473		
2	.83056641	.96331024	.99255145	.99853246			
3	.96240234	.99657440	.99970043	.99997344			
4	.99536133	.99983597	.99999458	.99999983			
5	.99975586	.99999619	.99999994				
2 7 0	.13348389	.29650545	.45663340	.59582070	.70829581	.79469930	.85858020
1	.44494629	.71559715	.86513076	.93932233	.97377519		
2	.75640869	.93506479	.98387891	.99617276			
3	.92944336	.99155807	.99903189	.99989043			
4	.98712158	.99938536	.99997105	.99999857			
5	.99865723	.99997616	.99999961				
6	.99993896	.99999952	1.00000000				
2 8 0	.10011292	.23439580	.37592543	.50792317	.62241199	.71685073	.79185026
1	.36708069	.63633424	.80619312	.90205796	.95250528		
2	.67854309	.89700443	.96945792	.99141809			
3	.88618469	.98249370	.99745115	.99964252			
4	.97270203	.99821919	.99988566	.99999264			
5	.99577332	.99989480	.99999733				
6	.99961853	.99999660	.99999997				
7	.99998474	.99999994	1.00000000				
2 9 0	.07508469	.18355144	.30481599	.42482712	.53536575	.63227361	.71413493
1	.30033875	.55717021	.73908883	.85402114	.92157455		
2	.60067749	.84973699	.94788072	.98294969			
3	.83427429	.96809000	.99425538	.99901746			
4	.95107269	.99574344	.99964228	.99997082			
5	.99000549	.99964780	.99998744				
6	.99865723	.99998263	.99999976				
7	.99989319	.99999952	1.00000000				
8	.99999619	.99999999	1.00000000				
2 10 0	.05631351	.14263227	.24400537	.34944906	.45161901	.54594319	.62993565
1	.24402523	.48106438	.66678645	.79628643	.88020177		
2	.52559280	.79465447	.91820709	.96932543			
3	.77587509	.94731776	.98853032	.99764544			
4	.92187309	.99119191	.99905727	.99990368			
5	.98027229	.99904287	.99995415				
6	.99649429	.99993373	.99999870				
7	.99958420	.99999721	.99999998				
8	.99997044	.99999993	1.00000000				
9	.99999905	1.00000000	1.00000000				

2.5 Column triplets and Sierpinski pyramids

Let us show that index triplets of perfect column triplets constitute the *Sierpinski pyramid*; see Sierpinski triangle (2012), and index triplets of imperfect column triplets constitute the generalized Serpinski pyramid defined below. Similarly to the Sierpinski triangle, the Sierpinski pyramid can be constructed in several ways, in particular, recursively. The construction is initialized with a single element, the front vertex in the bottom plot of Figure 5. At each step, the current configuration is quadruplicated by adding its three instances along the three coordinate axes.

Generalize the notion of Sierpinski pyramid to *Sierpinski pyramid of type i*. Define the Sierpinski pyramid of type 0 to be the empty set. For $i > 0$ the construction is initialized with the front vertex in the bottom plot of Figure 6. At each step the current configuration is quadruplicated by adding its three instances along the three coordinate axes. The other four cubes adjacent to the cube with the existing configuration are filled with the Sierpinski pyramids of type $i-1$. The original Sierpinski pyramid is of type 1.

Proposition 3 (Imperfect column triplets as generalized Sierpinski pyramids) *The index triplets of i-imperfect column triplets constitute the Sierpinski pyramid of type $i+1$.*

Proof. In the notation of Proposition 2, (011), (101), (110) and (111) sub-cubes correspond to triplets of m-columns with no two simultaneous 0s at the bottom. Hence, these column pairs are i-imperfect if and only if their upper $(m-1)$-segments make i-imperfect triplets, i.e. these four sub-cubes have the same structure of i-imperfect outcomes. The other four sub-cubes correspond to triplets of m-columns with at least two simultaneous 0s at the bottom. Consequently, these column triplets are i-imperfect if and only if their upper $(m-1)$-segments make $(i-1)$-imperfect triplets. This is exactly the way the Sierpinski pyramid of type $i+1$ is constructed. ■

2.6 Perfect and imperfect column triplets in the general case

First consider Bernoulli matrices with three columns. By definition of generalized Sierpinski pyramid and Proposition 3, the transition from m-columns to $(m+1)$-columns results in quadruplicating the number of i-imperfect outcomes $\overset{i+1}{x}_m$ (labeled by the type $i+1$ of the corresponding Sierpinski pyramid) and adding the number of $(i-1)$-imperfect outcomes denoted by $\overset{i}{x}_m$, also in quadruplicate:

$$
\begin{array}{lcll}
\overset{1}{x}_{m+1} & = & 4\,\overset{1}{x}_m & \text{(for Sierpinski pyramid of type 1)} \\
\overset{2}{x}_{m+1} & = & 4\,\overset{1}{x}_m + 4\,\overset{2}{x}_m & \text{(for Sierpinski pyramid of type 2)} \\
\overset{3}{x}_{m+1} & = & 4\,\overset{2}{x}_m + 4\,\overset{3}{x}_m & \text{(for Sierpinski pyramid of type 3)} \\
\multicolumn{4}{c}{\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots}
\end{array}
$$

initialized with $\overset{1}{x}_0 = \ldots = \overset{i+1}{x}_0 = 1$. In vector-matrix notation we have

$$\mathbf{x}_{m+1} = \mathbf{A}\mathbf{x}_m = \mathbf{A}^{m+1}\mathbf{1} \ ,$$

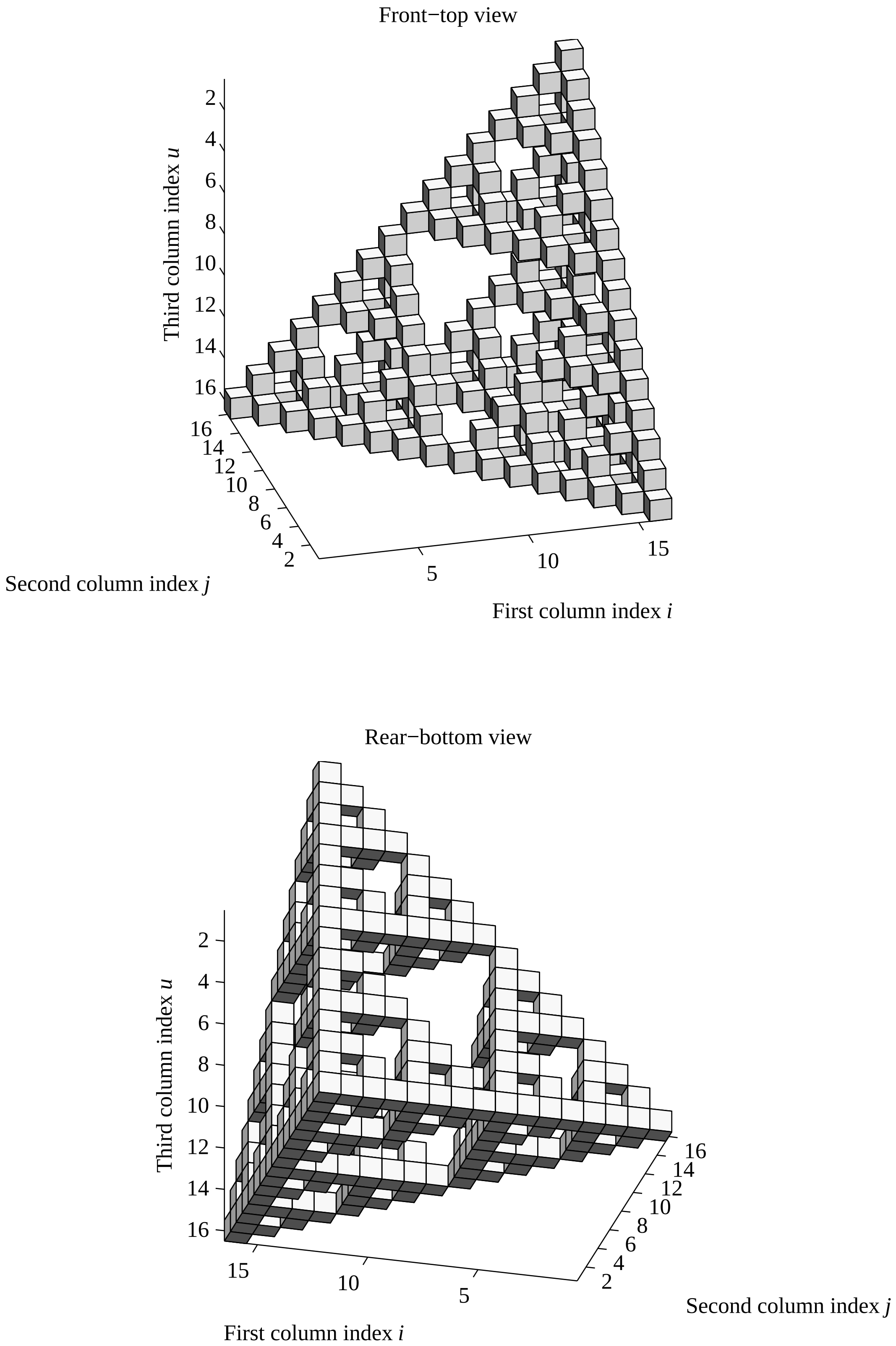


Figure 5: Perfect triplets of 4-columns (Sierpinski pyramid)

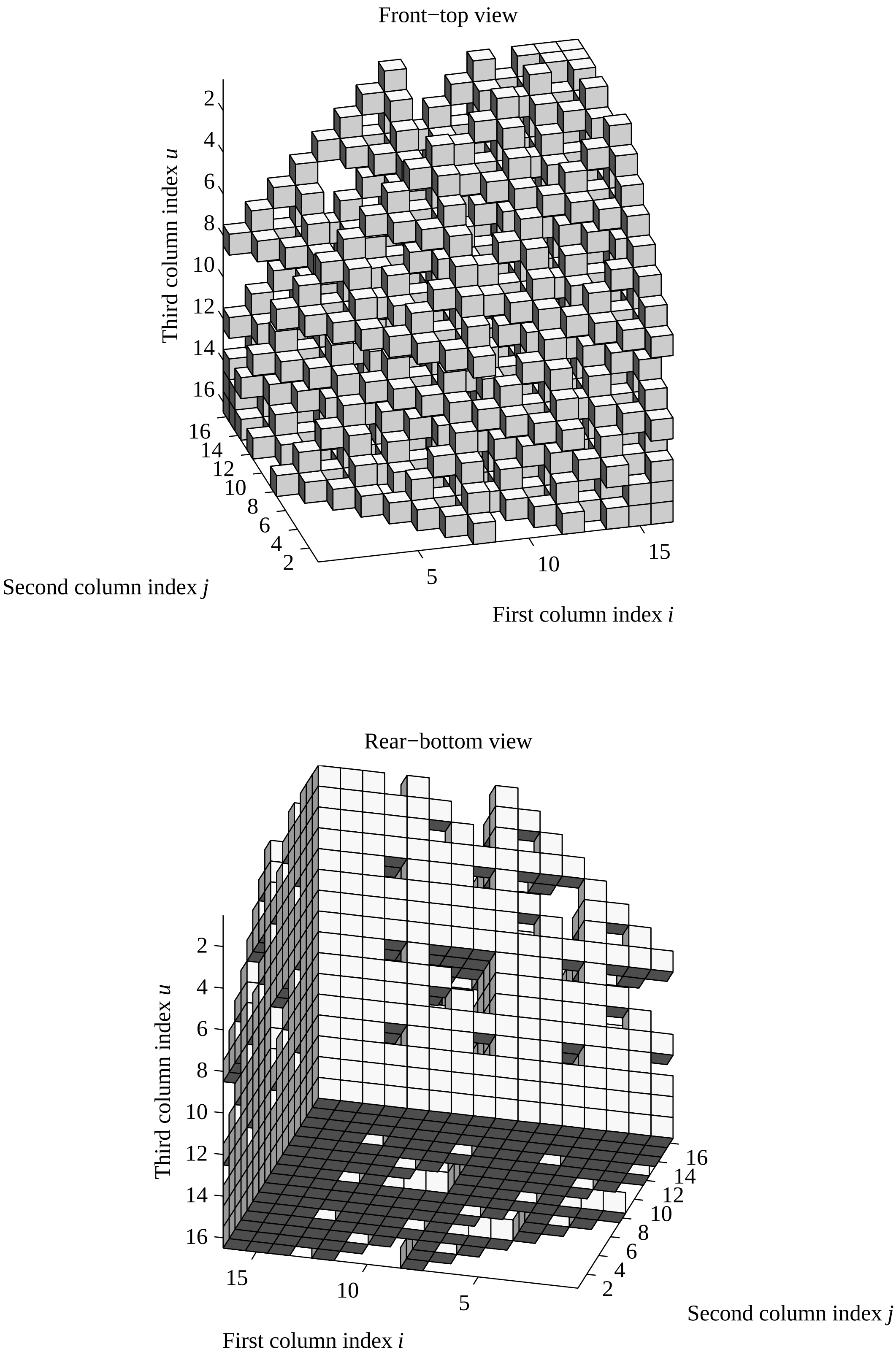


Figure 6: 1-imperfect triplets of 4-columns (Sierpinski pyramid of type 2)

where

$$\mathbf{x}_0 = \mathbf{1} = \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix}, \quad \mathbf{x}_m = \begin{pmatrix} \overset{1}{x}_m \\ \vdots \\ \overset{i+1}{x}_m \end{pmatrix}, \quad \text{and} \quad \mathbf{A} = \begin{pmatrix} 4 & 0 & 0 & \dots & 0 \\ 4 & 4 & 0 & \ddots & \vdots \\ 0 & 4 & 4 & \ddots & 0 \\ \vdots & & \ddots & \ddots & 0 \\ 0 & \dots & 0 & 4 & 4 \end{pmatrix}.$$

Since the cube of outcomes of Bernoulli $(m \times 3)$-matrix has $(2^m)^3 = 8^m$ elements,

$$\mathsf{P}(\text{Outcome of Bernoulli } (m \times 3)\text{-matrix is a perfect column triplet}) = \frac{\overset{1}{x}_m}{8^m} = \frac{4^m}{8^m}$$

$$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$$

$$\mathsf{P}(\text{Outcome of Bernoulli } (m \times 3)\text{-matrix is a } i\text{-imperfect column triplet}) = \frac{\overset{i+1}{x}_m}{8^m}.$$

Now extend the approach from Sections 2.3 and 2.4 to column triplets. A *perfect outcome* is a n-tuple of indices with three indices making a perfect triplet, that is, belonging to the Sierpinski pyramid. Perfect outcomes constitute $(n-3)$-dimensional hyperplanes stemming from elements of Serpinski pyramids in all the cube's *3D-faces* (bordered by three coordinate axes) — the union of $\binom{n}{3}$ intersecting Sierpinski 'hypercylinders'. For i-imperfect outcomes, the 'cylinder bases' are obviously Sierpinski pyramids of type $i+1$.

We say that a 3D-face of n-dimensional is of type j if its outcomes of interest constitute the Sierpinski pyramid of type j. The number of outcomes of interest in every n-dimensional cube depends on j-types of its 3D-faces (as cylinder bases) and their reciprocal position. All of these are characterized by multigraph *3D-adjacency* $(n \times n \times n)$*-matrix*

$$\mathbf{G} = \{e_{xyz}\}, \quad e_{xyz} = \begin{cases} j & \text{if the } xyz \text{ 3D-face of the cube is of type } j \\ 0 & \text{otherwise} \end{cases}.$$

The 3D-adjacency matrix is 'multi-symmetric', being completely characterized by the vector obtained from concatenating the columns in the layers of its bottom-left simplex. For a certain permutation of the graph vertices, this vector with $\frac{n(n-1)(n-2)}{6}$ elements attains its lexicographic maximum g which is called the *graph invariant.* The n-dimensional cubes with equal graph invariants have the same configuration of 'cylinder bases' and the same number of (im)perfect outcomes.

Similarly to Section 2.3, perfect and imperfect outcomes are counted with the help of recursive equations. The latter are derived by decomposing n-dimensional cubes of outcomes into 2^n sub-cubes with different 0–1 combinations of half-axes. The cubes are classified and labeled with graph invariants of their 3D adjacency matrices which are obtained by subtracting 3D reduction matrices $\mathbf{R}_c$ from the 3D matrix $\mathbf{G}$ of the large cube. The reduction matrices are of the same size as $\mathbf{G}$ and have zero elements, except for $e_{xyz} = 1$ if at least two out of x-half axis, y-half-axis, and z-half-axis are 0-half-axes (the corresponding triplets of m-columns have at least two simultaneous 0s at the bottom, implying that their upper $(m-1)$-segments have a reduced degree of imperfectness).

The algorithmic procedure for deriving recursive equations described in Section 2.4 and illustrated by Table 5 remains valid with the only difference that the adjacency and reduction matrices have dimensionality 3.

This procedure is implemented in a computer program. The recursive systems derived are described in Table 8, and the probabilities computed with these recursive systems are collected in Table 9. Some more probabilities are given in the Appendix.

Again, comparing the records in the penultimate column of Table 8, which are not interrupted by omission points, with the size of matrix $\mathbf{A}$ show that all elements of vector $\mathbf{x}_{m+1} = \mathbf{A}\mathbf{x}_m$ are different after a few successive m-iterations. That is, the sub-cubes with different multigraph invariants have different number of outcomes of interest. It disproves a possible impression that the method is overcomplicated.

As n and i increase, the computational complexity grows here even more rapidly than in case of column pairs. For instance, the recursive system for $n = 5$ and $i = 8$ has about 1.5 million equations with about 21 terms each, and their computer derivation takes four days. The recursive system for $n = 7$ and $i = 1$ has almost 200 thousand equations with about 75 terms each, and their computer derivation takes two days.

2.7 Summary

After the recursive system of equations has been derived, the probabilities of perfect and i-imperfect column pairs and triplets can be almost immediately computed for Bernoulli matrices with a quite large number m of rows. Therefore, applications of the Geometric method are not constrained by the height of Bernoulli matrix.

At the same time, the Geometric method is restricted to Bernoulli matrices with a rather small number n of columns. The computer performance is getting slower as new records, which back up the recursive equations, are appended to a large array (see Section 2.4). This operation requires overwriting the whole of the array, taking a noticeable time.

Besides, as the width n of Bernoulli matrix and the degree of imperfectness i increase, the representation of graph invariants as $(i+2)$-cimal numbers of length $\binom{n}{2}$ for column pairs and of length $\binom{n}{3}$ for column triplets requires larger inegers. Then the restricted accuracy of computer 64-bit arithmetics imply inaccuracies in graph invariants, resulting in errors in the system matrix $\mathbf{A}$ and, consequently, in output probabilities. The inaccuracy is surmounted if the graph invariants are represented, instead of integers, by vectors or concatenated integers, but it considerably slows the computer performance down.

Table 8: Description of recursive systems for counting perfect ($i = 0$) and i-imperfect outcomes of Bernoulli ($m \times n$)-matrices for column triplets ($k = 3$)

k n i	Matrix **A** of recursive system. Frames show the square sub-matrices related to the first key equations	Size of matrix **A** and vector $\mathbf{x}_m$	Average number of $\neq 0$ elements of **A** per row, also in %	Key equations	Number of different elements in vector $\mathbf{x}_m$ after successive iterations	Computer time to derive matrix **A**
				1	1	
				2	2	
				3	3	
				4	4	
				5	5	
				6	6	
				7	7	
				8	8	
				9	9	
				10	10	
				11	11	
				12	12	
				13	13	
				14	14	
				15	15	
				16	16	
				17	17	
				18	18	
				19	19	
				20	20	
				21	21	
				22	22	
				23	23	
				24	24	
3 3 48	$\begin{pmatrix} 4&0&0&0&0&0&0&0&0&0&\dots \\ 4&4&0&0&0&0&0&0&0&0&\dots \\ 0&4&4&0&0&0&0&0&0&0&\dots \\ 0&0&4&4&0&0&0&0&0&0&\dots \\ 0&0&0&4&4&0&0&0&0&0&\dots \\ 0&0&0&0&4&4&0&0&0&0&\dots \\ 0&0&0&0&0&4&4&0&0&0&\dots \\ 0&0&0&0&0&0&4&4&0&0&\dots \\ 0&0&0&0&0&0&0&4&4&0&\dots \\ 0&0&0&0&0&0&0&0&4&4&\dots \\ \dots&&&&&&&&&&\ddots \end{pmatrix}$	49	2.0 4.04%	25	25	0.01 s
				26	26	
				27	27	
				28	28	
				29	29	
				30	30	
				31	31	
				32	32	
				33	33	
				34	34	
				35	35	
				36	36	
				37	37	
				38	38	
				39	39	
				40	40	
				41	41	
				42	42	
				43	43	
				44	44	
				45	45	
				46	46	
				47	47	
				48	48	
				49	49	

k n i	Matrix **A** of recursive system. Frames show the square sub-matrices related to the first key equations	Size of matrix **A** and vector $\mathbf{x}_m$	Average number of $\neq 0$ elements of **A** per row, also in %	Key equations	Number of different elements in vector $\mathbf{x}_m$ after successive iterations	Computer time to derive matrix **A**
3 4 48	$\begin{pmatrix} 8 & 0 & 0 & 0 & \vert & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 4 & 6 & 0 & 0 & \vert & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 3 & 3 & 5 & 0 & \vert & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 0 & 6 & 0 & 5 & \vert & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ \hline 8 & 0 & 0 & 0 & & 8 & 0 & 0 & 0 & 0 & 0 & \dots \\ 6 & 2 & 0 & 0 & & 2 & 6 & 0 & 0 & 0 & 0 & \dots \\ 5 & 2 & 1 & 0 & & 1 & 2 & 5 & 0 & 0 & 0 & \dots \\ 0 & 6 & 0 & 0 & & 0 & 4 & 0 & 6 & 0 & 0 & \dots \\ 0 & 5 & 1 & 0 & & 0 & 2 & 2 & 1 & 5 & 0 & \dots \\ 0 & 5 & 0 & 1 & & 0 & 0 & 4 & 1 & 0 & 5 & \dots \\ \dots & & & & & & & & & & & \ddots \end{pmatrix}$	131,335	7.5 0.01%	 4 12 28 53 93 149 229 334 474 650 874 1,147 1,483 1,883 2,363 2,924 3,584 4,344 5,224 6,225 7,369 8,657 10,113 11,738 13,558 15,574 17,814 20,279 22,999 25,975 29,239 32,792 36,667 40,859 45,389 50,240 55,420 60,900 66,675 72,704 78,969 85,417 92,017 98,704 105,434 112,130 118,735 125,160 131,335	1 4 11 24 45 78 125 191 279 395 542 728 956 1,235 1,568 1,965 2,426 2,962 3,545 4,226 4,937 5,749 6,600 7,532 8,509 9,604 10,755 11,922 13,221 14,532 15,957 17,397 18,929 20,515 22,143 23,858 25,619 27,414 29,276 31,134 33,088 35,077 37,062 39,096 41,203 43,316 45,487 47,665 50,039 52,300 ⋮	4 m 31 s

k n i	Matrix **A** of recursive system. Frames show the square sub-matrices related to the first key equations	Size of matrix **A** and vector $\mathbf{x}_m$	Average number of $\neq 0$ elements of **A** per row, also in %	Key equations	Number of different elements in vector $\mathbf{x}_m$ after successive iterations	Computer time to derive matrix **A**
3 5 0	$\begin{pmatrix} 16 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 12 & 10 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 8 & 0 & 12 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 7 & 1 & 6 & 9 & 0 & 0 & 0 & 0 & 0 & 0 \\ 4 & 0 & 8 & 4 & 8 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 12 & 0 & 0 & 10 & 0 & 0 & 0 & 0 \\ 6 & 0 & 6 & 0 & 0 & 0 & 10 & 0 & 0 & 0 \\ 2 & 0 & 6 & 4 & 0 & 2 & 2 & 8 & 0 & 0 \\ 0 & 0 & 6 & 0 & 3 & 0 & 3 & 6 & 7 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 10 & 0 & 10 & 6 \end{pmatrix}$	10	3.2 32.00%	10	1 7 10	0.03 s
3 5 1	$\begin{pmatrix} 16 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 12 & 10 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 8 & 0 & 12 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 8 & 4 & 4 & 8 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 7 & 1 & 6 & 0 & 9 & 0 & 0 & 0 & 0 & 0 & \dots \\ 6 & 1 & 5 & 2 & 4 & 7 & 0 & 0 & 0 & 0 & \dots \\ 5 & 0 & 5 & 0 & 5 & 5 & 6 & 0 & 0 & 0 & \dots \\ 4 & 0 & 8 & 0 & 4 & 0 & 0 & 8 & 0 & 0 & \dots \\ 6 & 0 & 6 & 0 & 0 & 0 & 0 & 0 & 10 & 0 & \dots \\ 4 & 2 & 6 & 0 & 2 & 0 & 0 & 0 & 2 & 8 & \dots \\ \dots & & & & & & & & & & \ddots \end{pmatrix}$	96	8.5 8.87%	24 96	1 8 74 92 93 93 93 93 93 93 93 ⋮	0.10 s
3 5 2	$\begin{pmatrix} 16 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 12 & 10 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 8 & 0 & 12 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 8 & 4 & 4 & 8 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 7 & 1 & 6 & 0 & 9 & 0 & 0 & 0 & 0 & 0 & \dots \\ 6 & 1 & 5 & 2 & 4 & 7 & 0 & 0 & 0 & 0 & \dots \\ 5 & 0 & 5 & 0 & 5 & 5 & 6 & 0 & 0 & 0 & \dots \\ 4 & 0 & 8 & 0 & 4 & 0 & 0 & 8 & 0 & 0 & \dots \\ 6 & 0 & 6 & 0 & 0 & 0 & 0 & 0 & 10 & 0 & \dots \\ 4 & 2 & 6 & 0 & 2 & 0 & 0 & 0 & 2 & 8 & \dots \\ \dots & & & & & & & & & & \ddots \end{pmatrix}$	686	13.3 1.94%	33 276 686	1 8 111 561 662 666 667 667 667 667 667 ⋮	0.95 s
3 5 3	$\begin{pmatrix} 16 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 12 & 10 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 8 & 0 & 12 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 8 & 4 & 4 & 8 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 7 & 1 & 6 & 0 & 9 & 0 & 0 & 0 & 0 & 0 & \dots \\ 6 & 1 & 5 & 2 & 4 & 7 & 0 & 0 & 0 & 0 & \dots \\ 5 & 0 & 5 & 0 & 5 & 5 & 6 & 0 & 0 & 0 & \dots \\ 4 & 0 & 8 & 0 & 4 & 0 & 0 & 8 & 0 & 0 & \dots \\ 6 & 0 & 6 & 0 & 0 & 0 & 0 & 0 & 10 & 0 & \dots \\ 4 & 2 & 6 & 0 & 2 & 0 & 0 & 0 & 2 & 8 & \dots \\ \dots & & & & & & & & & & \ddots \end{pmatrix}$	3,787	16.6 0.44%	33 448 2, 045 3, 787	1 8 111 1,006 3,393 3,680 3,700 3,704 3,705 3,705 3,705 ⋮	4.55 s

k n i	Matrix **A** of recursive system. Frames show the square sub-matrices related to the first key equations	Size of matrix **A** and vector $\mathbf{x}_m$	Average number of $\neq 0$ elements of **A** per row, also in %	Key equations	Number of different elements in vector $\mathbf{x}_m$ after successive iterations	Computer time to derive matrix **A**
3 5 4	$\begin{pmatrix} 16 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 12 & 10 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 8 & 0 & 12 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 8 & 4 & 4 & 8 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 7 & 1 & 6 & 0 & 9 & 0 & 0 & 0 & 0 & 0 & \dots \\ 6 & 1 & 5 & 2 & 4 & 7 & 0 & 0 & 0 & 0 & \dots \\ 5 & 0 & 5 & 0 & 5 & 5 & 6 & 0 & 0 & 0 & \dots \\ 4 & 0 & 8 & 0 & 4 & 0 & 0 & 8 & 0 & 0 & \dots \\ 6 & 0 & 6 & 0 & 0 & 0 & 0 & 0 & 10 & 0 & \dots \\ 4 & 2 & 6 & 0 & 2 & 0 & 0 & 0 & 2 & 8 & \dots \\ \dots & & & & & & & & & & \ddots \end{pmatrix}$	16,859	18.5 0.11%	33 473 3, 699 10, 714 16, 859	1 8 111 1,060 6,919 15,663 16,512 16,572 16,588 16,592 16,593 ⋮	55.66 s
3 5 5	$\begin{pmatrix} 16 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 12 & 10 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 8 & 0 & 12 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 8 & 4 & 4 & 8 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 7 & 1 & 6 & 0 & 9 & 0 & 0 & 0 & 0 & 0 & \dots \\ 6 & 1 & 5 & 2 & 4 & 7 & 0 & 0 & 0 & 0 & \dots \\ 5 & 0 & 5 & 0 & 5 & 5 & 6 & 0 & 0 & 0 & \dots \\ 4 & 0 & 8 & 0 & 4 & 0 & 0 & 8 & 0 & 0 & \dots \\ 6 & 0 & 6 & 0 & 0 & 0 & 0 & 0 & 10 & 0 & \dots \\ 4 & 2 & 6 & 0 & 2 & 0 & 0 & 0 & 2 & 8 & \dots \\ \dots & & & & & & & & & & \ddots \end{pmatrix}$	62,317	19.7 0.03%	33 473 4, 536 19, 796 44, 113 62, 317	1 8 111 1,060 8,177 31,299 59,579 61,359 61,528 61,577 61,593 ⋮	7 m 9 s
3 5 6	$\begin{pmatrix} 16 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 12 & 10 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 8 & 0 & 12 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 8 & 4 & 4 & 8 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 7 & 1 & 6 & 0 & 9 & 0 & 0 & 0 & 0 & 0 & \dots \\ 6 & 1 & 5 & 2 & 4 & 7 & 0 & 0 & 0 & 0 & \dots \\ 5 & 0 & 5 & 0 & 5 & 5 & 6 & 0 & 0 & 0 & \dots \\ 4 & 0 & 8 & 0 & 4 & 0 & 0 & 8 & 0 & 0 & \dots \\ 6 & 0 & 6 & 0 & 0 & 0 & 0 & 0 & 10 & 0 & \dots \\ 4 & 2 & 6 & 0 & 2 & 0 & 0 & 0 & 2 & 8 & \dots \\ \dots & & & & & & & & & & \ddots \end{pmatrix}$	199,078	20.4 0.01%	33 473 4, 660 26, 447 81, 390 151, 040 199, 078	1 8 111 1,060 8,205 39,768 116,638 192,965 196,798 197,193 197,329 ⋮	1 h 19 m
3 5 7	$\begin{pmatrix} 16 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 12 & 10 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 8 & 0 & 12 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 8 & 4 & 4 & 8 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 7 & 1 & 6 & 0 & 9 & 0 & 0 & 0 & 0 & 0 & \dots \\ 6 & 1 & 5 & 2 & 4 & 7 & 0 & 0 & 0 & 0 & \dots \\ 5 & 0 & 5 & 0 & 5 & 5 & 6 & 0 & 0 & 0 & \dots \\ 4 & 0 & 8 & 0 & 4 & 0 & 0 & 8 & 0 & 0 & \dots \\ 6 & 0 & 6 & 0 & 0 & 0 & 0 & 0 & 10 & 0 & \dots \\ 4 & 2 & 6 & 0 & 2 & 0 & 0 & 0 & 2 & 8 & \dots \\ \dots & & & & & & & & & & \ddots \end{pmatrix}$	563,957	20.8 0.00%	33 473 4, 660 28, 902 114, 478 274, 040 449, 918 563, 957	1 8 111 1,060 8,205 40,554 157,720 361,821 551,523 559,001 559,874 ⋮	10 h 35 m

k n i	Matrix **A** of recursive system. Frames show the square sub-matrices related to the first key equations	Size of matrix **A** and vector $\mathbf{x}_m$	Average number of $\neq 0$ elements of **A** per row, also in %	Key equations	Number of different elements in vector $\mathbf{x}_m$ after successive iterations	Computer time to derive matrix **A**
3 5 8	$\begin{pmatrix} 16 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 12 & 10 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 8 & 0 & 12 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 8 & 4 & 4 & 8 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 7 & 1 & 6 & 0 & 9 & 0 & 0 & 0 & 0 & 0 & \dots \\ 6 & 1 & 5 & 2 & 4 & 7 & 0 & 0 & 0 & 0 & \dots \\ 5 & 0 & 5 & 0 & 5 & 5 & 6 & 0 & 0 & 0 & \dots \\ 4 & 0 & 8 & 0 & 4 & 0 & 0 & 8 & 0 & 0 & \dots \\ 6 & 0 & 6 & 0 & 0 & 0 & 0 & 0 & 10 & 0 & \dots \\ 4 & 2 & 6 & 0 & 2 & 0 & 0 & 0 & 2 & 8 & \dots \\ \dots & & & & & & & & & & \ddots \end{pmatrix}$	1,449,518	21.1 0.00%	33 473 4, 660 29, 148 133, 491 395, 660 799, 192 1, 198, 482 1, 449, 518	1 8 111 1,060 8,205 40,554 166,385 501,159 993,432 1,425,673 1,439,651 $\vdots$	4 d 1 h
3 6 0	$\begin{pmatrix} 32 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 32 & 16 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 24 & 0 & 20 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 16 & 0 & 0 & 24 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 18 & 2 & 6 & 10 & 14 & 0 & 0 & 0 & 0 & 0 & \dots \\ 8 & 0 & 8 & 16 & 8 & 12 & 0 & 0 & 0 & 0 & \dots \\ 16 & 0 & 8 & 8 & 0 & 0 & 16 & 0 & 0 & 0 & \dots \\ 14 & 0 & 2 & 12 & 0 & 0 & 0 & 18 & 0 & 0 & \dots \\ 12 & 1 & 2 & 11 & 2 & 0 & 0 & 10 & 13 & 0 & \dots \\ 12 & 0 & 6 & 6 & 0 & 0 & 0 & 12 & 0 & 14 & \dots \\ \dots & & & & & & & & & & \ddots \end{pmatrix}$	42	7.1 17.01%	42	1 14 42	0.39 s
3 6 1	$\begin{pmatrix} 32 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 32 & 16 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 24 & 0 & 20 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 18 & 0 & 18 & 14 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 16 & 0 & 12 & 16 & 10 & 0 & 0 & 0 & 0 & 0 & \dots \\ 16 & 0 & 0 & 0 & 0 & 24 & 0 & 0 & 0 & 0 & \dots \\ 18 & 2 & 6 & 0 & 0 & 10 & 14 & 0 & 0 & 0 & \dots \\ 8 & 0 & 8 & 0 & 0 & 16 & 8 & 12 & 0 & 0 & \dots \\ 16 & 0 & 8 & 0 & 0 & 8 & 0 & 0 & 16 & 0 & \dots \\ 16 & 0 & 8 & 0 & 0 & 8 & 0 & 0 & 0 & 16 & \dots \\ \dots & & & & & & & & & & \ddots \end{pmatrix}$	2,650	26.8 1.01%	317 2, 650	1 15 835 2,512 2,639 2,639 2,639 2,639 2,639 2,639 2,639 $\vdots$	21.10 s
3 6 2	$\begin{pmatrix} 32 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 32 & 16 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 24 & 0 & 20 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 18 & 0 & 18 & 14 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 16 & 0 & 12 & 16 & 10 & 0 & 0 & 0 & 0 & 0 & \dots \\ 16 & 0 & 0 & 0 & 0 & 24 & 0 & 0 & 0 & 0 & \dots \\ 18 & 2 & 6 & 0 & 0 & 10 & 14 & 0 & 0 & 0 & \dots \\ 8 & 0 & 8 & 0 & 0 & 16 & 8 & 12 & 0 & 0 & \dots \\ 16 & 0 & 8 & 0 & 0 & 8 & 0 & 0 & 16 & 0 & \dots \\ 16 & 0 & 8 & 0 & 0 & 8 & 0 & 0 & 0 & 16 & \dots \\ \dots & & & & & & & & & & \ddots \end{pmatrix}$	118,728	40.6 0.03%	1, 140 39, 644 118, 728	1 15 1,169 46,090 115,415 118,327 118,593 118,594 118,594 118,594 118,600 $\vdots$	2 h 29 m

k n i	Matrix **A** of recursive system. Frames show the square sub-matrices related to the first key equations	Size of matrix **A** and vector $\mathbf{x}_m$	Average number of $\neq 0$ elements of **A** per row, also in %	Key equations	Number of different elements in vector $\mathbf{x}_m$ after successive iterations	Computer time to derive matrix **A**
3 7 0	$\begin{pmatrix} 64 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 64 & 32 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 48 & 0 & 40 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 40 & 8 & 32 & 24 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 36 & 0 & 36 & 0 & 28 & 0 & 0 & 0 & 0 & 0 & \dots \\ 32 & 0 & 0 & 0 & 0 & 48 & 0 & 0 & 0 & 0 & \dots \\ 40 & 16 & 0 & 0 & 0 & 24 & 24 & 0 & 0 & 0 & \dots \\ 36 & 4 & 12 & 0 & 0 & 20 & 0 & 28 & 0 & 0 & \dots \\ 24 & 4 & 4 & 0 & 0 & 32 & 8 & 16 & 20 & 0 & \dots \\ 24 & 0 & 28 & 8 & 0 & 12 & 0 & 16 & 0 & 20 & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \ddots \end{pmatrix}$	262	19.0 7.25%	262	1 28 245 261 262	47.92 s
3 7 1	$\begin{pmatrix} 64 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 64 & 32 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 48 & 0 & 40 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 40 & 8 & 32 & 24 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 36 & 0 & 36 & 0 & 28 & 0 & 0 & 0 & 0 & 0 & \dots \\ 32 & 2 & 26 & 12 & 20 & 18 & 0 & 0 & 0 & 0 & \dots \\ 32 & 0 & 24 & 0 & 32 & 0 & 20 & 0 & 0 & 0 & \dots \\ 36 & 0 & 36 & 0 & 0 & 0 & 0 & 28 & 0 & 0 & \dots \\ 28 & 0 & 30 & 0 & 21 & 0 & 0 & 7 & 21 & 0 & \dots \\ 26 & 0 & 24 & 6 & 18 & 12 & 0 & 0 & 14 & 14 & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \ddots \end{pmatrix}$	194,236	75.3 0.04%	12, 531 194, 236	1 29 5,520 164,120 194,024 194,221 194,222 194,222 194,222 194,222 194,222 ⋮	2 d 12 h
3 8 0	$\begin{pmatrix} 128 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 128 & 64 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 96 & 0 & 80 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 80 & 16 & 64 & 48 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 56 & 4 & 72 & 56 & 34 & 0 & 0 & 0 & 0 & 0 & \dots \\ 72 & 0 & 72 & 0 & 0 & 56 & 0 & 0 & 0 & 0 & \dots \\ 64 & 0 & 0 & 0 & 0 & 0 & 96 & 0 & 0 & 0 & \dots \\ 80 & 32 & 0 & 0 & 0 & 0 & 48 & 48 & 0 & 0 & \dots \\ 48 & 16 & 0 & 0 & 0 & 0 & 72 & 48 & 36 & 0 & \dots \\ 72 & 8 & 24 & 0 & 0 & 0 & 40 & 0 & 0 & 56 & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \ddots \end{pmatrix}$	2,269	54.2 2.39%	2, 269	1 52 2,079 2,266 2,269	3 h 51 m

Table 9: Probabilities of perfect ($i = 0$) and i-imperfect column triplets ($k = 3$) in Bernoulli ($m \times n$)-matrices computed by the Geometric method (with recursive systems)

k m i	Width n of Bernoulli $(m \times n)$-matrix					
	3	4	5	6	7	8
3 1 0	.50000000	.68750000	.81250000	.89062500	.93750000	.96484375
3 2 0	.25000000	.44921875	.61132813	.72973633	.81225586	.86875916
1	.75000000	.90234375	.96484375	.98803711	.99609375	
3 3 0	.12500000	.27514648	.42376709	.55218887	.65610695	.73748273
1	.50000000	.74609375	.88024902	.94555664	.97567177	
2	.87500000	.96948242	.99340820	.99869156		
3 4 0	.06250000	.15995789	.27330971	.38646346	.49073754	.58249769
1	.31250000	.56596375	.74513435	.85620672	.92067240	
2	.68750000	.89280701	.96704102	.99048918		
3	.93750000	.99046326	.99876404			
3 5 0	.03125000	.08944607	.16627449	.25237670	.34059513	.42613960
1	.18750000	.39891815	.58403063	.72412922	.82220783	
2	.50000000	.76921082	.90341794	.96182809		
3	.81250000	.95708466	.99152851			
4	.96875000	.99701977	.99976826			
3 6 0	.01562500	.04860848	.09663510	.15557551	.22129020	.29024047
1	.10937500	.26506132	.42682210	.57051118	.68752910	
2	.34375000	.61914063	.79807311	.89883014		
3	.65625000	.88675117	.96696451			
4	.89062500	.98342246	.99792156			
5	.98437500	.99906868	.99995655			
3 7 0	.00781250	.02586606	.05421108	.09154705	.13604801	.18578765
1	.06250000	.16808704	.29376464	.42062332	.53730233	
2	.22656250	.46849843	.66322249	.79787705		
3	.50000000	.77972507	.91295352			
4	.77343750	.94755490	.98950133			
5	.93750000	.99376014	.99950709			
6	.99218750	.99970896	.99999185			
3 8 0	.00390625	.01355145	.02959583	.05192179	.07996495	.11292569
1	.03515625	.10272465	.19230925	.29245405	.39403851	
2	.14453125	.33612457	.51938407	.66882364		
3	.36328125	.64922988	.82507384			
4	.63671875	.88122084	.96569079			
5	.85546875	.97675404	.99684412			
6	.96484375	.99769716	.99988611			
7	.99609375	.99990905	.99999847			
3 9 0	.00195313	.00701787	.01582380	.02860887	.04534006	.06580189
1	.01953125	.06095828	.12079652	.19337858	.27304478	
2	.08984375	.23055155	.38529038	.52946132		
3	.25390625	.51328401	.70970503			
4	.50000000	.78520431	.91760455			
5	.74609375	.93939156	.98739264			
6	.91015625	.99004467	.99909197			
7	.98046875	.99916326	.99997422			
8	.99804688	.99997158	.99999971			
3 10 0	.00097656	.00360293	.00832623	.01541214	.02498582	.03706493
1	.01074219	.03532874	.07334450	.12257006	.18014215	
2	.05468750	.15228751	.27247535	.39726873		
3	.17187500	.38721516	.58058229			
4	.37695313	.66869562	.84132869			
5	.62304688	.87631803	.96414011			
6	.82812500	.97042244	.99562178			
7	.94531250	.99585327	.99974783			
8	.98925781	.99969980	.99999426			
9	.99902344	.99999112				

3 Algebraic method

3.1 Multinomial sums and exponential polynomials

We use the following notation for factorials, falling factorials (Graham, Knuth and Patashnik 1988, 47–48) and binomial coefficients, respectively:

$$\begin{aligned} n! &= 1 \cdot 2 \cdots n \\ n^{\underline{i}} &= \begin{cases} 1 & \text{if } i = 0 \\ n & \text{if } i = 1 \\ n(n-1)\cdots(n-i+1) & \text{if } 1 < i \le n \\ 0 & \text{if } i > n \end{cases} \\ \binom{n}{p} &= \frac{n!}{p!\,(n-p)!} \ . \end{aligned} \tag{6}$$

Proposition 4 (Multinomial coefficients; Feller 1968, p. 37)

Let $t_1, \ldots, t_p$ *be integers such that*

$$t_1 + \cdots + t_p = n, \qquad t_i \ge 0 \ .$$

The number of ways in which a population of n *elements can be divided into* p *ordered parts (partitioned into* p *sub-populations) of which the first contains* t_1 *elements, the second* t_2 *elements, etc., is given by the* multinomial coefficients

$$\frac{n!}{t_1! \ldots t_p!} \quad \textit{shortly denoted} \quad \frac{n!}{t_{1:p}\,!} \ . \tag{7}$$

This definition implies the following identity

$$(x_1 + \cdots + x_p)^n = \sum_{\substack{t_1 + \cdots + t_p = n \\ t_i \ge 0}} \frac{n!}{t_1! \ldots t_p!} x_1^{t_1} \ldots x_p^{t_p} \ . \tag{8}$$

In the multinomial sums below, the condition that at least one of $t_1, \ldots, t_p \ge 0$ is strictly positive is denoted by $t_1 + \cdots + t_p > 0$.

Proposition 5 (Sums of multinomial coefficients)

For nonnegative integers $i, j, k, n, p, r, s, t_i, v, w$ *it holds:*

$$\sum \frac{n!}{t_{1:p}\,!} = p^n \quad \left(\text{short form of} \sum_{\substack{t_1 + \cdots + t_p = n \\ t_i \ge 0}} \frac{n!}{t_1! \ldots t_p!} \right) \tag{9}$$

$$\sum_{t_1, \ldots, t_r = 0} \frac{n!}{t_{1:p}\,!} = (p - r)^n, \qquad r \le p \tag{10}$$

$$\sum_{t_1 > 0} \frac{n!}{t_{1:p}\,!} = p^n - (p-1)^n \tag{11}$$

$$\sum_{t_1, \ldots, t_r > 0} \frac{n!}{t_{1:p}\,!} = \sum_{i=0}^{r} (-1)^i \binom{r}{i} (p - i)^n, \qquad r \le p \tag{12}$$

$$\sum_{\substack{t_1,\ldots,t_r>0\\ t_{r+1}+\cdots+t_{r+s}>0}} \frac{n!}{t_{1:p}\,!} = \sum_{i=0}^{r}(-1)^i\binom{r}{i}(p-i)^n - \underbrace{\sum_{i=0}^{r}(-1)^i\binom{r}{i}(p-s-i)^n}_{=0\ \text{if}\ s=0}\ , \quad s+r\le p\ , \tag{13}$$

$$\sum_{t_1=1}\frac{n!}{t_{1:p}\,!} = n(p-1)^{n-1} \tag{14}$$

$$\sum_{t_1>1}\frac{n!}{t_{1:p}\,!} = p^n-(p-1)^n-n(p-1)^{n-1} \tag{15}$$

$$\sum_{\text{One of } t_1,\ldots,t_r>1}\frac{n!}{t_{1:p}\,!} = p^n-\sum_{i=0}^{r}\binom{r}{i}n^{\underline{i}}\,(p-r)^{n-i} \tag{16}$$

$$\sum_{\substack{t_1,\ldots,t_q=1\\ t_{q+1},\ldots,t_{q+r}>0\\ t_{q+r+1}+\cdots+t_{q+r+s}>0\\ t_{q+r+1},\ldots,t_{q+r+v}\le 1,\quad v\le s\\ t_{q+r+s+1},\ldots,t_{q+r+s+w}\le 1}}\frac{n!}{t_{1:p}\,!} =$$

$$= \begin{cases} \displaystyle\sum_{j=0}^{w}\binom{w}{j}n^{\underline{q+j}}\sum_{k=0}^{r}(-1)^k\binom{r}{k}(p-q-w-k)^{n-q-j} & \text{if}\quad v=s=0\\ \displaystyle\sum_{i=1}^{v}\binom{v}{i}\sum_{j=0}^{w}\binom{w}{j}n^{\underline{q+i+j}}\sum_{k=0}^{r}(-1)^k\binom{r}{k}(p-q-v-w-k)^{n-q-i-j} & \text{if}\quad 0<v=s\\ \displaystyle\sum_{i=0}^{v}\binom{v}{i}\sum_{j=0}^{w}\binom{w}{j}n^{\underline{q+i+j}}\sum_{k=0}^{r}(-1)^k\binom{r}{k}(p-q-v-w-k)^{n-q-i-j} & \\ \displaystyle\quad-\sum_{j=0}^{w}\binom{w}{j}n^{\underline{q+j}}\sum_{k=0}^{r}(-1)^k\binom{r}{k}(p-q-w-s-k)^{n-q-j} & \text{if}\quad v<s \end{cases} \tag{17}$$

Proof of Proposition 5

To prove (9) it suffices to substitute $x_1=\ldots=x_p=1$ into (8).

Prove (10):

$$\sum_{t_1,t_2,\ldots,t_r=0}\frac{n!}{t_{1:p}\,!} = \sum\frac{n!}{t_{1:p-r}\,!} \overset{(9)}{=} (p-r)^n\ .$$

Prove (11):

$$\sum_{t_1>0}\frac{n!}{t_{1:p}\,!} = \sum\frac{n!}{t_{1:p}\,!}-\sum_{t_1=0}\frac{n!}{t_{1:p}\,!} \overset{(9),\,(10)}{=} p^n-(p-1)^n\ .$$

To prove (12) first note that

$$\binom{r}{i}+\binom{r}{i-1} = \frac{r!}{i!\,(r-i)!}+\frac{r!}{(i-1)!\,(r+1-i)!}$$

$$\begin{aligned}
&= \frac{r!}{i!\,(r+1-i)!}[(r+1-i)+i] \\
&= \frac{(r+1)!}{i!\,(r+1-i)!} \\
&= \binom{r+1}{i} \qquad (18)
\end{aligned}$$

Now prove (12) by induction on r. For $p=1$ we have if from (11). Assume that it holds for r and prove it for $r+1$:

$$\begin{aligned}
\sum_{t_1,\ldots,t_{r+1}>0} \frac{n!}{t_{1:p}\,!} &= \sum_{t_1,\ldots,t_r>0} \frac{n!}{t_{1:p}\,!} - \sum_{\substack{t_1,\ldots,t_r>0\\ t_{r+1}=0}} \frac{n!}{t_{1:p}\,!} \\
&= \sum_{t_1,\ldots,t_r>0} \frac{n!}{t_{1:p}\,!} - \sum_{t_1,\ldots,t_r>0} \frac{n!}{t_{1:p-1}\,!} \qquad \overset{\text{By assumption of induction}}{\Rightarrow} \\
&= \sum_{i=0}^{r}(-1)^i \binom{r}{i}(p-i)^n - \sum_{i=0}^{r}(-1)^i \binom{r}{i}(p-1-i)^n \qquad \overset{\text{Replace } i \text{ in the 2nd sum by } j-1}{\Rightarrow} \\
&= \sum_{i=0}^{r}(-1)^i \binom{r}{i}(p-i)^n + \sum_{j=1}^{r+1} \underbrace{-(-1)^{j-1}}_{(-1)^j} \binom{r}{j-1}(p-j)^n \\
&= p^n + \sum_{i=1}^{r}(-1)^i \underbrace{\left[\binom{r}{i} + \binom{r}{i-1}\right]}_{=\binom{r+1}{i} \text{ by (18)}} (p-i)^n + (-1)^{r+1}[p-(r+1)]^n \\
&= \sum_{i=0}^{r+1}(-1)^i \binom{r+1}{i}(p-i)^n
\end{aligned}$$

Prove (13):

$$\begin{aligned}
\sum_{\substack{t_1,\ldots\, t_r>0\\ t_{r+1}+\cdots+t_{r+s}>0}} \frac{n!}{t_{1:p}\,!} &= \sum_{t_1,\ldots,t_r>0} \frac{n!}{t_{1:p}\,!} - \underbrace{\sum_{\substack{t_1,\ldots,t_r>0\\ t_{r+1},\ldots,t_{r+s}=0}} \frac{n!}{t_{1:p}\,!}}_{=0 \text{ if } s=0} \\
&= \sum_{t_1,\ldots,t_r>0} \frac{n!}{t_{1:p}\,!} - \underbrace{\sum_{t_1,\ldots,t_r>0} \frac{n!}{t_{1:p-s}\,!}}_{=0 \text{ if } s=0} \\
&\overset{(12)}{=} \sum_{i=0}^{r}(-1)^i \binom{r}{i}(p-i)^n - \underbrace{\sum_{i=0}^{r}(-1)^i \binom{r}{i}(p-s-i)^n}_{=0 \text{ if } s=0}\ .
\end{aligned}$$

Prove (14):

$$\sum_{t_1=1} \frac{n!}{t_{1:p}\,!} = \sum_{t_2+\ldots+t_p=n-1} \frac{n!}{t_{2:p}\,!}$$

$$
\begin{aligned}
&= n \sum_{t_2+\cdots+t_p=n-1} \frac{(n-1)!}{t_{2:p}\,!} \\
&\overset{(9)}{=} n(p-1)^{n-1}
\end{aligned}
$$

Prove (15):

$$
\begin{aligned}
\sum_{t_1>1} \frac{n!}{t_{1:p}\,!} &= \sum \frac{n!}{t_{1:p}\,!} - \sum_{t_1=0} \frac{n!}{t_{1:p}\,!} - \sum_{t_1=1} \frac{n!}{t_{1:p}\,!} \\
&\overset{(9),(10),(14)}{=} p^n - (p-1)^n - n(p-1)^{n-1}
\end{aligned}
$$

Prove (16). Taking into account that the number of subpopulations i with $t_i \geq 1$ cannot surpass n, obtain

$$
\begin{aligned}
\sum_{\text{One of } t_1,\ldots,t_r \geq 2} \frac{n!}{t_{1:p}\,!} &= \sum \frac{n!}{t_{1:p}\,!} - \sum_{t_1,\ldots,t_r\,=\,0,1} \frac{n!}{t_{1:p}\,!} \\
&= p^n - \sum_{i=0}^{\min(n,r)} \binom{r}{i} \sum_{\substack{t_1,\ldots,t_i\,=\,1 \\ t_{i+1},\ldots,t_r\,=\,0}} \frac{n!}{t_{1:p}\,!} \\
&= p^n - \sum_{i=0}^{\min(n,r)} \binom{r}{i} n(n-1)\cdots(n-i+1)(p-r)^{n-i} \\
&= p^n - \sum_{i=0}^{r} \binom{r}{i} n^{\underline{i}}\,(p-r)^{n-i} \ .
\end{aligned}
$$

Prove (17). Obviously,

$$
\sum_{\substack{t_1,\ldots,t_q\,=\,1 \\ t_{q+1},\ldots,t_{q+r}\,>\,0 \\ t_{q+r+1}+\cdots+t_{q+r+s}\,>\,0 \\ t_{q+r+1},\ldots,t_{q+r+v}\,\leq\,1 \\ t_{q+r+s+1},\ldots,t_{q+r+s+w}\,\leq\,1}} \frac{n!}{t_{1:p}\,!} = n^{\underline{q}} \sum_{\substack{t_{q+1},\ldots,t_{q+r}\,>\,0 \\ t_{q+r+1}+\cdots+t_{q+r+s}\,>\,0 \\ t_{q+r+1},\ldots,t_{q+r+v}\,\leq\,1 \\ t_{q+r+s+1},\ldots,t_{q+r+s+w}\,\leq\,1}} \frac{(n-q)!}{t_{1:p}\,!} \tag{19}
$$

Continue to compute (19) for three cases of parameters s and v.

- Case $v = s = 0$:

$$
\begin{aligned}
&= n^{\underline{q}} \sum_{\substack{t_1,\ldots,t_r\,>\,0 \\ t_{r+1},\ldots,t_{r+w}\,\leq\,1}} \frac{(n-q)!}{t_{1:p-q}\,!} \\
&= n^{\underline{q}} \sum_{j=0}^{w} \binom{w}{j} \sum_{\substack{t_1,\ldots,t_r\,>\,0 \\ t_{r+1},\ldots,t_{r+j}\,=\,1 \\ t_{r+j},\ldots,t_{r+w}\,=\,0}} \frac{(n-q)!}{t_{1:p-q}\,!} \\
&= \sum_{j=0}^{w} \binom{w}{j} n^{\underline{q+j}} \sum_{t_1,\ldots,t_r\,>\,0} \frac{(n-q-j)!}{t_{1:p-q-w}\,!} \\
&\overset{(12)}{=} \sum_{j=0}^{w} \binom{w}{j} n^{\underline{q+j}} \sum_{k=0}^{r} (-1)^k \binom{r}{k} (p-q-w-k)^{n-q-j} \ .
\end{aligned} \tag{20}
$$

- Case $v = s > 0$:

$$
\begin{aligned}
&= n^{\underline{q}} \sum_{i=1}^{v} \binom{v}{i} \sum_{\substack{t_1,\dots,t_r > 0 \\ t_{r+1},\dots,t_{r+i} = 1 \\ t_{r+i+1},\dots,t_{r+v} = 0 \\ t_{r+s+1},\dots,t_{r+s+w} \leq 1}} \frac{(n-q)!}{t_{1:p-q}\,!} \\
&= \sum_{i=1}^{v} \binom{v}{i} n^{\underline{q+i}} \sum_{\substack{t_1,\dots,t_r > 0 \\ t_{r+1},\dots,t_{r+w} = 0,1}} \frac{(n-q-i)!}{t_{1:p-q-v}\,!} \\
&= \sum_{i=1}^{v} \binom{v}{i} n^{\underline{q+i}} \sum_{j=0}^{w} \binom{w}{j} \sum_{\substack{t_1,\dots,t_r > 0 \\ t_{r+1},\dots,t_{r+j} = 1 \\ t_{r+j},\dots,t_{r+w} = 0}} \frac{(n-q-i)!}{t_{1:p-q-v}\,!} \\
&= \sum_{i=1}^{v} \binom{v}{i} \sum_{j=0}^{w} \binom{w}{j} n^{\underline{q+i+j}} \sum_{t_1,\dots,t_r > 0} \frac{(n-q-i-j)!}{t_{1:p-q-v-w}\,!} \\
&\overset{(12)}{=} \sum_{i=1}^{v} \binom{v}{i} \sum_{j=0}^{w} \binom{w}{j} n^{\underline{q+i+j}} \sum_{k=0}^{r} (-1)^k \binom{r}{k} (p-q-v-w-k)^{n-q-i-j} \ . \qquad (21)
\end{aligned}
$$

- Case $v < s$:

$$
= n^{\underline{q}} \left[\sum_{i=1}^{v} \binom{v}{i} \sum_{\substack{t_1,\dots,t_r > 0 \\ t_{r+1},\dots,t_{r+i} = 1 \\ t_{r+i+1},\dots,t_{r+v} = 0 \\ t_{r+s+1},\dots,t_{r+s+w} \leq 1}} \frac{(n-q)!}{t_{1:p-q}\,!} + \sum_{\substack{t_1,\dots,t_r > 0 \\ t_{r+1},\dots,t_{r+v} = 0 \\ t_{r+v+1}+\dots+t_{r+s} > 0 \\ t_{r+s+1},\dots,t_{r+s+w} \leq 1}} \frac{(n-q)!}{t_{1:p-q}\,!} \right] \qquad (22)
$$

Since the first sum in (22) is equal to (21), it remains to compute the second sum:

$$
\begin{aligned}
n^{\underline{q}} \sum_{\substack{t_1,\dots,t_r > 0 \\ t_{r+1},\dots,t_v = 0 \\ t_{r+v+1}+\dots+t_{r+s} > 0 \\ t_{r+s+1},\dots,t_{r+s+w} \leq 1}} \frac{(n-q)!}{t_{1:p-q}\,!} &= n^{\underline{q}} \sum_{j=0}^{w} \binom{w}{j} \sum_{\substack{t_1,\dots,t_r > 0 \\ t_{r+1}+\dots+t_{r+s-v} > 0 \\ t_{r+s+1},\dots,t_{r+s+j} = 1 \\ t_{r+s+j+1},\dots,t_{r+s+w} = 0}} \frac{(n-q)!}{t_{1:p-q-v}\,!} \\
&= \sum_{j=0}^{w} \binom{w}{j} n^{\underline{q+j}} \sum_{\substack{t_1,\dots,t_r > 0 \\ t_{r+1}+\dots+t_{r+s-v} > 0}} \frac{(n-q-j)!}{t_{1:p-q-v-w}\,!} \\
&\overset{(13)}{=} \sum_{j=0}^{w} \binom{w}{j} n^{\underline{q+j}} \\
&\quad \times \left[\sum_{k=0}^{r} (-1)^k \binom{r}{k} (p-q-v-w-k)^{n-q-j} \right. \\
&\quad \left. - \sum_{k=0}^{r} (-1)^k \binom{r}{k} (p-q-w-s-k)^{n-q-j} \right] \ . \qquad (23)
\end{aligned}
$$

Assembling (22) from (21) and (23), note that the first inner sum in (23) can be integrated into (21) by starting the summation on i with the index $i = 0$. Thereby for the case $v < s$ we obtain

$$\begin{aligned} = & \sum_{i=0}^{v} \binom{v}{i} \sum_{j=0}^{w} \binom{w}{j} n^{\underline{q+i+j}} \sum_{k=0}^{r} (-1)^k \binom{r}{k} (p-q-v-w-k)^{n-q-i-j} \\ & - \sum_{j=0}^{w} \binom{w}{j} n^{\underline{q+j}} \sum_{k=0}^{r} (-1)^k \binom{r}{k} (p-q-w-s-k)^{n-q-j} \;. \end{aligned} \tag{24}$$

Thus, (19) is computed with (20), (21) and (24), depending on parameters s and v:

$$\sum_{\substack{t_1,\ldots,t_q=1 \\ t_{q+1},\ldots,t_{q+r}>0 \\ t_{q+r+1}+\cdots+t_{q+r+s}>0 \\ t_{q+r+1},\ldots,t_{q+r+v}\le 1 \\ t_{q+r+s+1},\ldots,t_{q+r+s+w}\le 1}} \frac{n!}{t_{1:p}\,!} =$$

$$= \begin{cases} \sum_{j=0}^{w} \binom{w}{j} n^{\underline{q+j}} \sum_{k=0}^{r} (-1)^k \binom{r}{k} (p-q-w-k)^{n-q-j} & \text{if} \quad v=s=0 \\ \sum_{i=1}^{v} \binom{v}{i} \sum_{j=0}^{w} \binom{w}{j} n^{\underline{q+i+j}} \sum_{k=0}^{r} (-1)^k \binom{r}{k} (p-q-v-w-k)^{n-q-i-j} & \text{if} \quad 0<v=s \\ \sum_{i=0}^{v} \binom{v}{i} \sum_{j=0}^{w} \binom{w}{j} n^{\underline{q+i+j}} \sum_{k=0}^{r} (-1)^k \binom{r}{k} (p-q-v-w-k)^{n-q-i-j} & \\ \quad - \sum_{j=0}^{w} \binom{w}{j} n^{\underline{q+j}} \sum_{k=0}^{r} (-1)^k \binom{r}{k} (p-q-w-s-k)^{n-q-j} & \text{if} \quad v<s \end{cases}$$

as required ■

Although the exponential polynomials in (12), (13) and (17) are defined for any combinations of their parameters, the corresponding multinomial sums make no sense for $n < r$ and $n < q + r$. It turns however out that these polynomials are equal to zero for the 'prohibited' parameter values. This experimentally established property is practical for computer implementation, since it enables to apply the same polynomial regardless of the parameter value. We formulate this conjecture as proposition with no formal proof.

Proposition 6 (Self-constraining in multinomial formulas) *Exponential polynomials in (12), (13) and (17) are equal to zero if* $n < r$ *or* $n < q + r$*.*

3.2 Perfect column pairs in matrices with up to 3 rows

We continue to index types of m-columns as in tables T_m in Section 2.1. Till Section 3.4, 'Imperfect column pairs in the general case', the 'perfect outcomes' refer to column *pairs*.

Consider a Bernoulli $(m \times n)$-matrix. Then the number of its outcomes with t_i columns of the ith type is expressed by the multinomial coefficient (7)

$$\text{Number of outcomes with } t_i \text{ columns of the } i\text{th type} = \frac{n!}{t_{1:p}!} \ .$$

Perfect outcomes of 1-row Bernoulli matrix A perfect outcome requires a column of type 2 from Table 1:

$$\text{Number of perfect outcomes} \quad = \sum_{t_2>0} \frac{n!}{t_1!\,t_2!} \overset{(11)}{=} 2^n - 1 \ .$$

Perfect outcomes of 2-row Bernoulli matrix The outcomes with a column of type 4 from Table 2 are perfect. Besides, the outcomes with no column of type 4 but with columns of types 3 and 2 are perfect as well. All other outcomes, that is, with no columns of types 4 and 3 are not perfect. Hence,

$$\begin{aligned} \text{Number of perfect outcomes} \quad &= \sum_{t_4>0} \frac{n!}{t_{1:4}!} + \sum_{\substack{t_4=0\\ t_3>0\\ t_2>0}} \frac{n!}{t_{1:4}!} \\ &\overset{(11)}{=} 4^n - 3^n + \sum_{t_2t_3>0} \frac{n!}{t_{1:3}!} \\ &\overset{(12)}{=} 4^n - 3^n \quad + \quad 3^n - 2\cdot 2^n + 1 \\ &= 4^n - 2\cdot 2^n + 1 \ . \end{aligned}$$

In a perfect column pair, the column with the superior index is called *pivot*, and the column with the inferior index is called *match*. For example, in the perfect pair $(4, 5)$ of 3-columns from Table 3, column 5 is pivot and column 4 is match.

A column is called *self-sufficient* if it makes a perfect column pair with any other column.

The next proposition follows from the way the column types are indexed.

Proposition 7 (Properties of perfect column pairs)

1. *The 2^m-th m-column is self-sufficient.*
2. *In a perfect pair of m-columns, the pivot is from the second half of table T_m.*
3. *In a perfect pair of m-columns, at least one column is from an even quarter of table T_m (2nd quarter or 4th quarter), at least one column is from an even eighth of T_m, ..., and one column has an even index.*
4. *If a perfect pair of m-columns has no self-sufficient column (2^m-th column), the two columns are different.*

Perfect outcomes of 3-row Bernoulli matrix Note that a pivot can have several matching columns indexed $i_1, \ldots, i_p$. The existence of at least one match is therefore conditioned by the inequality $t_{i_1} + \ldots + t_{i_p} > 0$.

Construct disjoint events of perfect outcomes, starting from the maximal pivot as shown by the scheme below (for the indices of column types see Table 3):

$$
\begin{array}{lllll}
\multicolumn{5}{l}{\text{Certain event: all combinations of } \ t_1, \ldots, t_8 \ \text{ such that } \ \sum_{i=1}^{8} t_i = 3} \\
\downarrow \quad \searrow & & & & \\
t_8 > 0 & t_8 = 0 & & & \\
 & \downarrow \qquad\qquad \searrow & & & \\
 & t_7 > 0 & t_7 = 0 & & \\
 & t_2 + t_4 + t_6 > 0 & & & \\
 & & \downarrow \qquad \searrow & & \\
 & & t_6 > 0 & t_6 = 0 & \\
 & & t_3 + t_4 > 0 & & \\
 & & & \downarrow \quad \searrow & \\
 & & & t_5 > 0 & t_5 = 0 \\
 & & & t_4 > 0 & \\
 & & & & \downarrow \\
 & & & & \text{No more pivots; stop}
\end{array}
\tag{25}
$$

To run all the branches, start from the left branch and go down. As the bottom is attained go back till the first branching to the right, and go this branch down as well. Then again go back till the first branching to the right, and so on.

Here, the first event is $t_8 > 0$. By Item 1 of Proposition 7, all outcomes with a column of type 8 are perfect. After excluding this event, column 7 is regarded as the maximal pivot ($t_8 = 0,\ t_7 > 0$) with the match $t_2 + t_4 + t_6 > 0$. If the match is excluded ($t_2, t_4, t_6 = 0$) no other pivot with match exist (indeed, by Item 2 of Proposition 7 only column 5 could be pivot, but it would require column 4 as match, which is already excluded). Going the right-hand branch is under the condition $t_8, t_7 = 0$. Then column 6 is regarded as the maximal pivot ($t_8, t_7 = 0,\ t_6 > 0$) with the match $t_3 + t_4 > 0$. If $t_3, t_4 = 0$ no other pivot with match exist, and so on.

This partitioning into disjoint events of perfect outcomes enables to count the latter

$$
\begin{aligned}
&\text{Number of perfect outcomes} = \\
&= \sum_{t_8 > 0} \frac{n!}{t_{1:8}\,!} + \sum_{\substack{t_8 = 0 \\ t_7 > 0 \\ t_2 + t_4 + t_6 > 0}} \frac{n!}{t_{1:8}\,!} + \sum_{\substack{t_8, t_7 = 0 \\ t_6 > 0 \\ t_3 + t_4 > 0}} \frac{n!}{t_{1:8}\,!} + \sum_{\substack{t_8, t_7, t_6 = 0 \\ t_5 > 0 \\ t_4 > 0}} \frac{n!}{t_{1:8}\,!} \\
&\overset{(11)}{=} 8^n - 7^n + \sum_{\substack{t_7 > 0 \\ t_2 + t_4 + t_6 > 0}} \frac{n!}{t_{1:7}\,!} + \sum_{\substack{t_6 > 0 \\ t_3 + t_4 > 0}} \frac{n!}{t_{1:6}\,!} + \sum_{\substack{t_5 > 0 \\ t_4 > 0}} \frac{n!}{t_{1:5}\,!} \\
&\overset{(13)}{=} 8^n - 7^n \quad + \quad 7^n - 6^n - 4^n + 3^n \quad + \quad 6^n - 5^n - 4^n + 3^n \quad + \quad 5^n - 2 \cdot 4^n + 3^n \\
&= 8^n - 4 \cdot 4^n + 3 \cdot 3^n \ .
\end{aligned}
$$

3.3 Perfect column pairs in the general case

The disjoint events with perfect outcomes can be constructed by a branching algorithm which we exemplify with four-row Bernoulli matrices ($m = 4$).

Table 10: Lookup table $(k = 2,\ m = 4,\ i = 0)$. Columns in frames are self-sufficient

Pivot	Match
$\boxed{16}$	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
15	2 4 6 8 10 12 14
14	3 4 7 8 11 12
13	4 8 12
12	5 6 7 8
11	6 8
10	7 8
9	8

First of all make the *Lookup table* with pivots and matching columns, and the list *Self-sufficient columns*; see Table 10 where the only self-sufficient column is framed. Initialize the algorithm with the following three elements.

- List *Available columns* with column indices for selecting pivot and match. It is initialized with all but s column indices of self-sufficient columns. In our example of 4-columns, we have a single self-sufficient column 16, so that $s = 1$ and

$$Available\ columns = [1, \dots, 15]\ .$$

- *Polynomial* to count (im)perfect outcomes. It is initialized for the event containing at least one self-sufficient column, that is, expressing the multinomial sum $\sum_{t_1+\cdots+t_s>0} \frac{n!}{t_{1:2^m}!}$. In our example,

$$Polynomial = \sum_{t_{16}>0} \frac{n!}{t_{1:16}\,!} \overset{(11)}{=} 16^n - 15^n\ .$$

- List *Obligatory columns* for the column indices which can be required to be retained. It is initialized with the empty list

$$Obligatory\ columns = [\ ]\ .$$

The further operation is performed by a self-calling module (the nested calls are shown in **boldface**). Since *Polynomial* is initialized for the event with self-sufficient columns, other events have no self-sufficient columns, and by Item 4 of Proposition 7 *have multiple instances of neither pivots, nor match.*

Self-calling module for enumerating disjoint events with perfect column pairs

- *(Input).* Lists *Available columns* and *Obligatory columns.*
- *(Finding pivot and match to define a new disjoint event).* Find the maximal possible pivot and its match from *Available columns* using the *Lookup table.* If they are not found then exit the module (return).

  If they are found, define a new disjoint event for the given available and obligatory columns, pivot, and match. If one of matching columns belongs to *Obligatory columns* then match is guaranteed and the event is defined with the pivotal condition $t_{pivot} > 0$ only. Otherwise, if no matching column belongs to *Obligatory columns*, consider also the match-guaranteeing condition $\sum_{match} t_{match} > 0$. Compute the exponential polynomial for this event using (13) and add it to *Polynomial*.

In our example, the first encountering event results in the exponential polynomial

$$\sum_{\substack{t_{16}=0\\ t_{15}>0\\ t_2+t_4+t_6+t_8+t_{10}+t_{12}+t_{14}>0}} \frac{n!}{t_{1:16}\,!} \overset{(13)}{=} 15^n - 14^n - 8^n + 7^n.$$

- *(Branch 1: Pivot but no match).* If no matching column belongs to *Obligatory columns* then exclude all the matching columns from *Available columns*, add the pivot to *Obligatory columns*, and start a **new instance of the module**. Otherwise, if any of matching columns belongs to *Obligatory columns*, do nothing and just go to the next branch.
- *(Branch 2: No pivot).* If the pivot does not belong to *Obligatory columns* then delete the pivot from *Available columns* and start a **new instance of the module**. Otherwise, if the pivot belongs to *Obligatory columns*, exit the module (return).

Applying this algorithm to our example of Bernoulli $(4 \times n)$-matrices, we obtain:

$$\begin{aligned}
&\text{Number of perfect outcomes } = \\
&= \sum_{t_{16}>0} \frac{n!}{t_{1:16}\,!} + \sum_{\substack{t_{16}=0\\ t_{15}>0\\ t_2+t_4+t_6+t_8+t_{10}+t_{12}+t_{14}>0}} \frac{n!}{t_{1:16}\,!} + \sum_{\substack{t_{16},t_{15}=0\\ t_{14}>0\\ t_3+t_4+t_7+t_8+t_{11}+t_{12}>0}} \frac{n!}{t_{1:16}\,!} \\
&+ \sum_{\substack{t_{16},t_{15},t_{14}=0\\ t_{13}>0\\ t_4+t_8+t_{12}>0}} \frac{n!}{t_{1:16}\,!} + \underbrace{\sum_{\substack{t_{16},t_{15},t_{14}=0\\ t_{13}>0\\ t_4,t_8,t_{12}=0\\ t_{11}>0\\ t_6>0}} \frac{n!}{t_{1:16}\,!}}_{\substack{\text{Event with}\\ \text{obligatory column 13}}} + \underbrace{\sum_{\substack{t_{16},t_{15},t_{14}=0\\ t_{13}>0\\ t_4,t_8,t_{12}=0\\ t_{11}>0\\ t_6=0\\ t_{10}>0\\ t_7>0}} \frac{n!}{t_{1:16}\,!}}_{\substack{\text{Event with}\\ \text{obligatory}\\ \text{columns 13 and 11}}} \\
&+ \underbrace{\sum_{\substack{t_{16},t_{15},t_{14}=0\\ t_{13}>0\\ t_4,t_8,t_{12}=0\\ t_{11}=0\\ t_{10}>0\\ t_7>0}} \frac{n!}{t_{1:16}\,!}}_{\substack{\text{Event with}\\ \text{obligatory column 13}}} + \sum_{\substack{t_{16},\ldots,t_{13}=0\\ t_{12}>0\\ t_5+\ldots+t_8>0}} \frac{n!}{t_{1:16}\,!} + \sum_{\substack{t_{16},\ldots,t_{12}=0\\ t_{11}>0\\ t_6+t_8>0}} \frac{n!}{t_{1:16}\,!} \\
&\underbrace{\sum_{\substack{t_{16},\ldots,t_{12}=0\\ t_{11}>0\\ t_6,t_8=0\\ t_{10}>0\\ t_7>0}} \frac{n!}{t_{1:16}\,!}}_{\substack{\text{Event with}\\ \text{obligatory column 11}}} + \sum_{\substack{t_{16},\ldots,t_{11}=0\\ t_{10}>0\\ t_7+t_8>0}} \frac{n!}{t_{1:16}\,!} + \sum_{\substack{t_{16},\ldots,t_{10}=0\\ t_9>0\\ t_8>0}} \frac{n!}{t_{1:16}\,!}
\end{aligned}$$

$$\begin{aligned}
\overset{(11),(13)}{=} \quad & 16^n - 15^n \quad + \quad 15^n - 14^n - 8^n + 7^n \quad + \quad 14^n - 13^n - 8^n + 7^n \\
& + 13^n - 12^n - 10^n + 9^n \quad + \quad 10^n - 3 \cdot 9^n + 3 \cdot 8^n - 7^n \\
& + 9^n - 4 \cdot 8^n + 6 \cdot 7^n - 4 \cdot 6^n + 5^n \quad + \quad 9^n - 3 \cdot 8^n + 3 \cdot 7^n - 6^n \\
& + 12^n - 11^n - 8^n + 7^n \quad + \quad 11^n - 10^n - 9^n + 8^n \\
& + 9^n - 3 \cdot 8^n + 3 \cdot 7^n - 6^n \quad + \; 10^n - 9^n - 8^n + 7^n \quad + \; 9^n - 2 \cdot 8^n + 7^n \\
= \quad & 16^n - 12 \cdot 8^n + 16 \cdot 7^n - 6 \cdot 6^n + 5^n \ .
\end{aligned}$$

3.4 Imperfect column pairs in the general case

For counting imperfect outcomes, the algorithm from the previous paragraph remains intact, but its initialization requires two modifications. The *Lookup table* is extended to i-imperfect matching of columns, and self-sufficient columns are extended to *i-self-sufficient columns* which make i-imperfect column pairs with any other column; see Table 11.

Table 11: Lookup table ($k = 2,\ m = 4,\ i = 1$). Columns in frames are self-sufficient

Pivot	Match
[16]	1 2 3 4 5 6 7 [8] 9 10 11 [12] 13 [14] [15]
[15]	1 2 3 4 5 6 7 [8] 9 10 11 [12] 13 [14]
[14]	1 2 3 4 5 6 7 [8] 9 10 11 [12] 13
13	2 3 4 6 7 [8] 10 11 [12]
[12]	1 2 3 4 5 6 7 [8] 9 10 11
11	2 4 5 6 7 [8] 10
10	3 4 5 6 7 [8]
9	4 6 7 [8]
[8]	1 2 3 4 5 6 7
7	2 4 6
6	3 4
5	4

Item 4 of Proposition 7 remains valid, that is, *if a i-imperfect column pair includes no i-self-sufficient column then both columns are different.* Indeed, if a column is not i-self-sufficient than it has more than i zero elements, and two such columns have more than i rows with simultaneous zeros, implying no i-imperfect pair.

Therefore, after having considered the event with self-sufficient columns at the initialization step of the algorithm, the remaining events with i-imperfect outcomes have *multiple instances of neither pivots, nor match.* Thus, the algorithm can run with no further changes.

The exponential polynomials obtained are shown in Table 12, and the probabilities computed are given in Table 13. As one can see, the probabilities computed by the Geometric and Algebraic methods (from the common domains of Tables 7 and 13 are identical.

Table 12: Description of exponential polynomials for counting perfect ($i = 0$) and i-imperfect outcomes of Bernoulli ($m \times n$)-matrices for column pairs ($k = 2$)

k m i	Exponential polynomial (to compute probabilities multiply by 2^{-mn})	Number of terms in the exponential polynomial	Number of disjoint events constructed	Computer time
2 1 0	$2^n - 1$	2	1	0.00 s
2 2 0	$4^n - 2 \cdot 2^n + 1$	3	2	0.00 s
1	$4^n - 1$	2	1	0.00 s
2 3 0	$8^n - 4 \cdot 4^n + 3 \cdot 3^n$	3	4	0.00 s
1	$8^n - 3 \cdot 2^n + 2$	3	3	0.00 s
2	$8^n - 1$	2	1	0.00 s
2 4 0	$16^n - 12 \cdot 8^n + 16 \cdot 7^n - 6 \cdot 6^n + 5^n$	5	12	0.01 s
1	$16^n - 5^n - 6 \cdot 4^n + 6 \cdot 3^n$	4	8	0.01 s
2	$16^n - 4 \cdot 2^n + 3$	3	4	0.00 s
3	$16^n - 1$	2	1	0.00 s
2 5 0	$32^n - 81 \cdot 16^n + 185 \cdot 15^n - 150 \cdot 14^n + 50 \cdot 13^n - 5 \cdot 12^n$	6	81	0.11 s
1	$32^n - 5 \cdot 10^n - 10 \cdot 9^n + 20 \cdot 8^n - 10 \cdot 7^n + 4 \cdot 6^n$	6	40	0.05 s
2	$32^n - 6^n - 10 \cdot 4^n + 10 \cdot 3^n$	4	13	0.02 s
3	$32^n - 5 \cdot 2^n + 4$	3	5	0.01 s
4	$32^n - 1$	2	1	0.00 s
2 6 0	$64^n - 2646 \cdot 32^n + 11276 \cdot 31^n - 21120 \cdot 30^n + 23360 \cdot 29^n - 17340 \cdot 28^n + 9210 \cdot 27^n - 3555 \cdot 26^n + 975 \cdot 25^n - 180 \cdot 24^n + 20 \cdot 23^n - 22^n$	12	2,646	4.46 s
1	$64^n - 22^n - 35 \cdot 20^n - 70 \cdot 19^n + 210 \cdot 18^n - 45 \cdot 17^n - 90 \cdot 16^n - 15 \cdot 15^n + 60 \cdot 14^n - 15 \cdot 13^n$	10	635	0.96 s
2	$64^n - 6 \cdot 12^n - 20 \cdot 10^n + 60 \cdot 9^n - 65 \cdot 8^n + 30 \cdot 7^n$	6	97	0.14 s
3	$64^n - 7^n - 15 \cdot 4^n + 15 \cdot 3^n$	4	19	0.02 s
4	$64^n - 6 \cdot 2^n + 5$	3	6	0.01 s
5	$64^n - 1$	2	1	0.00 s
2 7 0	$128^n - 1422564 \cdot 64^n + 10552451 \cdot 63^n - 35759745 \cdot 62^n + 73300045 \cdot 61^n - 101419640 \cdot 60^n + 100106307 \cdot 59^n - 72673160 \cdot 58^n + 39473100 \cdot 57^n - 16196040 \cdot 56^n + 5049870 \cdot 55^n - 1202509 \cdot 54^n + 219492 \cdot 53^n - 30520 \cdot 52^n + 3115 \cdot 51^n - 210 \cdot 50^n + 7 \cdot 49^n$	17	1,422,564	54 m 44 s
1	$128^n - 7 \cdot 44^n - 175 \cdot 42^n - 490 \cdot 41^n - 525 \cdot 40^n + 4235 \cdot 39^n + 5285 \cdot 38^n - 39025 \cdot 37^n + 78300 \cdot 36^n - 94815 \cdot 35^n + 78729 \cdot 34^n - 44821 \cdot 33^n + 16254 \cdot 32^n - 3199 \cdot 31^n + 315 \cdot 30^n - 83 \cdot 29^n + 21 \cdot 28^n$	17	164,289	5 m 28 s
2	$128^n - 29^n - 56 \cdot 24^n + 35 \cdot 23^n - 210 \cdot 22^n + 630 \cdot 21^n - 665 \cdot 20^n + 546 \cdot 19^n - 455 \cdot 18^n + 280 \cdot 17^n - 245 \cdot 16^n + 175 \cdot 15^n - 35 \cdot 14^n$	13	2,971	4.94 s
3	$128^n - 7 \cdot 14^n - 35 \cdot 11^n + 105 \cdot 10^n - 84 \cdot 9^n - 15 \cdot 8^n + 35 \cdot 7^n$	7	200	0.29 s
4	$128^n - 8^n - 21 \cdot 4^n + 21 \cdot 3^n$	4	26	0.04 s
5	$128^n - 7 \cdot 2^n + 6$	3	7	0.02 s
6	$128^n - 1$	2	1	0.01 s

Table 13: Probabilities of perfect ($i = 0$) and i-imperfect column pairs ($k = 2$) in Bernoulli ($m \times n$)-matrices computed by the Algebraic method (with exponential polynomials)

k	m	i	Width n of Bernoulli $(m \times n)$-matrix							
			2	3	4	5	6	7	8	9
2	1	0	.75000000	.87500000	.93750000	.96875000	.98437500	.99218750	.99609375	.99804688
2	2	0	.56250000	.76562500	.87890625	.93847656	.96899414	.98443604	.99220276	.99609756
		1	.93750000	.98437500	.99609375	.99902344	.99975586	.99993896	.99998474	.99999619
2	3	0	.42187500	.65820313	.80932617	.89724731	.94584274	.97187853	.98554820	.99262745
		1	.84375000	.95703125	.98876953	.99713135	.99927521	.99981785	.99995434	.99998857
		2	.98437500	.99804688	.99975586	.99996948	.99999619	.99999952	.99999994	.99999999
2	4	0	.31640625	.55395508	.72706604	.83994007	.90894467	.94937097	.97234511	.98510658
		1	.73828125	.91528320	.97444153	.99255085	.99786454	.99939163	.99982666	.99995041
		2	.94921875	.99291992	.99906921	.99988079	.99998492	.99999810	.99999976	.99999997
		3	.99609375	.99975586	.99998474	.99999905	.99999994	1.00000000	1.00000000	1.00000000
2	5	0	.23730469	.45706177	.63682079	.76744083	.85591626	.91303641	.94862327	.97018599
		1	.63281250	.85913086	.94991684	.98295021	.99435489	.99816639	.99941260	.99981375
		2	.89648438	.98211670	.99709511	.99953550	.99992519	.99998772	.99999794	.99999964
		3	.98437500	.99890137	.99992752	.99999535	.99999971	.99999998	1.00000000	1.00000000
		4	.99902344	.99996948	.99999905	.99999997	1.00000000	1.00000000	1.00000000	1.00000000
2	6	0	.17797852	.37074661	.54485947	.68415703	.78781240	.86117292	.91116768	.94423140
		1	.53393555	.79124832	.91349691	.96588820	.98702473	.99520175	.99826584	.99938509
		2	.83056641	.96331024	.99255145	.99853246	.99971567	.99994552	.99998965	.99999804
		3	.96240234	.99657440	.99970043	.99997344	.99999755	.99999976	.99999998	1.00000000
		4	.99536133	.99983597	.99999458	.99999983	.99999999	1.00000000	1.00000000	1.00000000
		5	.99975586	.99999619	.99999994	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
2	7	0	.13348389	.29650545	.45663340	.59582070	.70829581	.79469930	.85858020	.90438661
		1	.44494629	.71559715	.86513076	.93932233	.97377519	.98902647	.99552985	.99821987
		2	.75640869	.93506479	.98387891	.99617276	.99912165	.99980385	.99995717	.99999082
		3	.92944336	.99155807	.99903189	.99989043	.99998768	.99999862	.99999985	.99999998
		4	.98712158	.99938536	.99997105	.99999857	.99999992	1.00000000	1.00000000	1.00000000
		5	.99865723	.99997616	.99999961	.99999999	1.00000000	1.00000000	1.00000000	1.00000000
		6	.99993896	.99999952	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000

3.5 Perfect column triplets in matrices with up to 3 rows

In the following sections 'perfect outcomes' refer to column *triplets*.

Perfect outcomes of 1-row Bernoulli matrix A perfect outcome requires at least two columns of type 2 form Table 1:

$$\text{Number of perfect outcomes} \;=\; \sum_{t_2>1} \frac{n!}{t_1!\,t_2!} \stackrel{(15)}{=} 2^n - 1 - n \;.$$

Perfect outcomes of 2-row Bernoulli matrix An outcome with multiple columns of type 4 from Table 2 is perfect, as well as an outcome with one column of type 4 and columns of types 3 and 2. An outcome with no columns of types 4 is not perfect. Hence,

$$\begin{aligned}
\text{Number of perfect outcomes} \;&=\; \sum_{t_4>1} \frac{n!}{t_{1:4}\,!} + \sum_{\substack{t_4\,=\,1\\ t_3\,>\,0\\ t_2\,>\,0}} \frac{n!}{t_{1:4}\,!} \\
&\stackrel{(15)}{=} 4^n - 3^n - n3^{n-1} + n \sum_{t_2 t_3>0} \frac{(n-1)!}{t_{1:3}\,!} \\
&\stackrel{(12)}{=} 4^n - 3^n - n3^{n-1} \quad + \quad n(3^{n-1} - 2\cdot 2^{n-1} + 1) \\
&= 4^n - 3^n - n2^n + n \;.
\end{aligned}$$

In a (im)perfect column triplet, the columns with the superior, intermediate, and inferior index are called, respectively, *pivot*, *sub-pivot*, and *match*. For example, in the perfect triplet $(6, 7, 8)$ of 3-columns from Table 3, column 8 is pivot, column 7 is sub-pivot, and column 6 is match.

An m-column is called *i-self-sufficient* if its two instances make a perfect triplet with any other m-column. For instance, the column of type 4 from Table 3 is 1-self-sufficient.

The next proposition follows from the way the column types are indexed.

Proposition 8 (Properties of perfect column triplets)

1. *The* 2^m*-th* m*-column makes a perfect triplet with any perfect pair of* m*-columns.*

2. *In a perfect triplet of* m*-columns, the pivot is from the fourth quarter of table* T_m*, and the sub-pivot is from the second half of* T_m*.*

3. *In a perfect triplet of* m*-columns, at least two columns are from even quarters of table* T_m *(2nd and 4th), at least two columns are from even eights of* T_m *(2nd, 4th, 6th, and 8th), ..., and at least two columns have even indices.*

4. *A perfect triplet of* m*-columns can have multiple instances of self-sufficient columns only.*

5. *Three* m*-columns make a perfect triplet if and only if any two of them make a perfect pair.*

Perfect outcomes of 3-row Bernoulli matrix Construct disjoint sets of perfect outcomes, starting from the maximal pivot 8 as shown by the scheme below:

$$
\begin{array}{llllll}
 & \text{Certain event: all combinations of } t_1,\ldots,t_8 \text{ such that } \sum_{i=1}^{8} = 3 & & & & \\
\swarrow & \downarrow & & & & \searrow \\
t_8 > 1 & t_8 = 1 & & & & t_8 = 0 \\
 & \downarrow & \searrow & & & \downarrow \\
 & t_7 > 0 & t_7 = 0 & & & t_7 > 0 \\
 & t_2 + t_4 + t_6 > 0 & & & & t_6 > 0 \\
 & & & & & t_5 > 0 \\
 & & \downarrow & \searrow & & \text{No more pivots; stop} \\
 & & t_6 > 0 & t_6 = 0 & & \\
 & & t_3 + t_4 > 0 & & & \\
 & & & \downarrow & & \\
 & & & t_5 > 0 & & \\
 & & & t_4 > 0 & &
\end{array}
$$

Here, the first event is $t_8 > 1$. By Item 1 of Proposition 8, all outcomes with multiple column 8 are perfect. After excluding this event, consider outcomes with its single instance $t_8 = 1$ as pivot. By Item 5 of Proposition 8, the sub-pivot and match must make a perfect pair. Therefore, this branch of the scheme is exactly as the branch $t_8 = 0$ in (25). The right-hand branch with no column 8 contains no self-sufficient column and consists of a single event with three different column types 7, 6, and 5. Hence,

$$
\begin{aligned}
&\text{Number of perfect outcomes} = \\
&= \sum_{t_8 \geq 2} \frac{n!}{t_{1:8}\,!} + \sum_{\substack{t_8 = 1 \\ t_7 > 0 \\ t_2 + t_4 + t_6 > 0}} \frac{n!}{t_{1:8}\,!} + \sum_{\substack{t_8 = 1 \\ t_7 = 0 \\ t_6 > 0 \\ t_3 + t_4 > 0}} \frac{n!}{t_{1:8}\,!} + \sum_{\substack{t_8 = 1 \\ t_7, t_6 = 0 \\ t_4 t_5 > 0}} \frac{n!}{t_{1:8}\,!} + \sum_{\substack{t_8 = 0 \\ t_7 t_6 t_4 > 0}} \frac{n!}{t_{1:8}\,!} \\
&\overset{(17)}{=} 8^n - 7^n - n7^{n-1} \quad + \quad n(7^{n-1} - 6^{n-1} - 4^{n-1} + 3^{n-1}) \\
&\quad + n(6^{n-1} - 5^{n-1} - 4^{n-1} + 3^{n-1}) \quad + \quad n(5^{n-1} - 2 \cdot 4^{n-1} + 3^{n-1}) \\
&\quad + 7^n - 3 \cdot 6^n + 3 \cdot 5^n - 4^n \\
&= 8^n - 3 \cdot 6^n + 3 \cdot 5^n - 4^n - 4n4^{n-1} + 3n3^{n-1} \quad .
\end{aligned}
$$

3.6 Perfect and imperfect column triplets in the general case

Constructing disjoint events for perfect and imperfect column triplets resembles that for column pairs. An important distinction is the existence of *jointly i-self-sufficient column pairs*. It means that two different columns make a i-imperfect column triplet with any other column. For instance, 3-columns 8 and 7 are not only self-sufficient but also jointly 1-self-sufficient.

As a consequence, the self-calling module of the branching algorithm has two blocks. One block deals with events with jointly self-sufficient column pairs. These are cases like for column pairs with dichotomous branching. After these events are exhausted, the second block deals with properly trinomial cases with pivots, sub-pivots, and match, resulting in quarichotomous branching. We exemplify all of these with counting 1-imperfect outcomes of tree-row Bernoulli matrices.

First of all compile the list of (i-)*Self-sufficient columns*, *Lookup table for single match* with sub-pivots making jointly self-sufficient pairs with a given pivot, and *Lookup table for double*

Table 14: Lookup table for single match ($k = 3,\ m = 3,\ i = 1$). Columns in frames are self-sufficient

Pivot	Sub-pivots making with pivot jointly 1-self-sufficient pairs
$\boxed{8}$	$\boxed{4}\ \boxed{6}\ \boxed{7}$

Table 15: Lookup table for double match ($k = 3,\ m = 3,\ i = 1$). Columns in frames are self-sufficient. Pivots and subpivots underlined are jointly self-sufficient

Pivot	Sub-pivot	Match
$\boxed{8}$	$\boxed{7}$	1 2 3 $\boxed{4}$ 5 $\boxed{6}$
$\boxed{8}$	$\boxed{6}$	1 2 3 $\boxed{4}$ 5
$\boxed{8}$	5	2 3 $\boxed{4}$
$\boxed{8}$	$\boxed{4}$	1 2 3
$\boxed{8}$	3	2
$\boxed{7}$	$\boxed{6}$	2 3 $\boxed{4}$
$\boxed{7}$	5	3 $\boxed{4}$
$\boxed{7}$	$\boxed{4}$	2
$\boxed{6}$	5	2 $\boxed{4}$
$\boxed{6}$	$\boxed{4}$	3
$\boxed{4}$	3	2

match with matching columns to a given pair (pivot, sub-pivot); see Tables 14 and 15. Next, initialize the algorithm with the following three elements.

- List *Available columns* for the column indices for selection of pivot, sub-pivot, and match. For column triplets, it is initialized with *all(!)* column indices. In our example with 3-columns

$$Available\ columns = [1, \ldots, 8]\ .$$

- *Polynomial* to count i-imperfect outcomes. It is initialized with the exponential polynomial for the event containing multiple instances of one of S self-sufficient columns. The latter have i or fewer zeros out of m elements, implying $S = \sum_{j=0}^{i} \binom{m}{j}$. By virtue of (16) the initialization looks as follows

$$Polynomial \quad = \quad \sum_{\text{One of } t_1, \ldots, t_S > 1} \frac{n!}{t_{1:2^m}!} = 2^{mn} - \sum_{s=0}^{S} \binom{S}{s} n^{\underline{s}} \left(2^m - S\right)^{n-s}\ ,$$

  which for our example with $m = 3,\ i = 0$, implying $S = 1$, gives

$$\sum_{t_8 > 1} \frac{n!}{t_{1:8}!} \overset{(11)}{=} 8^n - 7^n - n7^{n-1}\ .$$

- List *Obligatory columns* for the column indices which can be required to be retained. It is initialized with the empty list

$$Obligatory\ columns = [\,]\ .$$

The further operation is performed by a self-calling module (the nested calls are shown in **boldface**). Since *Polynomial* is initialized for the event with multiple self-sufficient columns, other encountering disjoint events have *single instances of self-sufficient pivots, sub-pivots, and matching columns.*

Self-calling module for enumerating disjoint events with perfect column triplets

- *(Input).* Input the lists *Available columns* and *Obligatory columns*.
- *(Preparing alternative blocks with alternative definitions of the next event)*
  - Find maximal pivot, maximal sub-pivot, and match from *Available columns*. If no triplet is found then there is no jointly self-sufficient column pair either, meaning that the branch is exhausted. Exit the module (return).

    In our example, the maximal pivot is column 8, maximal sub-pivot is column 7, and columns $1, \ldots, 6$ constitute the match.
  - Find maximal S-pivot and its S-sub-pivots from *Available columns*, which make jointly i-self-sufficient pairs.

    In our example, the maximal S-pivot is column 8, and columns $4, 6, 7$ make jointly with S-pivot i-self-sufficient pairs.
- *(1st Block—Jointly i-self-sufficient pivot and sub-pivots: If the S-pivot exists and its index is not inferior than that of the pivot found, then define the event with the S-pivot and S-sub-pivots regardless of match)*

  Define the new event with inequality/equality constraints, taking into account the membership of the S-pivot and S-sub-pivots in the lists *Self-sufficient columns* and *Obligatory columns*, as well as the number of other obligatory and available columns, including self-sufficient among them. All the columns from *Self-sufficient columns* must be additionally constrained to at most one instance (since the event with their multiple instances is considered while initializing *Polynomial*). If S-pivot belongs to *Obligatory columns*, no constraint to guarantee its inclusion is needed. Similarly, if one of S-sub-pivots belongs to *Obligatory columns*, no constraint to guarantee the inclusion of any S-sub-pivot is needed either. Using (17), express the multinomial sum corresponding to the new event by an exponential polynomial and add it to *Polynomial*.

  In our example, the first encountering event results in the exponential polynomial

$$\sum_{\substack{t_8 = 1 \\ t_4 + t_6 + t_7 > 0 \\ t_4, t_6, t_7 \leq 1}} \frac{n!}{t_{1:8}} \overset{(17)}{=} 3n^{\underline{2}} + 3n^{\underline{3}} + n^{\underline{4}} - 3 \ .$$

  - *(Branch 1: S-pivot but no S-sub-pivots).* If no S-sub-pivot belongs to *Obligatory columns* then exclude all S-sub-pivots from *Available columns*, add the S-pivot to *Obligatory columns*, and start a **new instance of the module**. Otherwise, if any of S-sub-pivots belongs to *Obligatory columns*, do nothing and just go to the next branch.
  - *(Branch 2: No S-pivot).* If the S-pivot does not belong to *Obligatory columns* then delete the S-pivot from *Available columns* and start a **new instance of the module**. Otherwise, if the S-pivot belongs to *Obligatory columns*, exit the module (return).

- *(2nd Block—Properly trinomial case: If there is no S-pivot, or its index is inferior than that of the pivot found, then define the event with the pivot, sub-pivot, and match)*

  Define the new event by inequality/equality constraints, taking into account the membership of the pivot, sub-pivot and matching columns in the lists *Self-sufficient columns* and *Obligatory columns*, as well as the number of other obligatory and available columns, including self-sufficient among them (to exclude multiple occurrence). All the columns from *Self-sufficient columns* must be additionally constrained to at most one instance (since the event with their multiple instances is considered while initializing *Polynomial*). If the pivot belongs to *Obligatory columns*, no constraint to guarantee its inclusion is needed. The same relates to the sub-pivot. If any of the matching columns belongs to *Obligatory columns*, no constraint to guarantee the inclusion of any matching column is needed either. Using (17), express the multinomial sum corresponding to the new event by an exponential polynomial and add it to *Polynomial*.

  – *(Branch 3: Pivot but no sub-pivot).* If the sub-pivot is not obligatory then exclude it from the list *Available columns*, add the pivot to *Obligatory columns*, and start a **new instance of the module**. Otherwise, if the sub-pivot is obligatory, do nothing and just go to the next branch.
  – *(Branch 4: Pivot and sub-pivot but no match).* If no matching column is obligatory then exclude all the matching columns from *Available columns*, add the pivot and sub-pivot to *Obligatory columns*, and start a **new instance of the module**. Otherwise, if any of the matching columns is obligatory, do nothing and just go to the next branch.
  – *(Branch 5: No pivot but sub-pivot).* If the pivot is not obligatory then exclude it from *Available columns*, add the sub-pivot to *Obligatory columns*, and start a **new instance of the module**. Otherwise, if the pivot is obligatory, do nothing and just go to the next branch.
  – *(Branch 6: Neither pivot nor sub-pivot).* If both pivot and sub-pivot are not obligatory then exclude them from *Available columns*, and start a **new instance of the module**. Otherwise, if either pivot or the sub-pivot is obligatory, exit the module (return).

Applying this algorithm to our example of Bernoulli $(3 \times n)$-matrices, we obtain:

$$\text{Number of 1-imperfect outcomes} = 8^n - 4^n - 6n3^{n-1} + 2n - 3n(n-1)2^{n-2} \ .$$

The exponential polynomials to count (im)perfect outcomes of Bernoulli matrices with up to seven rows and the output probabilities are shown in Tables 16 and 17. As one can see, the probabilities computed by the Geometric and Algebraic methods (from the common domains of Tables 9 and 17 are identical.

Table 16: Description of exponential polynomials for counting perfect ($i = 0$) and i-imperfect outcomes of Bernoulli ($m \times n$)-matrices for column triplets ($k = 3$)

k	m	i	Exponential polynomial (to compute probabilities multiply by 2^{-mn})	Number of terms in the exponential polynomial	Number of disjoint events constructed	Computer time
3	1	0	$2^n - 1 - n$	3	1	0.00 s
3	2	0	$4^n - 3^n - 2n2^{n-1} + n$	4	2	0.00 s
		1	$4^n - 1 - 3n - n^{\underline{2}}$	4	2	0.00 s
3	3	0	$8^n - 3 \cdot 6^n + 3 \cdot 5^n - 4^n - 4n4^{n-1} + 3n3^{n-1}$	6	5	0.01 s
		1	$8^n - 4^n - 6n3^{n-1} + 2n - 3n^{\underline{2}}2^{n-2}$	5	10	0.02 s
		2	$8^n - 1 - 7n - 6n^{\underline{2}} - n^{\underline{3}}$	5	6	0.01 s
3	4	0	$16^n - 10 \cdot 12^n + 15 \cdot 11^n - 6 \cdot 10^n - 12n8^{n-1} + 16n7^{n-1} - 6n6^{n-1} + n5^{n-1}$	8	18	0.03 s
		1	$16^n - 3 \cdot 9^n - 4 \cdot 8^n + 15 \cdot 7^n - 12 \cdot 6^n + 3 \cdot 5^n - 36n6^{n-1} + 43n5^{n-1} - 18n4^{n-1} + 6n3^{n-1} - 6n^{\underline{2}}4^{n-2}$	11	60	0.11 s
		2	$16^n - 5^n - 12n4^{n-1} - 6n3^{n-1} + 4n2^{n-1} + 3n - 24n^{\underline{2}}3^{n-2} - 3n^{\underline{2}} - 4n^{\underline{3}}2^{n-3}$	9	68	0.14 s
		3	$16^n - 1 - 15n - 25n^{\underline{2}} - 10n^{\underline{3}} - n^{\underline{4}}$	6	20	0.07 s
3	5	0	$32^n - 26^n - 50 \cdot 24^n + 120 \cdot 23^n - 100 \cdot 22^n + 35 \cdot 21^n - 5 \cdot 20^n - 81n16^{n-1} + 185n15^{n-1} - 150n14^{n-1} + 50n13^{n-1} - 5n12^{n-1}$	12	128	0.29 s
		1	$32^n - 30 \cdot 18^n - 10 \cdot 17^n + 134 \cdot 16^n - 120 \cdot 15^n + 15 \cdot 14^n + 10 \cdot 13^n - 40n13^{n-1} - 140n12^{n-1} + 350n11^{n-1} - 210n10^{n-1} + 20n9^{n-1} + 20n8^{n-1} - 10n7^{n-1} + 4n6^{n-1} - 10n^{\underline{2}}8^{n-2}$	16	664	1.43 s
		2	$32^n - 10 \cdot 12^n - 12 \cdot 11^n + 70 \cdot 10^n - 70 \cdot 9^n + 15 \cdot 8^n + 10 \cdot 7^n - 4 \cdot 6^n - 6n12^{n-1} + 21n11^{n-1} - 24n10^{n-1} - 114n9^{n-1} + 36n8^{n-1} + 237n7^{n-1} - 286n6^{n-1} + 150n5^{n-1} - 40n4^{n-1} + 10n3^{n-1} - 4n^{\underline{2}}11^{n-2} + 9n^{\underline{2}}10^{n-2} - 9n^{\underline{2}}9^{n-2} + 13n^{\underline{2}}8^{n-2} - 15n^{\underline{2}}7^{n-2} - 189n^{\underline{2}}6^{n-2} + 210n^{\underline{2}}5^{n-2} + \cdots$	34	967	2.13 s
		3	$32^n - 6^n - 20n5^{n-1} - 20n4^{n-1} + 10n2^{n-1} + 4n - 3n^{\underline{2}}6^{n-2} + 9n^{\underline{2}}5^{n-2} - 99n^{\underline{2}}4^{n-2} - 57n^{\underline{2}}3^{n-2} + 20n^{\underline{2}}2^{n-2} - 20n^{\underline{2}} - 3n^{\underline{3}}6^{n-3} + 6n^{\underline{3}}5^{n-3} - 3n^{\underline{3}}4^{n-3} - 80n^{\underline{3}}3^{n-3} - n^{\underline{4}}6^{n-4} + 5n^{\underline{4}}5^{n-4} - n^{\underline{4}}4^{n-4} - 3n^{\underline{4}}3^{n-4} - 5n^{\underline{4}}2^{n-4} + n^{\underline{5}}5^{n-5} - n^{\underline{5}}4^{n-5}$	23	476	1.24 s
		4	$32^n - 1 - 31n - 90n^{\underline{2}} - 65n^{\underline{3}} - 15n^{\underline{4}} - n^{\underline{5}}$	7	71	0.35 s
3	6	0	$64^n - 6 \cdot 52^n - 15 \cdot 51^n - 60 \cdot 50^n + 360 \cdot 49^n - 1006 \cdot 48^n + 1920 \cdot 47^n - 2175 \cdot 46^n + 1380 \cdot 45^n - 480 \cdot 44^n + 90 \cdot 43^n - 9 \cdot 42^n - 2646n32^{n-1} + 11276n31^{n-1} - 21120n30^{n-1} + 23360n29^{n-1} - 17340n28^{n-1} + 9210n27^{n-1} - 3555n26^{n-1} + 975n25^{n-1} - 180n24^{n-1} + 20n23^{n-1} - n22^{n-1}$	23	3,755	12.85 s
		1	$64^n - 42^n - 960 \cdot 37^n + 3555 \cdot 36^n - 6180 \cdot 35^n + 6570 \cdot 34^n - 3900 \cdot 33^n + 730 \cdot 32^n + 75 \cdot 31^n + 330 \cdot 30^n - 280 \cdot 29^n + 60 \cdot 28^n - n42^{n-1} + 2n41^{n-1} - 2n40^{n-1} + 3n39^{n-1} - 2n38^{n-1} + 2n36^{n-1} - 8n35^{n-1} + 10n34^{n-1} - 11n33^{n-1} + 18n32^{n-1} - 17n31^{n-1} + 7n30^{n-1} - n29^{n-1} + \cdots$	51	42,660	2 m 33 s
		2	$64^n - 15 \cdot 27^n - 30 \cdot 26^n - 45 \cdot 25^n - 610 \cdot 24^n + 2310 \cdot 23^n - 1923 \cdot 22^n - 315 \cdot 21^n + 495 \cdot 20^n + 525 \cdot 19^n - 510 \cdot 18^n + 132 \cdot 17^n - 15 \cdot 16^n - 15n31^{n-1} + 55n30^{n-1} - 88n29^{n-1} + 67n28^{n-1} + 25n27^{n-1} - 216n26^{n-1} + 515n25^{n-1} - 789n24^{n-1} + 906n23^{n-1} - 712n22^{n-1} + 256n21^{n-1} - 586n20^{n-1} + \cdots$	111	42,940	1 m 57 s
		3	$64^n - 10 \cdot 16^n - 15 \cdot 15^n - 180 \cdot 14^n + 960 \cdot 13^n - 1653 \cdot 12^n + 1350 \cdot 11^n - 525 \cdot 10^n + 60 \cdot 9^n + 15 \cdot 8^n - 3 \cdot 7^n - 22n18^{n-1} + 68n17^{n-1} - 102n16^{n-1} + 147n15^{n-1} - 180n14^{n-1} + 175n13^{n-1} - 1317n12^{n-1} + 1553n11^{n-1} + 1304n10^{n-1} - 3299n9^{n-1} + 1688n8^{n-1} + 453n7^{n-1} - 810n6^{n-1} + 360n5^{n-1} + \cdots$	107	23,068	56.65 s

k m i	Exponential polynomial (to compute probabilities multiply by 2^{-mn})	Number of terms in the exponential polynomial	Number of disjoint events constructed	Computer time
3 6 4	$64^n - 7^n - 30n6^{n-1} - 45n5^{n-1} - 20n4^{n-1} + 15n3^{n-1} + 18n2^{n-1} + 5n - 31n^{\underline{2}}7^{n-2} + 87n^{\underline{2}}6^{n-2} - 315n^{\underline{2}}5^{n-2} - 347n^{\underline{2}}4^{n-2} - 39n^{\underline{2}}3^{n-2} + 60n^{\underline{2}}2^{n-2} - 85n^{\underline{2}} - 117n^{\underline{3}}7^{n-3} + 242n^{\underline{3}}6^{n-3} - 131n^{\underline{3}}5^{n-3} - 531n^{\underline{3}}4^{n-3} - 368n^{\underline{3}}3^{n-3} + 65n^{\underline{3}}2^{n-3} + 15n^{\underline{3}} - 201n^{\underline{4}}7^{n-4} + 482n^{\underline{4}}6^{n-4} - 275n^{\underline{4}}5^{n-4} + \cdots$	59	3,880	11.70 s
5	$64^n - 1 - 63n - 301n^{\underline{2}} - 350n^{\underline{3}} - 140n^{\underline{4}} - 21n^{\underline{5}} - n^{\underline{6}} - 5n^{\underline{19}} + 8n^{\underline{20}} - 4n^{\underline{21}} - 12n^{\underline{22}} - 12n^{\underline{23}} - 36n^{\underline{24}} + 37n^{\underline{25}} + 64n^{\underline{26}} + 100n^{\underline{27}} + 4n^{\underline{28}} + 56n^{\underline{29}} + 152n^{\underline{30}} + 156n^{\underline{31}} + 695n^{\underline{32}} + 123n^{\underline{33}} + 98n^{\underline{34}} - 95n^{\underline{35}} + \cdots$	35	273	1.78 s
3 7 0	$128^n - 7\cdot 105^n - 56\cdot 104^n - 210\cdot 103^n - 1715\cdot 102^n + 7420\cdot 101^n - 8400\cdot 100^n + 5060\cdot 99^n - 9730\cdot 98^n + 19005\cdot 97^n - 46851\cdot 96^n + 139244\cdot 95^n - 274715\cdot 94^n + 347480\cdot 93^n - 302645\cdot 92^n + 191730\cdot 91^n - 90636\cdot 90^n + 31850\cdot 89^n - 8085\cdot 88^n + 1400\cdot 87^n - 147\cdot 86^n + 7\cdot 85^n - 1422564n64^{n-1} + 10552451n63^{n-1} - 35759745n62^{n-1} + \cdots$	38	1,612,114	2 h 59 m
1	$128^n - 28\cdot 84^n - 35\cdot 81^n - 840\cdot 80^n - 31752\cdot 79^n + 121800\cdot 78^n - 70560\cdot 77^n - 184590\cdot 76^n - 13720\cdot 75^n + 819924\cdot 74^n - 971145\cdot 73^n + 343210\cdot 72^n - 781980\cdot 71^n + 2428839\cdot 70^n - 3159156\cdot 69^n + 2243850\cdot 68^n - 950145\cdot 67^n + 216545\cdot 66^n + 11655\cdot 65^n - 32933\cdot 64^n + 13230\cdot 63^n - 1680\cdot 62^n - 1015\cdot 61^n + 630\cdot 60^n - 105\cdot 59^n + \cdots$	122	123,475,963	16 d 10 h
2	$128^n - 64^n - 70\cdot 57^n - 18410\cdot 56^n + 97860\cdot 55^n - 240240\cdot 54^n + 334320\cdot 53^n - 271950\cdot 52^n + 175980\cdot 51^n - 213780\cdot 50^n + 235830\cdot 49^n - 46620\cdot 48^n - 164990\cdot 47^n + 133350\cdot 46^n + 35735\cdot 45^n - 112392\cdot 44^n + 85470\cdot 43^n - 44450\cdot 42^n + 21945\cdot 41^n - 11900\cdot 40^n + 6517\cdot 39^n - 3080\cdot 38^n + 1050\cdot 37^n - 175\cdot 36^n - 257n73^{n-1} + \cdots$	324	101,382,529	10 d 23 h
3	$128^n - 135\cdot 36^n - 1155\cdot 34^n - 8232\cdot 33^n + 12425\cdot 32^n + 58940\cdot 31^n - 113190\cdot 30^n - 43290\cdot 29^n + 267414\cdot 28^n - 275415\cdot 27^n + 112350\cdot 26^n + 20580\cdot 25^n - 56322\cdot 24^n + 35518\cdot 23^n - 10875\cdot 22^n + 1470\cdot 21^n - 105\cdot 20^n + 21\cdot 19^n - 1023n49^{n-1} + 6612n48^{n-1} - 19690n47^{n-1} + 35964n46^{n-1} - 45338n45^{n-1} + 41824n44^{n-1} - 28607n43^{n-1} + \cdots$	446	7,796,210	8 h 16 m
4	$128^n - 35\cdot 20^n - 651\cdot 18^n + 1540\cdot 17^n + 3885\cdot 16^n - 17790\cdot 15^n + 26096\cdot 14^n - 18018\cdot 13^n + 3990\cdot 12^n + 2555\cdot 11^n - 2100\cdot 10^n + 588\cdot 9^n - 61\cdot 8^n - 197n24^{n-1} + 788n23^{n-1} - 1378n22^{n-1} + 1395n21^{n-1} - 886n20^{n-1} + 531n19^{n-1} - 337n18^{n-1} - 554n17^{n-1} - 677n16^{n-1} - 6115n15^{n-1} + 12861n14^{n-1} + 31608n13^{n-1} + \cdots$	372	965,160	47 m 25 s
5	$128^n - 8^n - 42n7^{n-1} - 84n6^{n-1} - 70n5^{n-1} + 42n3^{n-1} + 28n2^{n-1} + 6n - 196n^{\underline{2}}8^{n-2} + 523n^{\underline{2}}7^{n-2} - 927n^{\underline{2}}6^{n-2} - 1232n^{\underline{2}}5^{n-2} - 802n^{\underline{2}}4^{n-2} + 219n^{\underline{2}}3^{n-2} + 56n^{\underline{2}}2^{n-2} - 294n^{\underline{2}} - 1622n^{\underline{3}}8^{n-3} + 3255n^{\underline{3}}7^{n-3} - 1619n^{\underline{3}}6^{n-3} - 2174n^{\underline{3}}5^{n-3} - 3936n^{\underline{3}}4^{n-3} - 583n^{\underline{3}}3^{n-3} + 239n^{\underline{3}}2^{n-3} + 210n^{\underline{3}} - 6251n^{\underline{4}}8^{n-4} + \cdots$	190	40,584	2 m 17 s
6	$128^n - 1 - 127n - 966n^{\underline{2}} - 1701n^{\underline{3}} - 1050n^{\underline{4}} - 266n^{\underline{5}} - 28n^{\underline{6}} - n^{\underline{7}} - 4n^{\underline{12}} - 16n^{\underline{13}} + 12n^{\underline{14}} - 1824n^{\underline{15}} + 6964n^{\underline{16}} + 41632n^{\underline{17}} - 2299352n^{\underline{18}} + 4801438n^{\underline{19}} + 47450244n^{\underline{20}} + 131904732n^{\underline{21}} + 546358380n^{\underline{22}} - 5445507580n^{\underline{23}} - 39308717764n^{\underline{24}} - 4147411291n^{\underline{25}} + 200995172416n^{\underline{26}} + 112007060452n^{\underline{27}} + \cdots$	113	1,149	9.41 s

Table 17: Probabilities of perfect ($i = 0$) and i-imperfect column triplets ($k = 3$) in Bernoulli ($m \times n$)-matrices computed by the Algebraic method (with exponential polynomials)

k m i	Width n of Bernoulli ($m \times n$)-matrix							
	3	4	5	6	7	8	9	10
3 1 0	.50000000	.68750000	.81250000	.89062500	.93750000	.96484375	.98046875	.98925781
3 2 0	.25000000	.44921875	.61132813	.72973633	.81225586	.86875916	.90737152	.93393040
1	.75000000	.90234375	.96484375	.98803711	.99609375	.99876404	.99961853	.99988461
3 3 0	.12500000	.27514648	.42376709	.55218887	.65610695	.73748273	.80019028	.84815205
1	.50000000	.74609375	.88024902	.94555664	.97567177	.98919678	.99520132	.99785921
2	.87500000	.96948242	.99340820	.99869156	.99975586	.99995655	.99999255	.99999876
3 4 0	.06250000	.15995789	.27330971	.38646346	.49073754	.58249769	.66097559	.72683604
1	.31250000	.56596375	.74513435	.85620672	.92062546	.95666145	.97643659	.98719067
2	.68750000	.89280701	.96704102	.99048918	.99735689	.99928118	.99980669	.99994823
3	.93750000	.99046326	.99876404	.99985689	.99998474	.99999847	.99999985	.99999999
3 5 0	.03125000	.08944607	.16627449	.25237670	.34059513	.42613960	.50611353	.57896691
1	.18750000	.39891815	.58403063	.72412922	.82214329	.88755600	.92991492	.95679268
2	.50000000	.76921082	.90341794	.96181602	.98540344	.99452482	.99796478	.99924523
3	.81250000	.95708466	.99152851	.99847241	.99973840	.99995606	.99999253	.99999867
4	.96875000	.99701977	.99976826	.99998435	.99999905	.99999995	1.00000000	1.00000000
3 6 0	.01562500	.04860848	.09663510	.15557551	.22129020	.29024047	.35964297	.42742799
1	.10937500	.26506132	.42682210	.57051109	.68746276	.77756953	.84445699	.89279104
2	.34375000	.61914063	.79807311	.89882601	.95117467	.97704082	.98940236	.99517487
3	.65625000	.88675117	.96696451	.99105526	.99768827	.99941821	.99985497	.99996358
4	.89062500	.98342246	.99792156	.99976830	.99997558	.99999736	.99999967	.99999995
5	.98437500	.99906868	.99995655	.99999829	.99999994	1.00000000	1.00000000	1.00000000
3 7 0	.00781250	.02586606	.05421108	.09154705	.13604801	.18578767	.23895803	.29396719
1	.06250000	.16808704	.29376464	.42062329	.53723984	.63820408	.72205837	.78960331
2	.22656250	.46849843	.66322248	.79787613	.88326472	.93448100	.96403628	.98061241
3	.50000000	.77972507	.91295352	.96788341	.98868921	.99614568	.99871750	.99958047
4	.77343750	.94755490	.98950133	.99808440	.99967079	.99994523	.99999089	.99999841
5	.93750000	.99376014	.99950709	.99996632	.99999782	.99999984	.99999998	1.00000000
6	.99218750	.99970896	.99999185	.99999981	1.00000000	1.00000000	1.00000000	1.00000000

3.7 i-imperfect outcomes with i close to m

Since any pair or triplet of columns of Bernoulli ($m \times n$)-matrix is m-imperfect

$$\text{Number of } m\text{-imperfect outcomes} = 2^{mn} \ .$$

Column pairs A $(m-1)$-imperfect outcome requires at least one occurrence of 1 among all mn elements of the matrix. Hence,

$$\text{Number of } (m-1)\text{-imperfect outcomes} = 2^{mn} - 1 \ .$$

To guarantee a $(m-2)$-imperfect outcome, there should be two rows with 1s. Excluding the event with a single row with 1s and the event with no 1s, obtain

$$\begin{aligned}\text{Number of } (m-2)\text{-imperfect outcomes} &= \\ &= 2^{mn} - \binom{m}{m-1} \underbrace{(2^n - 1)}_{\substack{\text{One row}\\ \text{has 1s}}} \cdot \underbrace{1}_{\substack{\text{All other}\\ \text{rows} \equiv 0}} - 1 \\ &= 2^{mn} - m2^n + m - 1 \ .\end{aligned}$$

To have a $(m-3)$-imperfect outcome, there should be a column with two 1s and, additionally, a row with at least one 1.

$$\text{Number of } (m-3)\text{-imperfect outcomes} =$$

$$= 2^{mn} - \underbrace{(m+1)^n}_{\substack{\text{Every } m\text{-column has} \\ \text{at most one 1}}} - \binom{m}{2} \underbrace{(4^n - 3^n)}_{\substack{\text{Two rows} \\ \text{have column} \\ \text{with two 1s}}} \cdot \underbrace{1}_{\substack{\text{All other} \\ \text{rows} \equiv 0}}$$

$$= 2^{mn} - (m+1)^n - \frac{m(m-1)}{2}(4^n - 3^n) \ .$$

Column triplets A $(m-1)$-imperfect outcome requires at least one out of m rows of the matrix with two 1s. Hence,

$$\text{Number of } (m-1)\text{-imperfect outcomes} = 2^{mn} - \underbrace{(1+n)^m}_{\substack{\text{Every row} \\ \text{has no or} \\ \text{single 1}}} \ .$$

3.8 Summary

After the exponential polynomials have been derived, the probabilities of perfect and i-imperfect column pairs and triplets can be almost immediately computed for Bernoulli matrices with a quite large number n of columns. Therefore, the application of the method is not restricted by the width of Bernoulli matrix.

At the same time, the Algebraic method is restricted to Bernoulli matrices with a rather small number m of rows. The number of disjoint events and the length of lists like *Available columns* transmitted from one event to another grows exponentially as m increases, slowing the computer performance down. The computer limits are clearly seen for column triplets. As follows from Table 17, the final exponential polynomial for counting outcomes of 7-column Bernoulli matrices with 1-imperfect column triplets requires constructing over 123 million events/polynomials, which takes 16 days.

Besides, the accuracy of large binomial coefficients and falling factorials in the encountering exponential polynomials is restricted by the computer 64-bit arithmetics. The inaccuracies are accumulated and can result in erroneous output probabilities.

4 Probabilistic method

4.1 Inclusion-Exclusion principle

This chapter deals exclusively with *perfect* column pairs and triplets.

Label column k-tuples in a Bernoulli $(m \times n)$-matrix with k-tuples of corresponding column numbers $J = \{j_1, \ldots, j_k\}$ (not the column type indices!). Enumerate these k-tuples and identify J's with their numbers $1, 2, \ldots$. Denote by A_J the event that the Jth k-tuple is perfect. The probability of occurrence of a perfect k-tuple is expressed by the Inclusion–Exclusion Principle (Feller 1968, p. 99, and Held 1997, p. 42, 55–57); see also Inclusion-Exclusion Principle (2012):

$$\begin{aligned} \mathsf{P}(\bigcup_J A_J) &= \sum_J \mathsf{P}(A_J) - \sum_{J_1<J_2} \mathsf{P}(A_{J_1} \cap A_{J_2}) + \sum_{J_1<J_2<J_3} \mathsf{P}(A_{J_1} \cap A_{J_2} \cap A_{J_3}) - \ldots \\ &= \sum_S (-1)^{S-1} \sum_{J_1<J_2<\ldots<J_S} \mathsf{P}\left(A_{J_1} \cap A_{J_2} \cap \ldots \cap A_{J_S}\right) \qquad (26) \\ &= \sum_S (-1)^{S-1} \frac{1}{S!} \sum_{J_1\neq J_2\neq\ldots\neq J_S} \mathsf{P}\left(A_{J_1} \cap A_{J_2} \cap \ldots \cap A_{J_S}\right) \ , \qquad (27) \end{aligned}$$

where $J_1 \neq J_2 \neq \ldots \neq J_S$ means the pairwise difference. In (27), unlike (26), each non-ordered combination of S events A_J is included with its all $S!$ permutations.

4.2 First two sums in the Inclusion-Exclusion formula

First sum for column pairs Consider a column pair in Bernoulli $(m \times n)$-matrix and denote its sum along rows by m-vector $\mathbf{u} = \{u_i\}$. By independence, the four equiprobable outcomes of every element u_i of this random vector are

$$0+0=0 \qquad 0+1=1 \qquad 1+0=1 \qquad 1+1=2 \ ,$$

implying

$$\mathsf{P}(u_i \geq 1) = 3/4 \quad \text{for every} \quad i = 1, \ldots, m \ , \qquad (28)$$

whence $\mathsf{P}(\mathbf{u} \geq 1) = \left(\frac{3}{4}\right)^m$. Taking into account the number of ways two columns can be chosen from n, obtain

$$\sum_J \mathsf{P}(A_J) = \frac{n!}{2!\,(n-2)!} \left(\frac{3}{4}\right)^m .$$

Second sum for column pairs Let two pairs of m-columns be numbered 1 and 2. Find the probability that both pairs are perfect, that is, $\mathsf{P}(\mathbf{u}, \mathbf{v} \geq 1)$, where $\mathbf{u} = \{u_i\}$ is the sum of two vectors from pair 1, and $\mathbf{v} = \{v_i\}$ is the sum of two vectors from pair 2. For this purpose, define the *intersection profile* of column pairs 1 and 2 to be the sequence $\{x_1, x_2, x_{12}\}$, where

x_1 is the number of columns belonging to pair 1 but not to pair 2,

x_2 is the number of columns belonging to pair 2 but not to pair 1, and

x_{12} is the number of columns belonging simultaneously to pairs 1 and 2.

For two *different* column pairs, there can be two intersection profiles:

1. Two disjoint column pairs: $\{x_1, x_2, x_{12}\} = \{2, 2, 0\}$. By independence,

$$\mathsf{P}(u_i, v_i \geq 1) = \frac{3}{4} \cdot \frac{3}{4} = \frac{9}{16} \quad \text{for every} \quad i = 1, \ldots, m \;,$$
$$\mathsf{P}(\mathbf{u}, \mathbf{v} \geq 1) = \left(\frac{9}{16}\right)^m .$$

Taking into account the number of ways the given intersection profile can be chosen from n elements and the fact that two permutated column pairs (J_1, J_2) and (J_2, J_1) have the same intersection profile, obtain

$$\sum_{J_1,\, J_2\,:\,\text{disjoint}} \mathsf{P}(A_{J_1} \cap A_{J_2}) = \frac{n!}{2!\,2!\,0!\,(n-4)!} \cdot \left(\frac{9}{16}\right)^m .$$

2. Two column pairs with one common column: $\{x_1, x_2, x_{12}\} = \{1, 1, 1\}$. We have

$$\mathsf{P}(u_i, v_i \geq 1) = \underbrace{\frac{1}{2}}_{\substack{\text{Probability}\\ \text{that the}\\ \text{common}\\ \text{element}\\ =1}} + \underbrace{\frac{1}{2}}_{\substack{\text{Probability}\\ \text{that the}\\ \text{common}\\ \text{element}\\ =0}} \cdot \underbrace{\frac{1}{2}}_{\substack{\text{Probability}\\ \text{that another}\\ \text{element}\\ \text{of pair 1}\\ =1}} \cdot \underbrace{\frac{1}{2}}_{\substack{\text{Probability}\\ \text{that another}\\ \text{element}\\ \text{of pair 2}\\ =1}}$$
$$= \frac{5}{8} \quad \text{for every} \quad i = 1, \ldots, m \;,$$
$$\mathsf{P}(\mathbf{u}, \mathbf{v} \geq 1) = \left(\frac{5}{8}\right)^m .$$

Taking into account the number of ways the given intersection profile can be chosen from n elements, obtain

$$\sum_{J_1,\, J_2\,:\,\text{have one common column}} \mathsf{P}(A_{J_1} \cap A_{J_2}) = \frac{n!}{1!\,1!\,1!\,(n-3)!} \left(\frac{5}{8}\right)^m .$$

Thus, we have the second sum for (27)

$$\sum_{J_1 \neq J_2} \mathsf{P}(A_{J_1} \cap A_{J_2}) = \frac{n!}{2!\,2!\,(n-4)!} \left(\frac{9}{16}\right)^m + \frac{n!}{(n-3)!} \left(\frac{5}{8}\right)^m \;,$$

where the terms with factorials of negative integers (if $n < 4$) should be omitted.

First sum for column triplets Consider a column triplet in Bernoulli $(m \times n)$-matrix and denote its sum along rows by m-vector $\mathbf{u} = \{u_i\}$. By independence of elements, the eight equiprobable outcomes for every element u_i are

$$\begin{array}{cccc} 0+0+0=0 & 0+0+1=1 & 0+1+0=1 & 1+0+0=1 \\ 0+1+1=2 & 1+0+1=2 & 1+1+0=2 & 1+1+1=3 \;, \end{array}$$

implying $\mathsf{P}(u_i \geq 3/2) = 1/2$, whence $\mathsf{P}(\mathbf{u} \geq 3/2) = \left(\frac{1}{2}\right)^m$. Taking into account the number of ways three columns can be chosen from n, obtain

$$\sum_J \mathsf{P}(A_J) = \frac{n!}{3!\,(n-3)!} \left(\frac{1}{2}\right)^m .$$

Second sum for column triplets Let two *different* triplets of m-columns be numbered 1 and 2. Find the probability that both triplets are perfect, that is, $\mathsf{P}(\mathbf{u}, \mathbf{v} \geq 3/2)$, where $\mathbf{u} = \{u_i\}$ is the sum of three vectors from pair 1, and $\mathbf{v} = \{v_i\}$ is the sum of three vectors from pair 2. As previously, consider possible intersection profiles.

1. Two disjoint column triplets: $\{x_1, x_2, x_{12}\} = \{3, 3, 0\}$. By independence,

$$\begin{aligned} \mathsf{P}(u_i, v_i \geq 3/2) &= \frac{1}{2} \cdot \frac{1}{2} \\ &= \frac{1}{4} \quad \text{for every} \quad i = 1, \ldots, m \ , \\ \mathsf{P}(\mathbf{u}, \mathbf{v} \geq 3/2) &= \left(\frac{1}{4}\right)^m . \end{aligned}$$

Taking into account the number of ways the given intersection profile can be chosen from n elements, obtain

$$\sum_{J_1, J_2 \,:\, \text{disjoint}} \mathsf{P}(A_{J_1} \cap A_{J_2}) = \frac{n!}{3!\,3!\,0!\,(n-6)!} \cdot \left(\frac{1}{4}\right)^m .$$

2. Two column triplets with one common column: $\{x_1, x_2, x_{12}\} = \{2, 2, 1\}$. We have

$$\begin{aligned} \mathsf{P}(u_i, v_i \geq 3/2) &= \underbrace{\frac{1}{2}}_{\substack{\text{Probability}\\ \text{that the}\\ \text{common}\\ \text{element}\\ =1}} \cdot \underbrace{\frac{3}{4}}_{\substack{\text{Probability}\\ \text{that at least}\\ \text{one of two}\\ \text{other}\\ \text{elements}\\ \text{of triplet 1}\\ =1}} \cdot \underbrace{\frac{3}{4}}_{\substack{\text{Probability}\\ \text{that at least}\\ \text{one of two}\\ \text{other}\\ \text{elements}\\ \text{of triplet 2}\\ =1}} + \underbrace{\frac{1}{2}}_{\substack{\text{Probability}\\ \text{that the}\\ \text{common}\\ \text{element}\\ =0}} \cdot \underbrace{\frac{1}{4}}_{\substack{\text{Probability}\\ \text{that both}\\ \text{other}\\ \text{elements}\\ \text{of triplet 1}\\ =1}} \cdot \underbrace{\frac{1}{4}}_{\substack{\text{Probability}\\ \text{that both}\\ \text{other}\\ \text{elements}\\ \text{of triplet 2}\\ =1}} \\ &= \frac{5}{16} \quad \text{for every} \quad i = 1, \ldots, m \ , \\ \mathsf{P}(\mathbf{u}, \mathbf{v} \geq 3/2) &= \left(\frac{5}{16}\right)^m . \end{aligned}$$

Taking into account the number of ways the given intersection profile can be chosen from n elements, obtain

$$\sum_{J_1, J_2 \,:\, \text{have one common column}} \mathsf{P}(A_{J_1} \cap A_{J_2}) = \frac{n!}{2!\,2!\,1!\,(n-5)!} \left(\frac{5}{16}\right)^m .$$

3. Two column triplets with two common columns: $\{x_1, x_2, x_{12}\} = \{1, 1, 2\}$. We have

$$\begin{aligned} \mathsf{P}(u_i, v_i \geq 3/2) &= \underbrace{\frac{1}{4}}_{\substack{\text{Probability}\\ \text{that both}\\ \text{common}\\ \text{elements}\\ =1}} + \underbrace{\frac{1}{2}}_{\substack{\text{Probability}\\ \text{that one of two}\\ \text{common}\\ \text{elements}\\ =1}} \cdot \underbrace{\frac{1}{2}}_{\substack{\text{Probability}\\ \text{that the third}\\ \text{element}\\ \text{of triplet } i\\ =1}} \cdot \underbrace{\frac{1}{2}}_{\substack{\text{Probability}\\ \text{that the third}\\ \text{element}\\ \text{of triplet } j\\ =1}} \\ &= \frac{3}{8} \quad \text{for every} \quad i = 1, \ldots, m \ , \\ \mathsf{P}(\mathbf{u}, \mathbf{v} \geq 3/2) &= \left(\frac{3}{8}\right)^m . \end{aligned}$$

Taking into account the number of ways the given intersection profile can be chosen from n elements, obtain

$$\sum_{J_1,\,J_2\,:\,\text{have two common columns}} \mathsf{P}(A_{J_1} \cap A_{J_2}) = \frac{n!}{1!\,1!\,2!\,(n-4)!}\left(\frac{3}{8}\right)^m .$$

Thus, we have the second sum for (27)

$$\sum_{J_1 \neq J_2} \mathsf{P}(A_{J_1} \cap A_{J_2}) = \frac{n!}{3!\,3!\,0!\,(n-6)!}\left(\frac{1}{4}\right)^m + \frac{n!}{2!\,2!\,1!\,(n-5)!}\left(\frac{5}{16}\right)^m + \frac{n!}{1!\,1!\,2!\,(n-4)!}\left(\frac{3}{8}\right)^m,$$

where the terms with factorials of negative integers (if $n < 6$) should be omitted.

4.3 The S-th sum in the Inclusion-Exclusion formula

Intersection subsets In the previous section we considered intersection profiles $\{x_1, x_2, x_{12}\}$ of two sets J_1, J_2. The intersection subsets, even if empty, are indexed with 1, 2, and 12, indicating the belonging of their elements to a particular set or sets. Similarly, intersection profiles of three sets J_1, J_2, J_3 are denoted by $\{x_1, x_2, x_{12}, x_3, x_{13}, x_{23}, x_{123}\}$. Let us recursively construct the index sequences $T = [1], [2], [12], \ldots$ of intersection subsets of S sets:

$\emptyset$ is used to initialize the index sequence (needed for the recursion construction only).

1 is joined to the empty sequence, resulting in the sequence $T = [1]$.

2 is joined to each of preceding sequences, including the empty one, resulting in two sequences [2] and [1 2].

...

t is joined to each of preceding sequences as shown below

		Index sequences T
		[ ]
$\{[\ ]\}, 1$	$\to$	[1]
$\{[\ ], [1]\}, 2$	$\to$	[2], [1 2]
$\{[\ ], [1], [2], [1\,2]\}, 3$	$\to$	[3], [1 3], [2 3], [1 2 3]
....	....	....

The sequences on the right-hand side of the schema are labels of all 2^S possible intersection subsets of S sets, including the empty label for the empty intersection. Omit the empty sequence, and enumerate the labels in the order they are constructed. Now they can be used as index sequences $\{T : T = [t_1\, t_2\, \ldots\,]\}$, or simply as indices $T = 1, \ldots, 2^S - 1$ of intersection subsets.

Intersection profiles Consider S sets numbered $1, \ldots, S$. Their *intersection profile* is a $(2^S - 1)$-sequence of non-negative integers $\{x_T\}$, where each x_T is the number of elements in the intersection subset I_T with $T = [t_1\, t_2\, \ldots\,]$; see Figure 7.

The intersection profiles of different column k-tuples are subordinated to certain restrictions. For instance, the intersection profile of two column pairs

$$\{x_1, x_2, x_{12}\} = \{0, 0, 2\}$$

Intersection subsets of k-tuples J_1, J_2, J_3	I_1	I_2	I_{12}	I_3	I_{13}	I_{23}	I_{123}
Intersection profile $\{x_T\}$	x_1	x_2	x_{12}	x_3	x_{13}	x_{23}	x_{123}
Allocation $\{y_T\}$ ($y_T \leq x_T$)	y_1	y_2	y_{12}	y_3	y_{13}	y_{23}	y_{123}
Probability of allocation $\{y_T\}$ in $\{x_T\}$	$\binom{x_1}{y_1}\frac{1}{2^{x_1}}$	$\binom{x_2}{y_2}\frac{1}{2^{x_2}}$	$\binom{x_{12}}{y_{12}}\frac{1}{2^{x_{12}}}$	$\binom{x_3}{y_3}\frac{1}{2^{x_3}}$	$\binom{x_{13}}{y_{13}}\frac{1}{2^{x_{13}}}$	$\binom{x_{23}}{y_{23}}\frac{1}{2^{x_{23}}}$	$\binom{x_{123}}{y_{123}}\frac{1}{2^{x_{123}}}$

Figure 7: Intersection profile of three k-tuples, allocation, and its probability

is impossible, because it implies equal pairs. The intersection profile of three column triplets

$$\{x_1, x_2, x_{12}, x_3, \ldots\} = \{3, 3, 0, 3, \ldots\}$$

must have $x_{13}, x_{23}, x_{123} = 0$, otherwise some triplets have more than three elements. The restrictions on column k-tuples of Bernoulli $(m \times n)$-matrix are as follows:

$$\begin{aligned} x_T &\geq 0 \quad \text{for all} \quad T && (29)\\ \sum_T x_T &\leq n \quad \text{(total number of columns in all } k\text{-tuples} \leq n) && (30)\\ \sum_{T: t \in T} x_T &= k \quad \text{for every} \quad t \quad \text{(the } t\text{-th } k\text{-tuple has exactly } k \text{ elements)} && (31)\\ \sum_{T: t_1, t_2 \in T} x_T &\leq k-1 \quad \text{for all} \quad t_1 \neq t_2 \quad (k\text{-tuples } t_1 \text{ and } t_2 \text{ are different})\ . && (32) \end{aligned}$$

Perfect allocations Let S different column k-tuples of Bernoulli $(1 \times n)$-matrix have intersection profile $\{x_T\}$. Every outcome of the Bernoulli matrix results in an *allocation* defined to be the $(2^S - 1)$-sequence $\{y_T\}$, where each y_T is the number of 1s occurred in the T-th intersection subset; see Figure 7. An allocation is said to be *perfect* if 1s predominate in every k-tuple considered (meaning that all the k-tuples are perfect):

$$\begin{aligned} 0 \leq y_T \leq x_T & \quad \text{for all} \quad T \quad \text{(no more 1s than elements in } x_T) && (33)\\ \sum_{T: t \in T} y_T \geq k/2, & \quad \text{for all} \quad t = 1, \ldots, S \quad \text{(1s predominate in every } k\text{-tuple).} && (34) \end{aligned}$$

The probability of any perfect allocation in intersection profile $\{x_T\}$ is obviously

$$\sum_{\substack{\text{Allocations} \\ \{y_T\} \text{ in } \{x_T\} \\ \text{satisfying (33)–(34)}}} \prod_T \binom{x_T}{y_T} \frac{1}{2^{x_T}} = \frac{1}{2^{N_{\{x_T\}}}} \sum_{\substack{\text{Allocations} \\ \{y_T\} \text{ in } \{x_T\} \\ \text{satisfying (33)–(34)}}} \prod_T \binom{x_T}{y_T}\ , \tag{35}$$

where

$$N_{\{x_T\}} = \sum_T x_T \quad \text{(total number of columns in the given } k\text{-tuples)}\ .$$

The S-th sum of the Inclusion-Exclusion formula By independence of rows of Bernoulli $(m \times n)$-matrix, the probability that its every row has a perfect allocation is the mth power of (35). Hence, for ordered k-tuples we obtain

$$\sum_{J_1<J_2<\ldots<J_S} \mathsf{P}\left(A_{J_1} \cap \ldots \cap A_{J_S}\right) = \sum_{J_1<J_2<\ldots<J_S} \left[\sum_{\substack{\text{Allocations } \{y_T\}\\ \text{in intersection}\\ \text{profile } \{x_T\} \text{ of}\\ k\text{-tuples } J_1,\ldots,J_S\\ \text{satisfying (33)–(34)}}} \frac{1}{2^{N_{\{x_T\}}}} \prod_T \binom{x_T}{y_T} \right]^m . \tag{36}$$

For non-ordered k-tuples proceed as in Section 4.2. Taking into account (35) and the number of ways a given intersection profile $\{x_T\}$ can be chosen from n elements, obtain

$$\sum_{J_1 \neq J_2 \neq \ldots \neq J_S} \mathsf{P}\left(A_{J_1} \cap \ldots \cap A_{J_S}\right) =$$

$$= \sum_{\substack{\text{Intersection}\\ \text{profiles}\\ \{x_T\} \text{ of } S\\ k\text{-tuples}\\ \text{satisfying}\\ \text{(29)–(32)}}} \frac{n!}{\underbrace{x_1! \ldots x_{2^S-1}!}_{\substack{\text{Intersection profile}\\ \{x_t\} \text{ of } S\ k\text{-tuples}}} \; (\underbrace{n - N_{\{x_T\}}}_{\substack{\text{Number of}\\ \text{columns}\\ \text{outside the } S\\ k\text{-tuples}}})! \, 2^{mN_{\{x_T\}}}} \left[\sum_{\substack{\text{Allocations } \{y_T\}\\ \text{in intersection}\\ \text{profile } \{x_T\} \text{ of}\\ k\text{-tuples } J_1,\ldots,J_S\\ \text{satisfying (33)–(34)}}} \prod_T \binom{x_T}{y_T} \right]^m . \tag{37}$$

Shortly denote the total number $N_{\{x_T\}}$ of columns in intersection profile $\{x_T\}$ by N. Then $N \leq \min(n, kS)$ and $S \leq \binom{N}{k}$ — otherwise some k-tuples would coincide. Besides,

$$\frac{n!}{\underbrace{x_1! \ldots x_{2^S-1}!}_{\substack{\text{Intersection profile}\\ \{x_t\} \text{ of } S\ k\text{-tuples}}} \; \underbrace{(n-N)}_{\substack{\text{Number of}\\ \text{columns}\\ \text{outside the } S\\ k\text{-tuples}}}\ !} \overset{\substack{\text{Multiply and}\\ \text{divide by } N!}}{=} \binom{n}{N} \frac{N!}{x_{1:2^S-1}!} .$$

Substitute all of these in (37), regroup summation with respect to N, and obtain

$$\sum_{J_1 \neq J_2 \neq \ldots \neq J_S} \mathsf{P}\left(A_{J_1} \cap \ldots \cap A_{J_S}\right) =$$

$$= \sum_{N : \binom{N}{k} \geq S}^{N \leq \min(n,kS)} \binom{n}{N} \frac{1}{2^{mN}} \sum_{\substack{\text{Intersection}\\ \text{profiles } \{x_T\} \text{ of } S\\ k\text{-tuples with}\\ \text{totally } N \text{ columns}\\ \text{satisfying (29)–(32)}}} \frac{N!}{x_{1:T}!} \left[\sum_{\substack{\text{Allocations } \{y_T\}\\ \text{in } \{x_T\}\\ \text{satisfying (33)–(34)}}} \prod_t \binom{x_t}{y_t} \right]^m . \tag{38}$$

Table 18: Computational complexity of finding exact probabilities of perfect column pairs ($k = 2$) in Bernoulli ($m \times n$)-matrices by the Inclusion-Exclusion formula

Width n of matrix B	Number S of sums in the Inclusion-Exclusion formula (number of column pairs)	Total number of combinations of column pairs	Total number of perfect allocations	Computation time
2	1	1	2	0.00 s
3	3	7	25	0.00 s
4	6	63	378	0.03 s
5	10	1,023	10,013	0.62 s
6	15	32,767	490,712	32.06 s

Table 19: Computational complexity of finding exact probabilities of perfect column triplets ($k = 3$) in Bernoulli ($m \times n$)-matrices by the Inclusion-Exclusion formula

Width n of matrix B	Number S of sums in the Inclusion-Exclusion formula (number of column triplets)	Total number of combinations of column triplets	Total number of perfect allocations	Computation time
3	1	1	2	0.00 s
4	4	15	63	0.01 s
5	10	1,023	6,848	0.62 s
6	20	1,048,575	8,290,887	20 m 22 s

4.4 Exact computations

Compute probabilities by the Inclusion-Exclusion formula with sums (36):

- Enumerate all k-tuples of n elements. Their number $\binom{n}{k}$ is shown in the second column of Tables 18 and 19.
- For every $S = 1, \ldots, \binom{n}{k}$ enumerate all combinations of S different k-tuples.
  - For each k-tuple find the intersection profile $\{x_T\}$. The total number of these combinations/intersection profiles $\sum_{S=1}^{\binom{n}{k}} \binom{\binom{n}{k}}{S} = 2^{\binom{n}{k}} - 1$ is shown in the third column of Tables 18 and 19.
    - ∗ For each intersection profile $\{x_T\}$ find all perfect allocations $\{y_T\}$ satisfying (33)–(34). The total number of perfect allocations processed is shown in the penultimate column of Tables 18 and 19.
  - Compute the Sth sum of the Inclusion-Exclusion formula by (36).
- Add all the S sums computed.

The exact probabilities $\mathsf{P}\left(\bigcup A_i\right)$ for Bernoulli matrices of width $n \leq 6$ computed this way are shown in Tables 20 and 21. The probabilities of perfect column pairs in Table 20 are identical to the ones computed with Geometric and Algebraic methods; see Tables 7 and 13. The same holds for perfect column triplets; cf. Table 21 with Tables 9 and 17.

Table 20: Probabilities of perfect column pairs ($k = 2$) in Bernoulli ($m \times n$)-matrices computed by the Inclusion-Exclusion formula

k	m	Width n of Bernoulli ($m \times n$)-matrix				
		2	3	4	5	6
2	1	.75000000	.87500000	.93750000	.96875000	.98437500
2	2	.56250000	.76562500	.87890625	.93847656	.96899414
2	3	.42187500	.65820313	.80932617	.89724731	.94584274
2	4	.31640625	.55395508	.72706604	.83994007	.90894467
2	5	.23730469	.45706177	.63682079	.76744083	.85591626
2	6	.17797852	.37074661	.54485947	.68415703	.78781240
2	7	.13348389	.29650545	.45663340	.59582070	.70829581
2	8	.10011292	.23439580	.37592543	.50792317	.62241199
2	9	.07508469	.18355144	.30481599	.42482712	.53536575
2	10	.05631351	.14263227	.24400537	.34944906	.45161901
2	11	.04223514	.11014066	.19323001	.28332781	.37442440
2	12	.03167635	.08461506	.15164311	.22689331	.30573822
2	13	.02375726	.06473252	.11811019	.17979540	.24638047
2	14	.01781795	.04935154	.09141364	.14120784	.19630835
2	15	.01336346	.03751882	.07037994	.11006967	.15490608
2	16	.01002260	.02845674	.05394884	.08525589	.12123568
2	17	.00751695	.02154203	.04120302	.06568579	.09422735
2	18	.00563771	.01628167	.03137294	.05038297	.07280873
2	19	.00422828	.01228971	.02382769	.03850151	.05598300
2	20	.00317121	.00926644	.01805900	.02933051	.04286870

Table 21: Probabilities of perfect column triplets ($k = 3$) in Bernoulli ($m \times n$)-matrices computed by the Inclusion-Exclusion formula

k	m	Width n of Bernoulli ($m \times n$)-matrix			
		3	4	5	6
3	1	.50000000	.68750000	.81250000	.89062500
3	2	.25000000	.44921875	.61132813	.72973633
3	3	.12500000	.27514648	.42376709	.55218887
3	4	.06250000	.15995789	.27330971	.38646346
3	5	.03125000	.08944607	.16627449	.25237670
3	6	.01562500	.04860848	.09663510	.15557551
3	7	.00781250	.02586606	.05421108	.09154705
3	8	.00390625	.01355145	.02959583	.05192179
3	9	.00195313	.00701787	.01582380	.02860887
3	10	.00097656	.00360293	.00832623	.01541214
3	11	.00048828	.00183772	.00432787	.00815887
3	12	.00024414	.00093276	.00222866	.00426117
3	13	.00012207	.00047169	.00113951	.00220247
3	14	.00006104	.00023787	.00057947	.00112935
3	15	.00003052	.00011970	.00029347	.00057557
3	16	.00001526	.00006014	.00014816	.00029198
3	17	.00000763	.00003018	.00007462	.00014759
3	18	.00000381	.00001513	.00003752	.00007441
3	19	.00000191	.00000758	.00001884	.00003744
3	20	.00000095	.00000380	.00000945	.00001881

4.5 Approximations with the first sums

As n increases, computations with the Inclusion-Exclusion formula rapidly become excessive. For instance, the probabilities of perfect column triplets in Bernoulli matrices of width $n = 7$ are computed with the Geometric method in 48 sec (see the row 3 7 0 in Table 8), whereas the Probabilistic method fails to do it in a day. An alternative to excessive computations with the Inclusion-Exclusion formula is using its first S sums (38) as approximations.

By general reasons, if the number m of rows in Bernoulli $(m \times n)$-matrix is large then the occurrence of numerous perfect column k-tuples is less probable than the occurrence of fewer ones. It implies that for a fixed number of columns n and a sufficiently large m the S-th sums (38) decrease after a certain sum number S. Besides, the larger m the less probable perfect k-tuples of m-columns, meaning that S-sums vanish as m increases.

Since the sums in the Inclusion-Exclusion formula decrease and have alternating signs, the probability $\mathsf{P}\left(\bigcup A_i\right)$ can be approximated with the first S sums. Under these prerequisites, the true value of probabilities $\mathsf{P}\left(\bigcup A_i\right)$ is always located between two successive approximations obtained with $S-1$ and S sums. The (m, n)-domain of $(m \times n)$-sizes of Bernoulli matrices, where the Inclusion-Exclusion formula with S sums provides accurate approximations, can be found as follows:

- Approximate probabilities of perfect column pairs (triplets) in Bernoulli $(m \times n)$-matrices by the first S sums (38) with factorials. For the resulting probability approximations $p_{mn} \approx \Sigma^1_{mn} + \cdots + \Sigma^S_{mn}$ find the (m, n)-domain where the sums decrease, that is, where $\left|\Sigma^{S-1}_{mn}\right| > \left|\Sigma^S_{mn}\right|$.
- Further restrict the (m, n)-domain to sufficiently small S-th sums, say, $\left|\Sigma^S_{mn}\right| < 0.025 p_{mn}$. Then the approximations p_{mn} are 2.5%-accurate. Here, relative accuracy looks more adequate than absolute because the probabilities exponentially decrease in m, and one interpolation method in the next chapter relies on their products.

Note that sums (38) are computed for parameters $N \leq kS$ regardless of width n of Bernoulli matrix. However, the number of encountering intersection profiles and allocations can be large even for the first sums. The computational complexity of the method for column pairs and triplets is summarized in Tables 22 and 23.

Let us compare the complexity of exact and approximate formulas. Consider computing *exact* probabilities of perfect column triplets in Bernoulli matrix with six columns; see Table 19. There are 20 combinations of triplets and, correspondingly, $S = 20$ sums in the exact Inclusion-Exclusion formula with the number of perfect allocations 8.3 Million. The processing time is 20 min. Now look at computing *approximate* probabilities with $S = 6$ sums but for Bernoulli matrices with an unconstrained number of columns; see Table 23. The intersection profiles of six column triplets can contain up to 18 columns (instead of six in the previous case), so that the number of perfect allocations is over 76 million. The processing time is over two hours.

Table 22: Computational complexity of approximating probabilities of perfect column pairs in Bernoulli $(m \times n)$-matrices by the Inclusion-Exclusion formula with up to 6 sums ($k = 2,\ s \leq 6$)

The sth sum (s column pairs considered jointly)	Number of intersection profiles	Number of perfect allocations	Computation time
1	1	2	0.06 s
2	2	9	0.06 s
3	9	75	0.06 s
4	70	953	0.08 s
5	794	16,647	0.48 s
6	12,055	376,544	10.37 s

Table 23: Computational complexity of approximating probabilities of perfect column triplets in Bernoulli $(m \times n)$-matrices by the Inclusion-Exclusion formula with up to 6 sums ($k = 3,\ s \leq 6$)

The sth sum (s column triplets considered jointly)	Number of intersection profiles	Number of perfect allocations	Computation time
1	1	2	0.06 s
2	3	14	0.06 s
3	29	267	0.07 s
4	666	10,613	0.58 s
5	28,344	726,728	45.82 s
6	1,935,054	76,316,184	2 h 2 m

4.6 Summary

The exact version of the Inclusion-Exclusion formula is computationally restricted to finding probabilities of perfect column pairs and triplets in Bernoulli matrices with up to six columns (at our computer facility). This method is less efficient than the Geometric one, which enables to advance up to matrices with eight columns and, what is even more important, is appropriate for managing i-imperfect column pairs and triplets.

The approximate version of the Probabilistic method offers however some advantages. Unlike Geometric and Algebraic methods, it is not restricted to Bernoulli matrices with a few columns or a few rows but is appropriate for quite large matrices.

5 Interpolation methods

5.1 Missing probabilities

All the three methods considered so far have computational constraints. Referring to output probability tables for *perfect* column pairs and triplets like Table 21, the Geometric method provides a few first columns and the Algebraic method provides a few first rows, making a kind of the table frame. The approximate version of Probabilistic method contributes to the table bottom-triangular domain for (m, n)-sizes of Bernoulli matrices having the row-to-column ratio about $m : n > 2$.

As for *imperfect* column pairs and triplets, the Algebraic and Geometric methods can handle only low degrees of imperfectness, with a minor exception for matrices with 3–4 columns feasible by the Geometric method. The Probabilistic method is not appropriate for that at all.

The missing probabilities exhibit certain regularities. By general reasons, the unknown probabilities exponentially vanish as the vertical size m of Bernoulli matrix increases, and their distance to probability 1 vanishes exponentially as the horizontal size n of the matrix increases. These regularities enable to restore the missing probabilities by interpolation between the known values.

5.2 Interpolation of perfect outcomes on column height m

Table 24 shows probabilities of perfect column pairs in Bernoulli $(m \times n)$-matrices. The exact probabilities computed by the Geometric method occupy the first eight columns, and the ones computed by the Algebraic method constitute the upper seven rows. They are shown in roman font style. The 2.5%-accurate approximations obtained by the Inclusion-Exclusion formula are in the bottom triangular section of the table. They are shown in *italics*.

The **boldfaced** table elements in the upper-triangular section are interpolated column-by-column between exact values in the upper seven rows and approximations below. The interpolation is done with the cubic spline techniques implemented in the MATLAB program `interp1`, that is, with piecewise-polynomial functions of third degree (Stoer and Bulirsch 2002, pp. 93–106; Stoer and Bulirsch 2007, pp. 112–148); see also Spline (mathematics) (2012) and Spline Interpolation (2012). Since the decrease along columns is exponential, the interpolation is performed for the logarithm of the column elements.

The relief of the table is visualized in Figure 8, where the interpolated values are shown by **thick** segments. The 'vertical' curves show that the table elements vanish along columns. The 'horizontal' curves, corresponding to rows, approach probability 1 as the column number n increases.

Table 24: Sheet A. Probabilities of perfect column pairs ($k = 2,\ i = 0$) in Bernoulli ($m \times n$)-matrices. The exact probabilities are shown by the roman font style (columns 2–8 and rows 1–7). The accurately approximated probabilities computed by the Inclusion–Exclusion formula with $S = 6$ sums are in *italics* (below the table diagonal). Inaccurate approximations, which differ by more than 2.5% form the ones computed with $S = 5$ sums, are replaced by interpolated values **boldfaced**. The interpolation is performed column-by-column with the cubic spline method

k m	Width n of Bernoulli ($m \times n$)-matrix							
	2	3	4	5	6	7	8	9
2 1	.75000000	.87500000	.93750000	.96875000	.98437500	.99218750	.99609375	.99804688
2 2	.56250000	.76562500	.87890625	.93847656	.96899414	.98443604	.99220276	.99609756
2 3	.42187500	.65820313	.80932617	.89724731	.94584274	.97187853	.98554820	.99262745
2 4	.31640625	.55395508	.72706604	.83994007	.90894467	.94937097	.97234511	.98510658
2 5	.23730469	.45706177	.63682079	.76744083	.85591626	.91303641	.94862327	.97018599
2 6	.17797852	.37074661	.54485947	.68415703	.78781240	.86117292	.91116768	.94423140
2 7	.13348389	.29650545	.45663340	.59582070	.70829581	.79469930	.85858020	.90438661
2 8	.10011292	.23439580	.37592543	.50792317	.62241199	.71685073	.79185026	**.84669472**
2 9	.07508469	.18355144	.30481599	.42482712	.53536575	.63227361	.71413493	**.77447120**
2 10	.05631351	.14263227	.24400537	.34944906	.45161901	.54594319	.62993565	**.69314321**
2 11	.04223514	.11014066	.19323001	.28332781	.37442440	.46227659	.54408316	**.60787398**
2 12	.03167635	.08461506	.15164311	.22689331	.30573822	.38461535	.46088803	**.52313119**
2 13	.02375726	.06473252	.11811019	.17979540	.24638047	.31507086	.38363189	**.44243322**
2 14	.01781795	.04935154	.09141364	.14120784	.19630835	.25462875	.31440448	**.36826335**
2 15	.01336346	.03751882	.07037994	.11006967	.15490608	.20338866	.25419391	**.30211795**
2 16	.01002260	.02845674	.05394884	.08525589	.12123568	.16083996	.20311397	**.24464447**
2 17	.00751695	.02154203	.04120302	.06568579	.09422735	.12611192	.16067127	**.19582526**
2 18	.00563771	.01628167	.03137294	.05038297	.07280873	.09817027	.12601086	*.15517082*
2 19	.00422828	.01228971	.02382769	.03850151	.05598300	.07595496	.09811105	*.12186509*
2 20	.00317121	.00926644	.01805900	.02933051	.04286870	.05846631	.07592094	*.09491296*
2 21	.00237841	.00698061	.01366293	.02228604	.03271354	.04481128	.05844711	*.07343745*
2 22	.00178381	.00525472	.01032191	.01689680	.02489223	.03422200	.04480064	*.05651919*
2 23	.00133786	.00395310	.00778841	.01278765	.01889538	.02605651	.03421619	*.04330848*
2 24	.00100339	.00297237	.00587081	.00966324	.01431449	.01978954	.02605339	*.03306568*
2 25	.00075254	.00223399	.00442162	.00729305	.01082604	.01499839	.01978789	*.02516966*
2 26	.00056441	.00167845	.00332783	.00549846	.00817631	.01134735	.01499753	*.01911146*
2 27	.00042331	.00126068	.00250315	.00414185	.00616794	.00857258	.01134691	*.01448139*
2 28	.00031748	.00094666	.00188192	.00311768	.00464839	.00646848	.00857237	*.01095413*
2 29	.00023811	.00071072	.00141430	.00234535	.00350037	.00487588	.00646837	*.00827415*
2 30	.00017858	.00053349	.00106251	.00176345	.00263411	.00367231	.00487583	*.00624240*
2 31	.00013394	.00040040	.00079800	.00132537	.00198112	.00276389	.00367229	*.00470489*
2 32	.00010045	.00030048	.00059920	.00099577	.00148931	.00207897	.00276388	*.00354315*
2 33	.00007534	.00022547	.00044984	.00074792	.00111916	.00156302	.00207897	*.00266644*
2 34	.00005650	.00016917	.00033765	.00056162	.00084073	.00117464	.00156302	*.00200552*
2 35	.00004238	.00012692	.00025341	.00042164	.00063140	.00088247	.00117464	*.00150770*
2 36	.00003178	.00009522	.00019017	.00031650	.00047408	.00066278	.00088247	*.00113301*
2 37	.00002384	.00007143	.00014269	.00023754	.00035589	.00049767	.00066278	*.00085115*
2 38	.00001788	.00005358	.00010706	.00017826	.00026713	.00037362	.00049767	*.00063924*
2 39	.00001341	.00004019	.00008032	.00013376	.00020048	.00028044	.00037362	*.00047997*
2 40	.00001006	.00003015	.00006026	.00010036	.00015044	.00021047	.00028044	*.00036032*
2 41	.00000754	.00002261	.00004520	.00007530	.00011288	.00015794	.00021047	*.00027046*
2 42	.00000566	.00001696	.00003391	.00005649	.00008469	.00011851	.00015794	*.00020298*
2 43	.00000424	.00001272	.00002544	.00004238	.00006354	.00008892	.00011851	*.00015232*
2 44	.00000318	.00000954	.00001908	.00003179	.00004767	.00006671	.00008892	*.00011429*
2 45	.00000239	.00000716	.00001431	.00002385	.00003576	.00005005	.00006671	*.00008575*
2 46	.00000179	.00000537	.00001073	.00001789	.00002682	.00003754	.00005005	*.00006433*
2 47	.00000134	.00000403	.00000805	.00001342	.00002012	.00002816	.00003754	*.00004826*
2 48	.00000101	.00000302	.00000604	.00001006	.00001509	.00002113	.00002816	*.00003620*

Table 24: Sheet B. Probabilities of perfect column pairs ($k = 2,\ i = 0$) in Bernoulli ($m \times n$)-matrices. The exact probabilities are shown by the roman font style (columns 2–8 and rows 1–7). The accurately approximated probabilities computed by the Inclusion–Exclusion formula with $S = 6$ sums are in *italics* (below the table diagonal). Inaccurate approximations, which differ by more than 2.5% form the ones computed with $S = 5$ sums, are replaced by interpolated values **boldfaced**. The interpolation is performed column-by-column with the cubic spline method

k m	Width n of Bernoulli ($m \times n$)-matrix						
	10	11	12	13	14	15	16
2 1	.99902344	.99951172	.99975586	.99987793	.99993896	.99996948	.99998474
2 2	.99804783	.99902368	.99951178	.99975587	.99987793	.99993897	.99996948
2 3	.99625873	.99810874	.99904664	.99952042	.99975912	.99987915	.99993942
2 4	.99207073	.99581803	.99781157	.99886225	.99941173	.99969725	.99984481
2 5	.98296031	.99038813	.99463956	.99704029	.99838020	.99912043	.99952571
2 6	.96556449	.97904511	.98741265	.99252615	.99560849	.99744394	.99852504
2 7	.93641152	.95832546	.97304482	.98277197	.98910788	.99318181	.99577088
2 8	**.88873706**	**.91915685**	**.94079515**	**.95590980**	**.96637153**	**.97354040**	**.97785729**
2 9	**.82453450**	**.86235253**	**.89050256**	**.91103270**	**.92595058**	**.93669881**	**.94271203**
2 10	**.74871056**	**.79226495**	**.82596775**	**.85145839**	**.87074071**	**.88520237**	**.89281043**
2 11	**.66623780**	**.71353982**	**.75143037**	**.78102481**	**.80419506**	**.82216240**	**.83119938**
2 12	**.58169502**	**.63066675**	**.67115382**	**.70371641**	**.72999453**	**.75096713**	**.76121274**
2 13	**.49894512**	**.54763018**	**.58907825**	**.62333497**	**.65174485**	**.67500713**	**.68620026**
2 14	**.42096129**	**.46768380**	**.50857082**	**.54324506**	**.57272949**	**.59744199**	**.60929669**
2 15	**.34978734**	**.39324804**	**.43228218**	**.46620839**	**.49573499**	**.52102542**	**.53324772**
2 16	**.28660194**	**.32591313**	**.36210175**	**.39430647**	**.42295273**	**.44799555**	**.46029955**
2 17	**.23185043**	**.26652012**	**.29919286**	**.32894005**	**.35595121**	**.38002830**	**.39215031**
2 18	**.18540907**	**.21528946**	**.24408386**	**.27088699**	**.29570565**	**.31824492**	**.32995460**
2 19	**.14675367**	**.17196970**	**.19679048**	**.22039799**	**.24266852**	**.26326076**	**.27436867**
2 20	*.11511248*	**.13598446**	**.15694815**	**.17731059**	**.19686389**	**.21526120**	**.22562265**
2 21	*.08958964*	*.10656303*	**.12393824**	**.14116558**	**.15799075**	**.17409095**	**.18360641**
2 22	*.06926272*	*.08284786*	*.09699770*	**.11131395**	**.12552329**	**.13934559**	**.14795821**
2 23	*.05326387*	*.06398035*	*.07531109*	**.08700746**	**.09880030**	**.11045688**	**.11814776**
2 24	*.04078400*	*.04915022*	*.05808432*	*.06746975*	**.07709879**	**.08676620**	**.09354837**
2 25	*.03111770*	*.03759827*	*.04456681*	*.05196082*	*.05969066*	**.06758379**	**.07349544**
2 26	*.02367314*	*.02866270*	*.03405490*	*.03981561*	*.04589725*	*.05223295*	**.05733069**
2 27	*.01796612*	*.02178932*	*.02593646*	*.03038859*	*.03512006*	*.04009562*	**.04443300**
2 28	*.01360766*	*.01652588*	*.01970033*	*.02312036*	*.02677192*	*.03063619*	**.03423772**
2 29	*.01028944*	*.01250996*	*.01493076*	*.01754573*	*.02034711*	*.02332471*	**.02624666**
2 30	*.00776966*	*.00945501*	*.01129552*	*.01328767*	*.01542710*	*.01770822*	**.02003098**
2 31	*.00586024*	*.00713676*	*.00853269*	*.01004598*	*.01167414*	*.01341409*	**.01522925**
2 32	*.00441588*	*.00538110*	*.00643774*	*.00758461*	*.00882028*	*.01014302*	**.01154230**
2 33	*.00332488*	*.00405369*	*.00485223*	*.00571979*	*.00665553*	*.00765848*	**.00872634**
2 34	*.00250179*	*.00305146*	*.00365415*	*.00430942*	*.00501679*	*.00577571*	*.00658549*
2 35	*.00188143*	*.00229560*	*.00274998*	*.00324430*	*.00377828*	*.00435160*	*.00496388*
2 36	*.00141426*	*.00172609*	*.00206834*	*.00244087*	*.00284351*	*.00327605*	*.00373829*
2 37	*.00106269*	*.00129731*	*.00155492*	*.00183543*	*.00213874*	*.00246473*	*.00281328*
2 38	*.00079826*	*.00097470*	*.00116848*	*.00137956*	*.00160787*	*.00185335*	*.00211592*
2 39	*.00059948*	*.00073210*	*.00087780*	*.00103654*	*.00120829*	*.00139301*	*.00159065*
2 40	*.00045010*	*.00054974*	*.00065924*	*.00077857*	*.00090771*	*.00104662*	*.00119530*
2 41	*.00033788*	*.00041273*	*.00049499*	*.00058466*	*.00068171*	*.00078613*	*.00089791*
2 42	*.00025360*	*.00030981*	*.00037159*	*.00043895*	*.00051186*	*.00059033*	*.00067434*
2 43	*.00019032*	*.00023252*	*.00027892*	*.00032950*	*.00038426*	*.00044320*	*.00050631*
2 44	*.00014281*	*.00017449*	*.00020932*	*.00024730*	*.00028842*	*.00033269*	*.00038009*
2 45	*.00010716*	*.00013093*	*.00015708*	*.00018559*	*.00021646*	*.00024969*	*.00028528*
2 46	*.00008040*	*.00009824*	*.00011786*	*.00013926*	*.00016243*	*.00018738*	*.00021410*
2 47	*.00006032*	*.00007371*	*.00008843*	*.00010449*	*.00012188*	*.00014060*	*.00016066*
2 48	*.00004525*	*.00005530*	*.00006634*	*.00007839*	*.00009144*	*.00010550*	*.00012055*

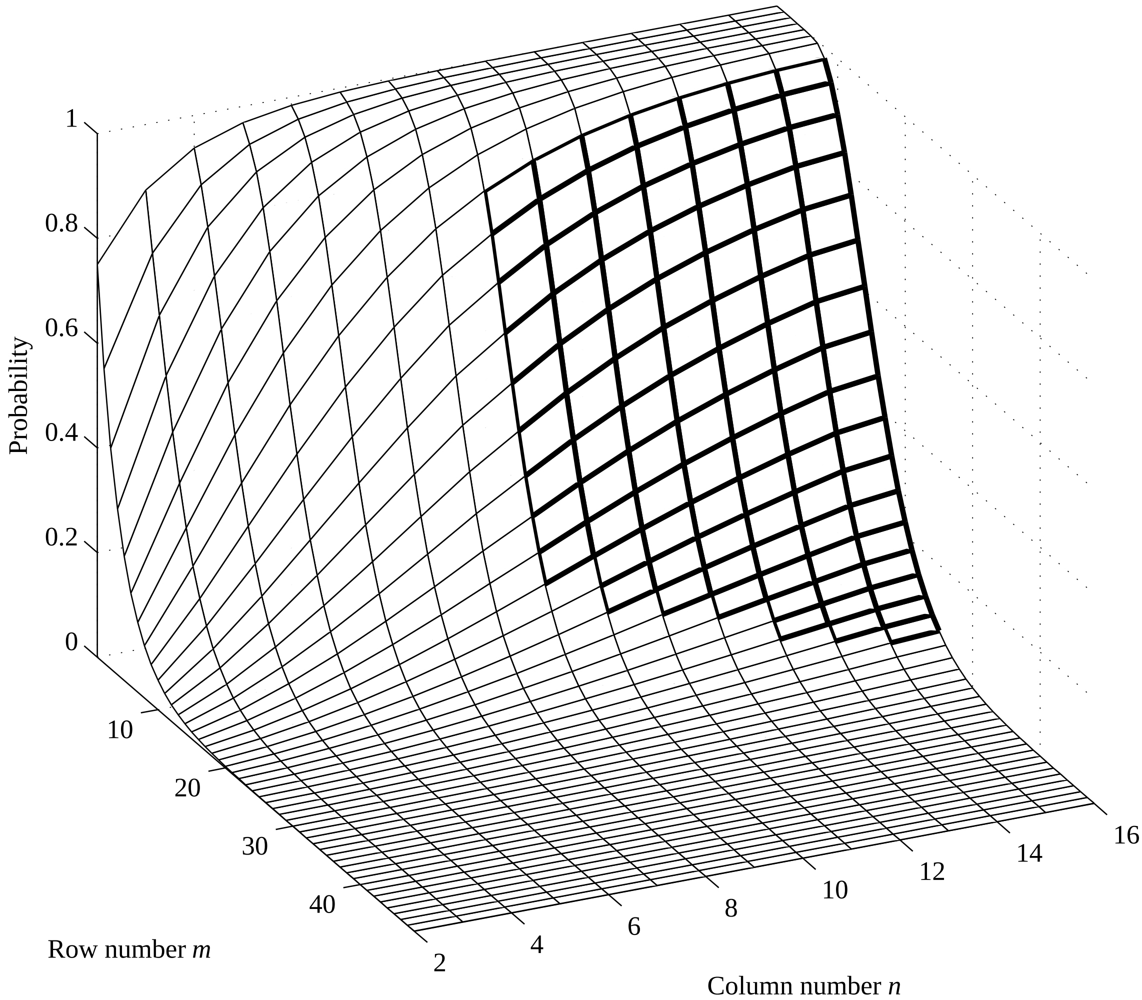
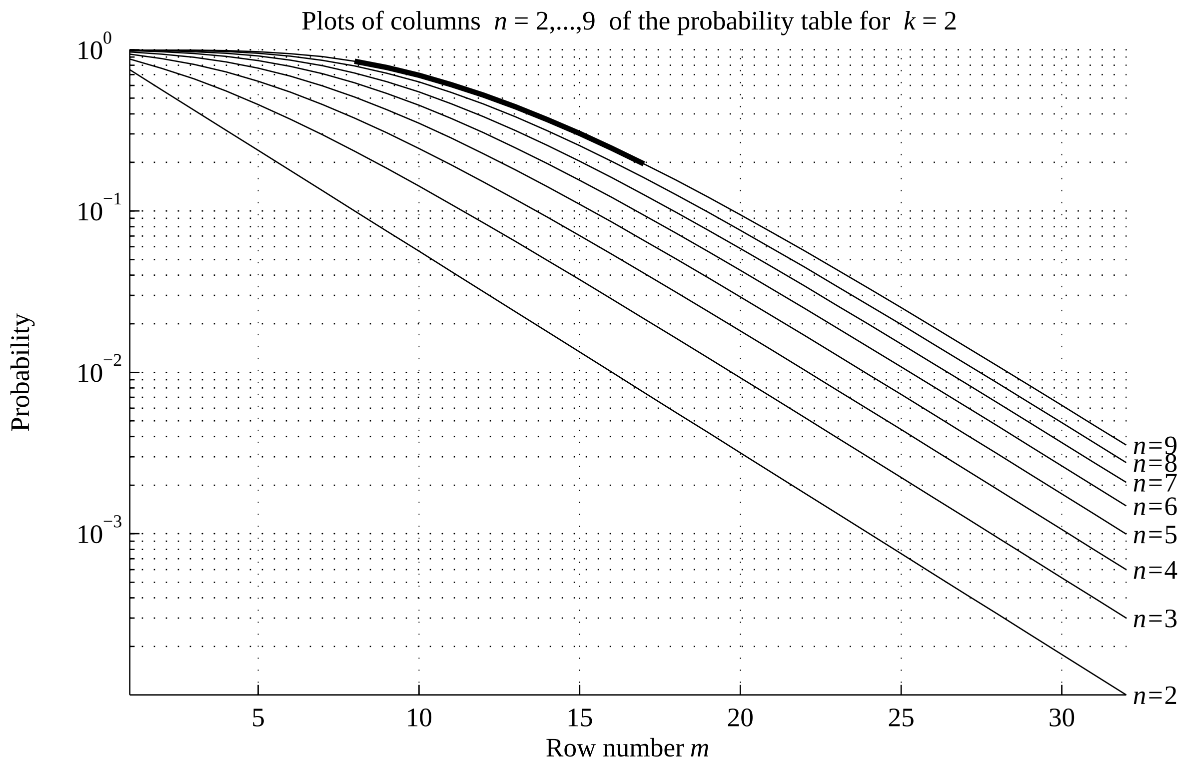


Figure 8: 3D relief of the probability Table 24 (for $k = 2,\ i = 0$) with the interpolated segments shown by **thick** curves

Figure 9 visualizes log-scaled columns and rows of the table. Each column plot has a round head and (almost) line tails, illustrating the exponential decrease. Every exactly computed plot of rows $m = 1, \ldots, 7$ has decreasing convexity, tending to linearity (in log-scale), corresponding to exponential approach to probability 1. The plots of rows with number $m > 7$ have interpolated tails. They are convex as long as the interpolated elements (in corresponding columns) are few. As the number of interpolated elements in columns increases, the row plots have 'implausible' inflection points, violating the exponential approach to 1. It is caused by flattened interpolated heads of the column plots, which should have been more round (concave). These 'implausible' plots indicate at the necessity to improve the interpolation technique to obtain more accurate approximations, which, however, is beyond the scope of the given study.

The next Table 25 shows the probabilities of perfect column *triplets* in Bernoulli matrices. Its layout is the same as that of Table 24. The accompanying Figures 10 and 11 are similar to Figures 8 and 9. However, since the interpolated elements are fewer, the 'implausible' inflection of row curves is not seen at all.



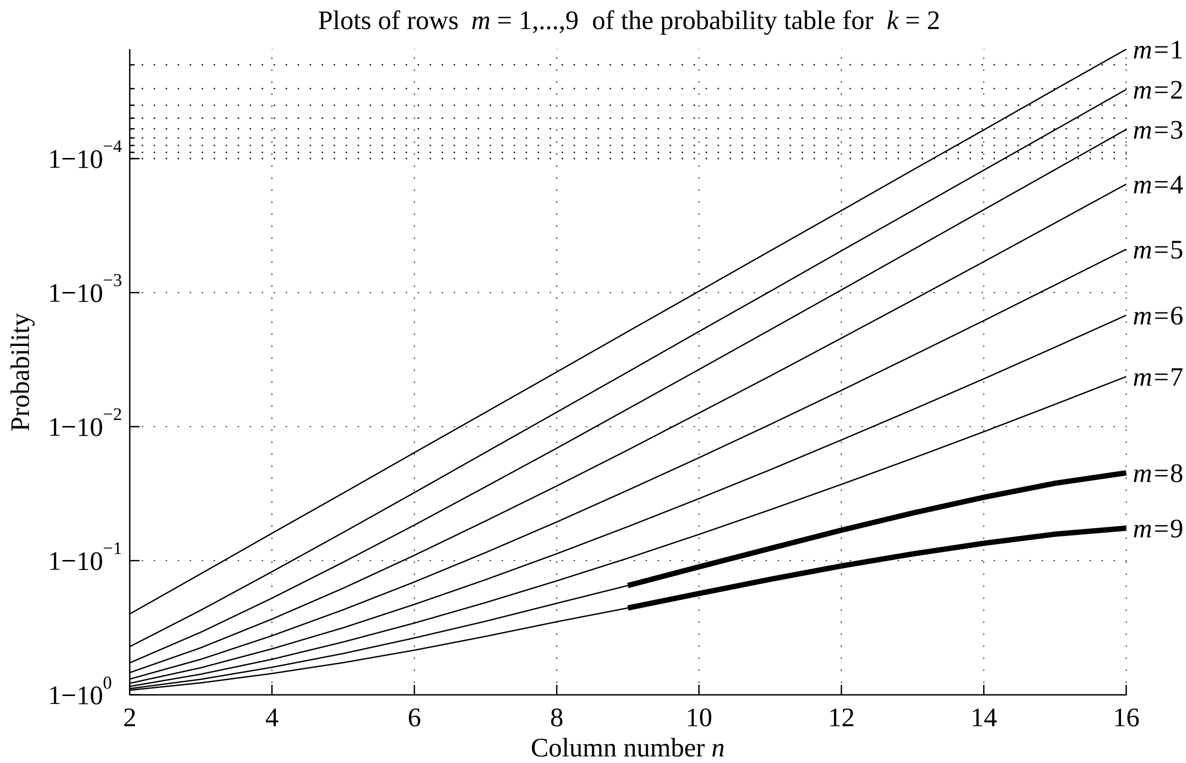


Figure 9: Plots of the logarithmically scaled first columns and first rows of Table 24 of probabilities of perfect column pairs ($k = 2,\ i = 0$) with the interpolated segments shown by **thick** curves

Table 25: Sheet A. Probabilities of perfect column triplets ($k = 3,\ i = 0$) in Bernoulli ($m \times n$)-matrices. The exact probabilities are shown by the roman font style (columns 3–8 and rows 1–7). The accurately approximated probabilities computed by the Inclusion–Exclusion formula with $S = 6$ sums are in *italics* (below the table diagonal). Inaccurate approximations, which differ by more than 2.5% form the ones computed with $S = 5$ sums, are replaced by interpolated values **boldfaced**. The interpolation is performed column-by-column with the cubic spline method

k	m	Width n of Bernoulli ($m \times n$)-matrix							
		3	4	5	6	7	8	9	10
3	1	.50000000	.68750000	.81250000	.89062500	.93750000	.96484375	.98046875	.98925781
3	2	.25000000	.44921875	.61132813	.72973633	.81225586	.86875916	.90737152	.93393040
3	3	.12500000	.27514648	.42376709	.55218887	.65610695	.73748273	.80019028	.84815205
3	4	.06250000	.15995789	.27330971	.38646346	.49073754	.58249769	.66097559	.72683604
3	5	.03125000	.08944607	.16627449	.25237670	.34059513	.42613960	.50611353	.57896691
3	6	.01562500	.04860848	.09663510	.15557551	.22129020	.29024047	.35964297	.42742799
3	7	.00781250	.02586606	.05421108	.09154705	.13604801	.18578765	.23895803	.29396719
3	8	.00390625	.01355145	.02959583	.05192179	.07996495	.11292569	**.15030873**	**.19070050**
3	9	.00195313	.00701787	.01582380	.02860887	.04534006	.06580189	**.09022072**	**.11761338**
3	10	.00097656	.00360293	.00832623	.01541214	.02498582	.03706493	**.05201094**	**.06939694**
3	11	.00048828	.00183772	.00432787	.00815887	.01346445	.02032335	**.02898384**	**.03942116**
3	12	.00024414	.00093276	.00222866	.00426117	.00713022	.01090990	**.01571437**	**.02169455**
3	13	.00012207	.00047169	.00113951	.00220247	.00372507	.00576038	**.00834304**	**.01163941**
3	14	.00006104	.00023787	.00057947	.00112935	.00192587	.00300258	**.00436561**	**.00612629**
3	15	.00003052	.00011970	.00029347	.00057557	.00098771	.00154961	*.00226603*	**.00318329**
3	16	.00001526	.00006014	.00014816	.00029198	.00050346	.00079367	*.00117084*	*.00164321*
3	17	.00000763	.00003018	.00007462	.00014759	.00025542	.00040413	*.00059908*	*.00084551*
3	18	.00000381	.00001513	.00003752	.00007441	.00012912	.00020486	*.00030465*	*.00043141*
3	19	.00000191	.00000758	.00001884	.00003744	.00006510	.00010350	*.00015426*	*.00021894*
3	20	.00000095	.00000380	.00000945	.00001881	.00003275	.00005216	*.00007786*	*.00011069*
3	21	.00000048	.00000190	.00000473	.00000944	.00001645	.00002623	*.00003921*	*.00005581*
3	22	.00000024	.00000095	.00000237	.00000473	.00000826	.00001317	*.00001971*	*.00002808*
3	23	.00000012	.00000048	.00000119	.00000237	.00000414	.00000661	*.00000989*	*.00001411*
3	24	.00000006	.00000024	.00000059	.00000119	.00000207	.00000331	*.00000496*	*.00000708*
3	25	.00000003	.00000012	.00000030	.00000059	.00000104	.00000166	*.00000249*	*.00000355*
3	26	.00000001	.00000006	.00000015	.00000030	.00000052	.00000083	*.00000125*	*.00000178*
3	27	.00000001	.00000003	.00000007	.00000015	.00000026	.00000042	*.00000062*	*.00000089*
3	28	.00000000	.00000001	.00000004	.00000007	.00000013	.00000021	*.00000031*	*.00000045*
3	29	.00000000	.00000001	.00000002	.00000004	.00000007	.00000010	*.00000016*	*.00000022*
3	30	.00000000	.00000000	.00000001	.00000002	.00000003	.00000005	*.00000008*	*.00000011*
3	31	.00000000	.00000000	.00000000	.00000001	.00000002	.00000003	*.00000004*	*.00000006*
3	32	.00000000	.00000000	.00000000	.00000000	.00000001	.00000001	*.00000002*	*.00000003*
3	33	.00000000	.00000000	.00000000	.00000000	.00000000	.00000001	*.00000001*	*.00000001*
3	34	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	*.00000000*	*.00000001*
3	35	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	*.00000000*	*.00000000*
3	36	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	*.00000000*	*.00000000*
3	37	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	*.00000000*	*.00000000*
3	38	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	*.00000000*	*.00000000*
3	39	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	*.00000000*	*.00000000*
3	40	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	*.00000000*	*.00000000*
3	41	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	*.00000000*	*.00000000*
3	42	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	*.00000000*	*.00000000*
3	43	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	*.00000000*	*.00000000*
3	44	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	*.00000000*	*.00000000*
3	45	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	*.00000000*	*.00000000*
3	46	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	*.00000000*	*.00000000*
3	47	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	*.00000000*	*.00000000*
3	48	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	*.00000000*	*.00000000*

Table 25: Sheet B. Probabilities of perfect column triplets ($k = 3,\ i = 0$) in Bernoulli ($m \times n$)-matrices. The exact probabilities are shown by the roman font style (columns 3–8 and rows 1–7). The accurately approximated probabilities computed by the Inclusion–Exclusion formula with $S = 6$ sums are in *italics* (below the table diagonal). Inaccurate approximations, which differ by more than 2.5% form the ones computed with $S = 5$ sums, are replaced by interpolated values **boldfaced**. The interpolation is performed column-by-column with the cubic spline method

k	m	Width n of Bernoulli ($m \times n$)-matrix					
		11	12	13	14	15	16
3	1	.99414063	.99682617	.99829102	.99908447	.99951172	.99974060
3	2	.95239639	.96539468	.97465602	.98132761	.98617879	.98973327
3	3	.88471509	.91254806	.93371826	.94980919	.96202954	.97130156
3	4	.78136257	.82603278	.86231031	.89154963	.91495750	.93358279
3	5	.64405187	.70130374	.75102324	.79372946	.83006016	.86070366
3	6	.49213106	.55277591	.60876972	.65981562	.70584123	.74694029
3	7	.34947075	.40436962	.45779233	.50907133	.55771770	.60339607
3	8	**.23288424**	**.27708502**	**.32198510**	**.36629022**	**.41076216**	**.45420457**
3	9	**.14679688**	**.17920271**	**.21333702**	**.24744230**	**.28371656**	**.32032996**
3	10	**.08814653**	**.11011057**	**.13398773**	**.15797262**	**.18491987**	**.21290854**
3	11	**.05077729**	**.06470268**	**.08026706**	**.09594144**	**.11443885**	**.13414993**
3	12	**.02826012**	**.03659993**	**.04615177**	**.05579618**	**.06766137**	**.08060132**
3	13	**.01530322**	**.02006129**	**.02562850**	**.03127764**	**.03845668**	**.04645159**
3	14	**.00812008**	**.01072542**	**.01383079**	**.01701186**	**.02114234**	**.02582963**
3	15	**.00425178**	**.00562992**	**.00729903**	**.00903682**	**.01131282**	**.01393950**
3	16	*.00221247*	**.00292064**	**.00379038**	**.00471935**	**.00592804**	**.00734413**
3	17	*.00114741*	*.00150730*	**.00194897**	**.00243899**	**.00306099**	**.00379970**
3	18	*.00058782*	*.00077622*	*.00099847*	*.00125561*	**.00156714**	**.00194191**
3	19	*.00029906*	*.00039605*	*.00051121*	*.00064570*	*.00080045*	**.00098612**
3	20	*.00015146*	*.00020095*	*.00025991*	*.00032905*	*.00040905*	*.00050050*
3	21	*.00007646*	*.00010157*	*.00013155*	*.00016679*	*.00020767*	*.00025457*
3	22	*.00003850*	*.00005120*	*.00006637*	*.00008424*	*.00010500*	*.00012885*
3	23	*.00001936*	*.00002575*	*.00003341*	*.00004244*	*.00005293*	*.00006501*
3	24	*.00000972*	*.00001294*	*.00001679*	*.00002134*	*.00002663*	*.00003272*
3	25	*.00000487*	*.00000649*	*.00000843*	*.00001071*	*.00001338*	*.00001644*
3	26	*.00000244*	*.00000325*	*.00000423*	*.00000537*	*.00000671*	*.00000825*
3	27	*.00000122*	*.00000163*	*.00000212*	*.00000269*	*.00000336*	*.00000414*
3	28	*.00000061*	*.00000082*	*.00000106*	*.00000135*	*.00000169*	*.00000207*
3	29	*.00000031*	*.00000041*	*.00000053*	*.00000068*	*.00000084*	*.00000104*
3	30	*.00000015*	*.00000020*	*.00000027*	*.00000034*	*.00000042*	*.00000052*
3	31	*.00000008*	*.00000010*	*.00000013*	*.00000017*	*.00000021*	*.00000026*
3	32	*.00000004*	*.00000005*	*.00000007*	*.00000008*	*.00000011*	*.00000013*
3	33	*.00000002*	*.00000003*	*.00000003*	*.00000004*	*.00000005*	*.00000007*
3	34	*.00000001*	*.00000001*	*.00000002*	*.00000002*	*.00000003*	*.00000003*
3	35	*.00000000*	*.00000001*	*.00000001*	*.00000001*	*.00000001*	*.00000002*
3	36	*.00000000*	*.00000000*	*.00000000*	*.00000001*	*.00000001*	*.00000001*
3	37	*.00000000*	*.00000000*	*.00000000*	*.00000000*	*.00000000*	*.00000000*
3	38	*.00000000*	*.00000000*	*.00000000*	*.00000000*	*.00000000*	*.00000000*
3	39	*.00000000*	*.00000000*	*.00000000*	*.00000000*	*.00000000*	*.00000000*
3	40	*.00000000*	*.00000000*	*.00000000*	*.00000000*	*.00000000*	*.00000000*
3	41	*.00000000*	*.00000000*	*.00000000*	*.00000000*	*.00000000*	*.00000000*
3	42	*.00000000*	*.00000000*	*.00000000*	*.00000000*	*.00000000*	*.00000000*
3	43	*.00000000*	*.00000000*	*.00000000*	*.00000000*	*.00000000*	*.00000000*
3	44	*.00000000*	*.00000000*	*.00000000*	*.00000000*	*.00000000*	*.00000000*
3	45	*.00000000*	*.00000000*	*.00000000*	*.00000000*	*.00000000*	*.00000000*
3	46	*.00000000*	*.00000000*	*.00000000*	*.00000000*	*.00000000*	*.00000000*
3	47	*.00000000*	*.00000000*	*.00000000*	*.00000000*	*.00000000*	*.00000000*
3	48	*.00000000*	*.00000000*	*.00000000*	*.00000000*	*.00000000*	*.00000000*

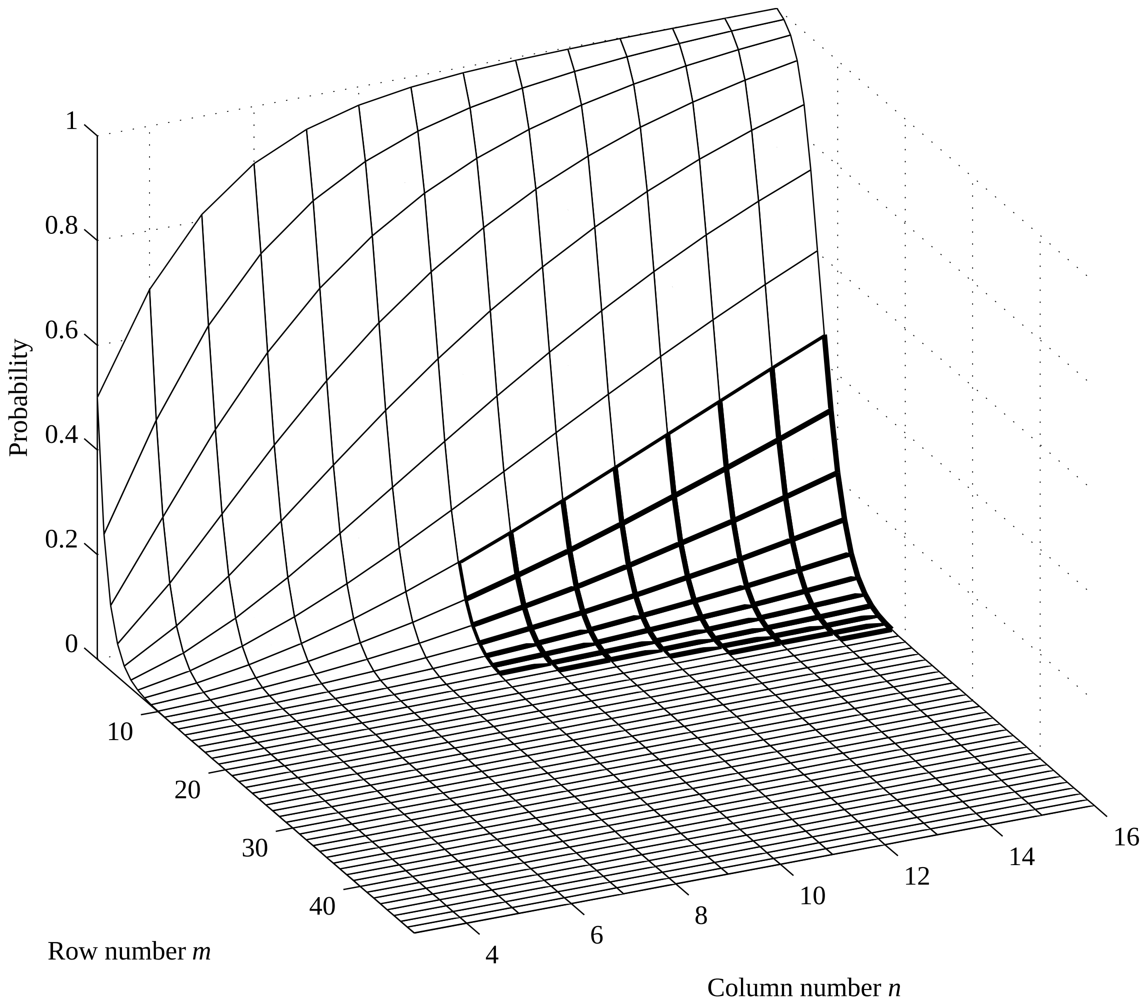


Figure 10: 3D relief of the probability Table 25 (for $k = 3,\ i = 0$) with the interpolated segments shown by **thick** curves

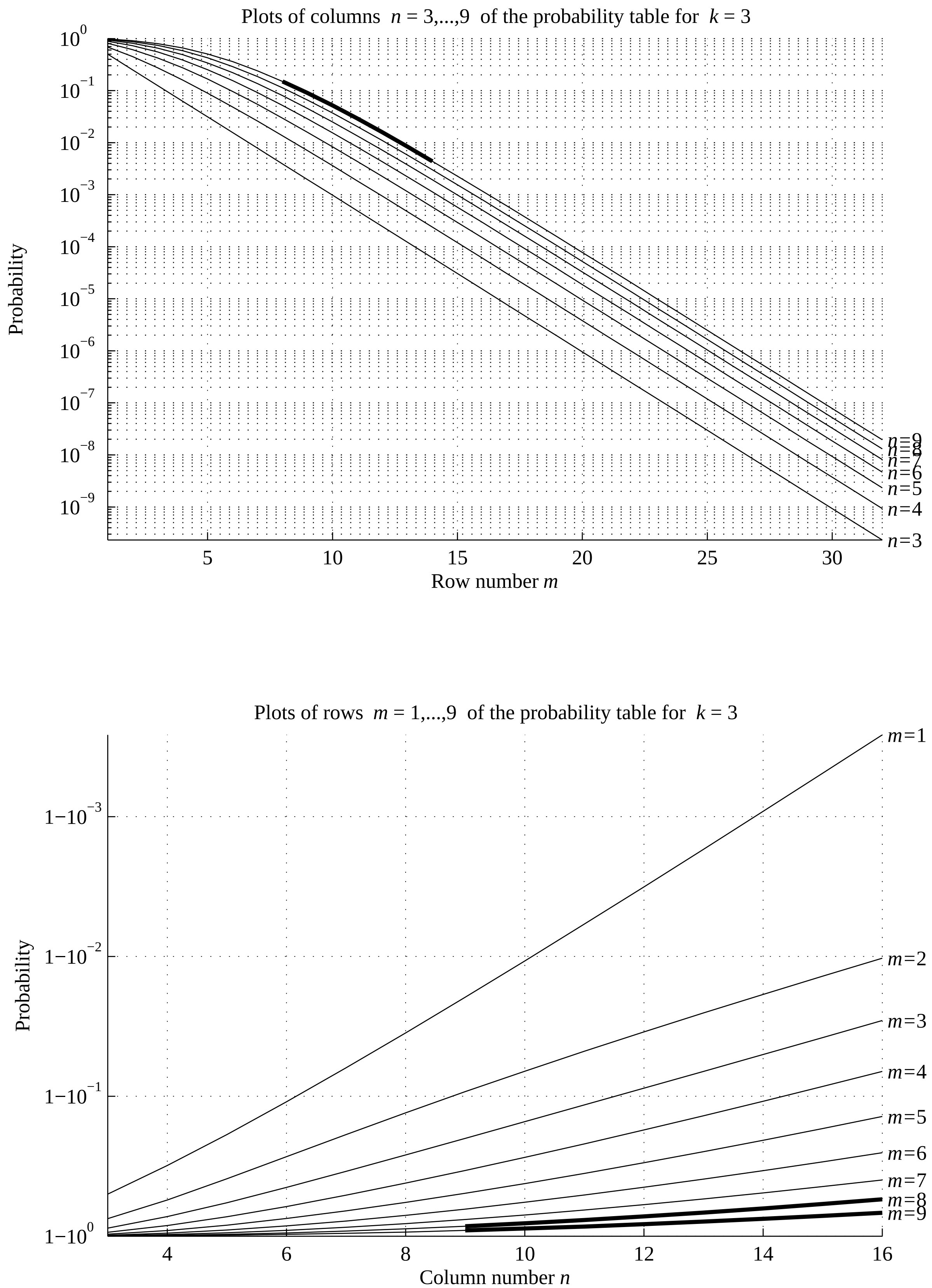


Figure 11: Plots of the logarithmically scaled first columns and first rows of Table 25 of probabilities of perfect column triplets ($k = 3,\ i = 0$) with the interpolated segments shown by **thick** curves

5.3 Interpolation of probabilities on degree of imperfectness i

Restrict attention to probabilities of imperfect column *pairs*. The consideration of imperfect column triplets is identical.

We dispose the probabilities of outcomes of Bernoulli $(m \times n)$-matrices with the following degrees of imperfectness i:

- $i = -1$ (impossible outcome): probability 0.
- $i = 0$ (perfect outcomes): exact, accurately approximated, and interpolated probabilities collected in Table 24.
- $i = 1, 2, \ldots$ (imperfect outcomes): exact probabilities for certain (m, n)-combinations computed by Geometric and/or Algebraic method; see Table 7 and 13. For Bernoulli matrices with 2–3 columns, the Geometric method gives exact probabilities for the degrees of imperfectness up to $i = 48$, and for four-column matrices — up to $i = 16$.
- $i = m-2, m-1$ ('very imperfect' outcomes): exact probabilities computed by the Algebraic method; see Section 3.7.
- $i = m$ (certain outcome): probability 1.

The upper plot of Figure 12 exhibits probability curves as functions of i. Already for small vertical size m of Bernoulli matrix, these S-shaped curves resemble cumulative binomial and β-distribution functions. To emphasize the similarity, the horizontal axis of the plot is complemented with point $i = -1$ where the probabilities are zeros.

The reference to the binomial distribution and β-distributions is not accidental. As follows from (28), i-imperfect outcomes of a two-column Bernoulli matrix have the binomial probability. According to Abramovitz and Stegun (1972, 26.5.7 and 26.5.24), the latter is expressed by the incomplete β-function, so that

$$\begin{aligned} \mathsf{P}(i\text{-imperfect outcome of Bernoulli } (m \times 2)\text{-matrix}) &= \sum_{j=0}^{i} \binom{m}{j} \left(\frac{1}{4}\right)^j \left(\frac{3}{4}\right)^{m-j} && (39) \\ &= I_{\frac{3}{4}}(m-i, i+1) \ , && (40) \end{aligned}$$

where the incomplete β-function

$$I_x(a,b) = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} \int_0^x t^{a-1}(1-t)^{b-1}\, dt, \quad a > 0, \quad b > 0, \quad 0 \le x \le 1 \ . \tag{41}$$

Naturally, some features of binomial and β-distributions manifest themselves in more complex cases as well.

Thus for $i = -1, \ldots, m$, some probabilities are known, particularly, for the first and last i's. It enables interpolating between the known probabilities by fitting binomial and β-curves. For binomial fit, use function (39) with two unknown parameters M and p $\left(\text{instead of } m \text{ and } \frac{1}{4}\right)$ in the form of incomplete β-function (40)

$$y_{\text{Bnml}}(i) = \begin{cases} \sum_{j=0}^{i} \binom{M}{j} (1-p)^j p^{M-j} = I_p(M-i, i+1) & \text{if} \quad i = 0, \ldots, M \\ 1 & \text{if} \quad i > M \end{cases} \ . \tag{42}$$

For β-fit, use function (41) with two unknown parameters a and b

$$y_\beta(i) = I_{\frac{i+1}{m+1}}(a, b), \quad i = -1, \ldots, m \ . \tag{43}$$

There is a minor difference in using these two functions. In (43), the properly fitting domain $i = -1, \ldots, m$ is fixed, whereas M in (42) is one of unknown parameters. Besides, in (43) parameters a and b are independent, whereas in (42) they are bound by the constant sum $a + b = (M - i) + (i + 1) = M + 1$.

The interpolation by fitting binomial and β-curves is implemented with the MATLAB program `lsqcurvefit` from the *Optimization toolbox*, which fits non-linear functions in the 'least-squares sense'. The fitting results are exemplified in the middle and bottom plots of Figure 12. In fact, all points here are known (taken from the upper plot), but the first and last points are used for fitting, and others serve to control the interpolation accuracy. The fitted segments are shown by thin curves, interpolated segments by **thick** ones, and the known probabilities from the upper plot are shown by stars $*$. Tables 26 and 27 display fit and interpolation errors for the case depicted in Figure 12 (in the left-hand halves of the tables) and for the case of more numerous fitting points (for $i \leq 5$ instead of $i \leq 3$). Correspondingly, the fit accuracy is worse, and the interpolation accuracy is better.

Figure 13 and Table 28 show how the interpolation performed along the imperfectness i-dimension of the probabilities looks from the viewpoint of the n-dimension — along the variable width of Bernoulli matrix. As seen in Figure 13, where the corresponding curves are marked with 'Bnml' and β labels, the accuracy improves as the width n of Bernoulli matrix increases.

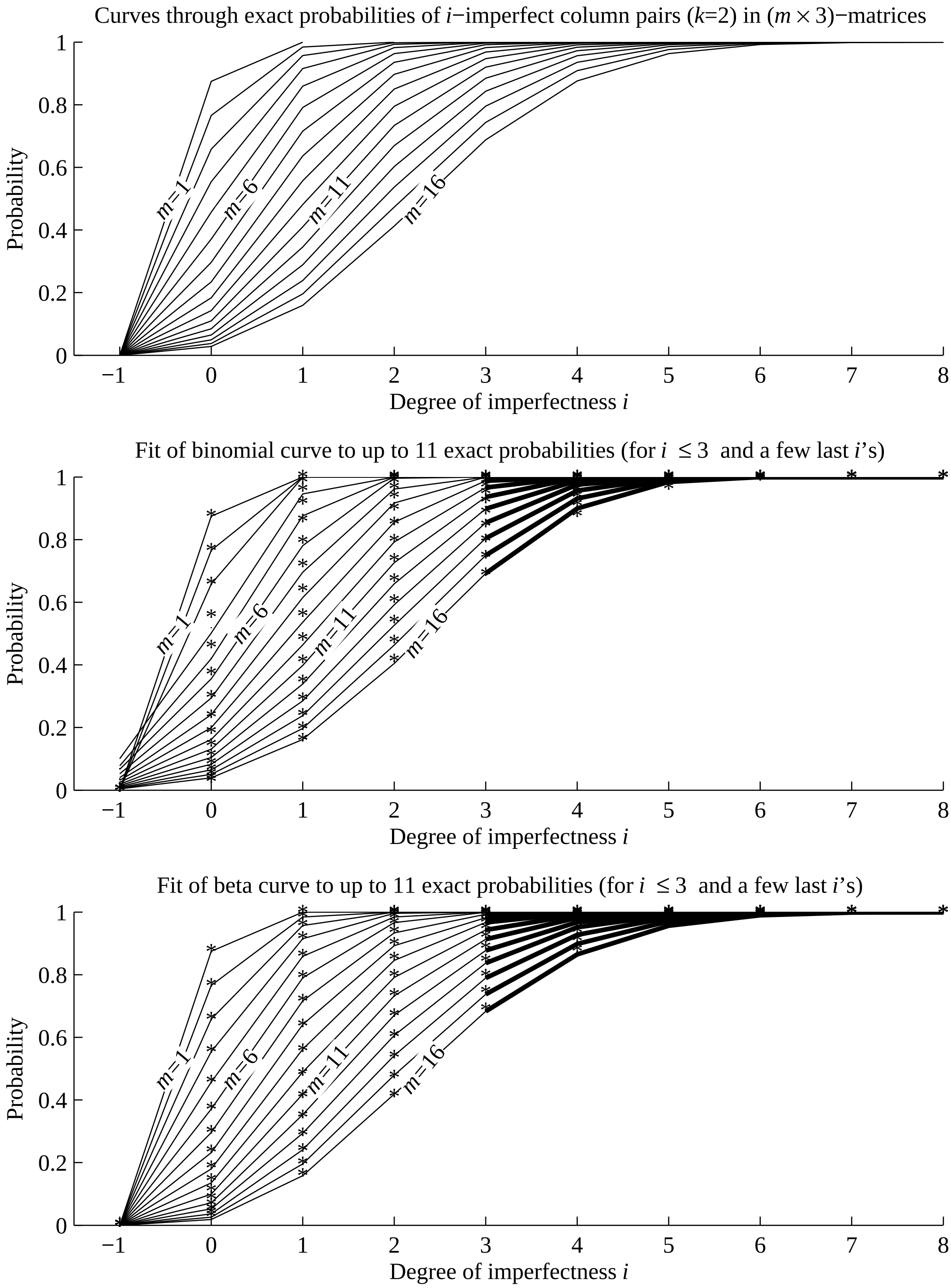


Figure 12: Interpolation of probabilities of i-imperfect column pairs ($k = 2$) in Bernoulli ($m \times 3$)-matrices. The domain of i-imperfectness is conditionally extended to -1, where the probability is equal to 0. The exact probabilities in the middle and bottom charts are shown by stars (*). Every interpolating curve is fitted to the first 5 points (probabilities of i-imperfect column pairs for $i = -1, \ldots, 3$) and a few last i's. The interpolated curves for $i > 3$ are shown by **thick** segments. The interpolation errors are shown by dotted lines

Table 26: Errors of binomial interpolation of probabilities of i-imperfect column pairs ($k = 2$) in Bernoulli ($m \times 3$)-matrices. The errors of fit to exact probabilities are shown in roman font style. The interpolation errors are in **bold**

i	Fit up to 11 points ($i \leq 3$ and a few last i's)				Fit up to 13 points ($i \leq 5$ and a few last i's)			
	$m = 1$	$m = 6$	$m = 11$	$m = 16$	$m = 1$	$m = 6$	$m = 11$	$m = 16$
−1	0.0008	0.0663	0.0189	0.0042	0.0008	0.0663	0.0193	0.0051
0	−0.0000	−0.0153	0.0205	0.0106	−0.0000	−0.0153	0.0217	0.0148
1	0.0000	−0.0108	−0.0120	0.0028	0.0000	−0.0108	−0.0112	0.0098
2		0.0324	−0.0075	−0.0083		0.0324	−0.0091	−0.0076
3		0.0034	0.0166	0.0046		0.0034	0.0144	−0.0077
4		0.0002	**0.0133**	**0.0235**		0.0002	0.0125	0.0066
5		0.0000	**0.0022**	**0.0207**		0.0000	0.0022	0.0112
6		0.0000	**0.0002**	**0.0078**		0.0000	**0.0002**	**0.0058**
7			**0.0000**	**0.0013**			**0.0000**	**0.0013**
8			0.0000	**0.0002**			0.0000	**0.0002**
9			0.0000	**0.0000**			0.0000	**0.0000**
10			0.0000	**0.0000**			0.0000	**0.0000**
11			0.0000	0.0000			0.0000	0.0000
12				0.0000				0.0000
13				0.0000				0.0000
14				0.0000				0.0000
15				0.0000				0.0000
16				0.0000				0.0000
Total square error:	0.0000	0.0058	0.0013	0.0002	0.0000	0.0058	0.0014	0.0006
Fit/**Interpolation**			**0.0002**	**0.0010**			**0.0000**	**0.0000**
Maximal error:	0.0008	0.0663	0.0205	0.0106	0.0008	0.0663	0.0217	0.0148
Fit/**Interpolation**			**0.0133**	**0.0235**			**0.0002**	**0.0058**

Table 27: Errors of beta interpolation of probabilities of i-imperfect column pairs ($k = 2$) in Bernoulli ($m \times 3$)-matrices. The errors of fit to exact probabilities are shown in roman font style. The interpolation errors are in **bold**

i	Fit up to 11 points ($i \leq 3$ and a few last i's)				Fit up to 13 points ($i \leq 5$ and a few last i's)			
	$m = 1$	$m = 6$	$m = 11$	$m = 16$	$m = 1$	$m = 6$	$m = 11$	$m = 16$
−1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0	−0.0000	0.0005	−0.0107	−0.0103	−0.0000	0.0005	−0.0110	−0.0118
1	0.0000	−0.0015	0.0087	−0.0013	0.0000	−0.0015	0.0086	−0.0046
2		0.0027	−0.0013	0.0062		0.0027	−0.0010	0.0069
3		0.0023	−0.0066	−0.0045		0.0023	−0.0063	0.0008
4		0.0002	**−0.0022**	**−0.0110**		0.0002	−0.0021	−0.0051
5		0.0000	**0.0001**	**−0.0069**		0.0000	0.0001	−0.0034
6		0.0000	**0.0001**	**−0.0019**		0.0000	**0.0001**	**−0.0006**
7			**0.0000**	**−0.0001**			**0.0000**	**0.0002**
8			0.0000	**0.0001**			0.0000	**0.0001**
9			0.0000	**0.0000**			0.0000	**0.0000**
10			0.0000	**0.0000**			0.0000	**0.0000**
11			0.0000	0.0000			0.0000	0.0000
12				0.0000				0.0000
13				0.0000				0.0000
14				0.0000				0.0000
15				0.0000				0.0000
16				0.0000				0.0000
Total square error:	0.0000	0.0000	0.0002	0.0002	0.0000	0.0000	0.0002	0.0002
Fit/**Interpolation**			**0.0000**	**0.0002**			**0.0000**	**0.0000**
Maximal error:	0.0000	0.0027	0.0107	0.0103	0.0000	0.0027	0.0110	0.0118
Fit/**Interpolation**			**0.0022**	**0.0110**			**0.0001**	**0.0006**

5.4 Interpolation of probabilities on the matrix width n

For Bernoulli matrices with 2–3 columns exact probabilities are known for the degrees of imperfectness up to $i = 48$, and for four-column matrices — up to $i = 16$; see Table 7. Taking into account that the probabilities grow along the rows of the table and their distance to probability 1 decreases quasi-exponentially, interpolate the missing probabilities by fitting exponential curves to the first known points and the infinite probability 1.

For this purpose, consider fitting functions with unknown parameters a and b

$$y_{\text{Exp}}(n) = 1 - a2^{-bn}, \quad n = 2, 3, \ldots.$$

This type of interpolation is also implemented with the MATLAB program `lsqcurvefit` from the *Optimization toolbox* applied along the n-dimension of the probability array.

The probabilities interpolated are shown in Figure 13 and Table 28. As seen in Figure 13, where the corresponding curves are marked with 'Expn' label, the accuracy improves as the width n of Bernoulli matrix increases.

5.5 Interpolation with the Inclusion-Exclusion formula

As mentioned in Section 5.3, i-imperfect outcomes of a two-column Bernoulli matrix are binomially distributed, and Bernoulli $(m \times n)$-matrices with $n > 2$ columns to a certain extent inherit this property. Indeed, the quasi-linear curve tails in the log-scaled upper plot of Figure 9 imply that 'tails' of every n-th column of Table 24 can be represented by a product of almost equal factors $p_1 \cdots p_m$. The factors p_j are ratios of subsequent elements of the nth column of Table 24 and can be interpreted as probabilities of 'independent' contribution of the jth row to the perfect outcome. By the Central Limit Theorem (2012), an average 'independently' contributing row is characterized by the mean

$$p = \frac{p_1 + \cdots + p_m}{m}$$

and a i-imperfect outcome with i 'not contributing' rows occurs with the approximating binomial probability

$$\mathsf{P}\{i\text{-imperfect outcome of Bernoulli } (m \times n)\text{-matrix}\} \approx \binom{m}{i} p^{m-i}(1-p)^i \ .$$

The probabilities interpolated this way from the ones obtained by the Inclusion-Exclusion formula are illustrated in Figure 13 and Table 28, where they are marked by the 'In-Ex' label.

5.6 Bringing interpolated probabilities together

The results of the four interpolations, labeled 'Bnml', 'β', 'Expn', and 'In-Ex' are exemplified in Figure 13. The three plots show interpolated probabilities of i-imperfect outcomes of a eight-row Bernoulli matrix of variable width n for $i = 2, 3, 4$. These and some other interpolated probabilities are collected in Table 28, where exact probabilities are given with eight decimals, and interpolated probabilities with two decimals.

Following the 'error correction' practice by taking the mean of observed values, the missing probabilities are estimated as the mean of the four interpolation results. These final estimates constitute tables of Appendix 2.

The case of column *triplets* is identical to the case of column pairs. The final estimates of probabilities of occurrence of perfect and imperfect column triplets in Bernoulli matrices are obtained in the same way and presented in Appendix 2.

5.7 Summary

Thus, for Bernoulli matrices with a few columns, exact probabilities are computed. For wider matrices, approximate probabilities are interpolated.

It should be noted that the interpolation methods considered are partially heuristic and do not guarantee reliable estimates of unknown probabilities. The situation is getting more critical if the probabilities interpolated are relatively numerous, if they are 'distant' from the known ones, and/or when the known probabilities are rather few. Therefore, the estimates obtained should be regarded rather as a conditional reference.

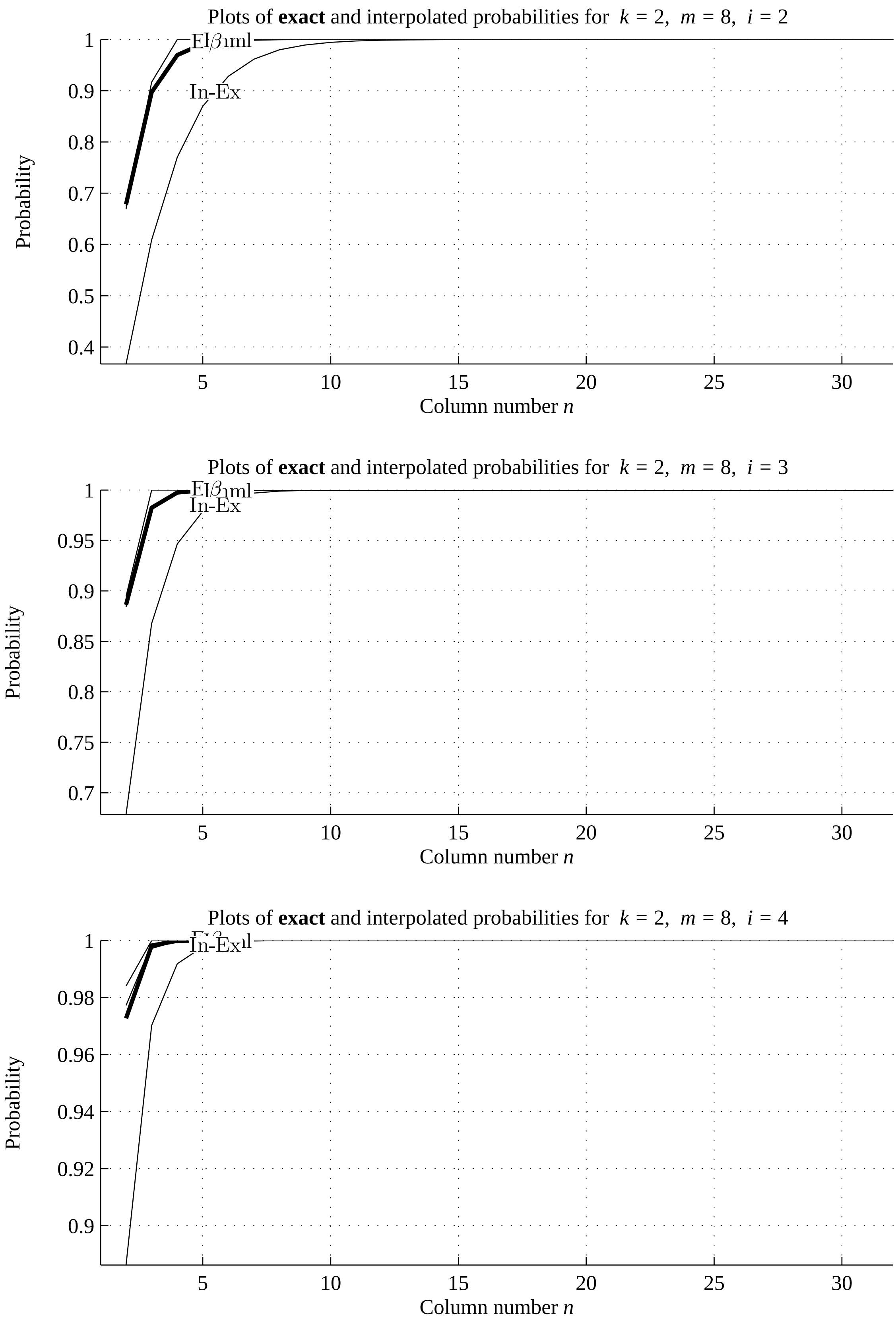


Figure 13: Plots of interpolated probabilities of i-imperfect column pairs ($k = 2$) in Bernoulli ($m \times n$)-matrices. The interpolation is made by exponential, binomial, and beta curve fit and with the Inclusion-Exclusion formula and denoted respectively as Expn, Bnml, β, and In-Ex. The exact probabilities, if known, are shown by **thick** segments.

Table 28: Exact and interpolated probabilities of perfect ($i = 0$) and i-imperfect column pairs ($k = 2$) in Bernoulli ($m \times n$)-matrices. The exact probabilities are given with 8 decimals. The interpolated probabilities are given with 2 decimals in four versions: Expn, Bnml, β, and In-Ex — computed respectively with exponential, binomial, and beta curve fit and with the Inclusion-Exclusion formula

k m i	Width n of Bernoulli ($m \times n$)-matrix								
	2	3	4	5	6	7	8	9	10
2 8 0	.10011292	.23439580	.37592543	.50792317	.62241199	.71685073	.79185026	.84669472	.88873706
	.05 Expn	.26 Expn	.42 Expn	.54 Expn	.64 Expn	.72 Expn	.78 Expn	.83 Expn	.87 Expn
	.12 Bnml	.24 Bnml	.35 Bnml	.46 Bnml	.62 Bnml	.72 Bnml	.79 Bnml	.85 Bnml	.89 Bnml
	.09 β	.23 β	.38 β	.51 β	.62 β	.72 β	.79 β	.85 β	.89 β
	.10 In-Ex	.23 In-Ex	.38 In-Ex	.51 In-Ex	.62 In-Ex	.72 In-Ex	.79 In-Ex	.85 In-Ex	.89 In-Ex
1	.36708069	.63633424	.80619312	.90205796	.95250528				
	.36 Expn	.65 Expn	.81 Expn	.89 Expn	.94 Expn	.97 Expn	.98 Expn	.99 Expn	.99 Expn
	.36 Bnml	.61 Bnml	.82 Bnml	.92 Bnml	1.00 Bnml	1.00 Bnml	1.00 Bnml	1.00 Bnml	1.00 Bnml
	.37 β	.64 β	.81 β	.90 β	.95 β	.97 β	.98 β	.99 β	1.00 β
	.10 In-Ex	.24 In-Ex	.38 In-Ex	.51 In-Ex	.63 In-Ex	.72 In-Ex	.79 In-Ex	.85 In-Ex	.89 In-Ex
2	.67854309	.89700443	.96945792	.99141809					
	.68 Expn	.90 Expn	.97 Expn	.99 Expn	1.00 Expn	1.00 Expn	1.00 Expn	1.00 Expn	1.00 Expn
	.67 Bnml	.92 Bnml	1.00 Bnml	1.00 Bnml	1.00 Bnml	1.00 Bnml	1.00 Bnml	1.00 Bnml	1.00 Bnml
	.68 β	.89 β	.97 β	.99 β	1.00 β	1.00 β	1.00 β	1.00 β	1.00 β
	.37 In-Ex	.61 In-Ex	.77 In-Ex	.87 In-Ex	.93 In-Ex	.96 In-Ex	.98 In-Ex	.99 In-Ex	.99 In-Ex
3	.88618469	.98249370	.99745115	.99964252					
	.89 Expn	.98 Expn	1.00 Expn	1.00 Expn	1.00 Expn	1.00 Expn	1.00 Expn	1.00 Expn	1.00 Expn
	.89 Bnml	1.00 Bnml	1.00 Bnml	1.00 Bnml	1.00 Bnml	1.00 Bnml	1.00 Bnml	1.00 Bnml	1.00 Bnml
	.88 β	.98 β	1.00 β	1.00 β	1.00 β	1.00 β	1.00 β	1.00 β	1.00 β
	.68 In-Ex	.87 In-Ex	.95 In-Ex	.98 In-Ex	.99 In-Ex	1.00 In-Ex	1.00 In-Ex	1.00 In-Ex	1.00 In-Ex
4	.97270203	.99821919	.99988566	.99999264					
	.97 Expn	1.00 Expn	1.00 Expn	1.00 Expn	1.00 Expn	1.00 Expn	1.00 Expn	1.00 Expn	1.00 Expn
	.98 Bnml	1.00 Bnml	1.00 Bnml	1.00 Bnml	1.00 Bnml	1.00 Bnml	1.00 Bnml	1.00 Bnml	1.00 Bnml
	.98 β	1.00 β	1.00 β	1.00 β	1.00 β	1.00 β	1.00 β	1.00 β	1.00 β
	.89 In-Ex	.97 In-Ex	.99 In-Ex	1.00 In-Ex	1.00 In-Ex	1.00 In-Ex	1.00 In-Ex	1.00 In-Ex	1.00 In-Ex
5	.99577332	.99989480	.99999733						
	1.00 Expn	1.00 Expn	1.00 Expn	1.00 Expn	1.00 Expn	1.00 Expn	1.00 Expn	1.00 Expn	1.00 Expn
	1.00 Bnml	1.00 Bnml	1.00 Bnml	1.00 Bnml	1.00 Bnml	1.00 Bnml	1.00 Bnml	1.00 Bnml	1.00 Bnml
	1.00 β	1.00 β	1.00 β	1.00 β	1.00 β	1.00 β	1.00 β	1.00 β	1.00 β
	.97 In-Ex	1.00 In-Ex	1.00 In-Ex	1.00 In-Ex	1.00 In-Ex	1.00 In-Ex	1.00 In-Ex	1.00 In-Ex	1.00 In-Ex
6	.99961853	.99999660	.99999997						
	1.00 Expn	1.00 Expn	1.00 Expn	1.00 Expn	1.00 Expn	1.00 Expn	1.00 Expn	1.00 Expn	1.00 Expn
	1.00 Bnml	1.00 Bnml	1.00 Bnml	1.00 Bnml	1.00 Bnml	1.00 Bnml	1.00 Bnml	1.00 Bnml	1.00 Bnml
	1.00 β	1.00 β	1.00 β	1.00 β	1.00 β	1.00 β	1.00 β	1.00 β	1.00 β
	1.00 In-Ex	1.00 In-Ex	1.00 In-Ex	1.00 In-Ex	1.00 In-Ex	1.00 In-Ex	1.00 In-Ex	1.00 In-Ex	1.00 In-Ex
7	.99998474	.99999994	1.00000000						
	1.00 Expn	1.00 Expn	1.00 Expn	1.00 Expn	1.00 Expn	1.00 Expn	1.00 Expn	1.00 Expn	1.00 Expn
	1.00 Bnml	1.00 Bnml	1.00 Bnml	1.00 Bnml	1.00 Bnml	1.00 Bnml	1.00 Bnml	1.00 Bnml	1.00 Bnml
	1.00 β	1.00 β	1.00 β	1.00 β	1.00 β	1.00 β	1.00 β	1.00 β	1.00 β
	1.00 In-Ex	1.00 In-Ex	1.00 In-Ex	1.00 In-Ex	1.00 In-Ex	1.00 In-Ex	1.00 In-Ex	1.00 In-Ex	1.00 In-Ex

6 Example: Evaluation of German parties and coalitions

Let us come back to the questions posed in Introduction. Apply the results obtained to evaluate the representative capacity of five eligible German parties (CDU/CSU—conservative union, SPD—social democrats, FDP—neo-liberals, Greens—ecologists, and Left-Party—left social democrats) and their possible coalitions at the time of Bundestag elections 2009.

Table 29 visualized by Figure 14 shows positions of the five parties on 32 topical policy issues as well as outcomes of polls of public opinion on these issues; see Tangian (2012) for references to data sources. We are going to estimate the statistical significance of representative capacity of the parties and their coalitions understood as capability to express the majority opinion.

To explain the figure, consider the top question: '2. Introduce nation-wide minimal wage'. Each party is depicted by a rectangle, whose length is proportional to the number of the party seats in the Bundestag. The 'No/Yes' party opinion on the question is reflected by the location of the rectangle to the left side or to the right side from the central vertical axis, respectively. A Bundestag majority is attained if the cumulative length of party rectangles surpasses the 50%-threshold (marked with dotted lines). The outcomes of the related public surveys are shown by the blue bars with the length normalized to 100% (abstaining respondents are ignored). Their bias from the center indicates at the prevailing public opinion.

For every question, a given party represents either a majority, or a minority of the population (identified with the fraction in the opinion polls). For instance, the CDU/CSU (black rectangle) with the 'No' answer to the top question '2. Introduce nation-wide minimal wage' represents the opinion of 43% of the population against 52%; see Table 29 for exact figures. After normalization, we obtain that its *representativeness* for question 2 is

$$r_{\text{CDU/CSU},2} = \frac{43}{43+52} \cdot 100\% \approx 45\% \ .$$

Similarly, with the 'No' answer to the next question '17. Relax protection against dismissals', the CDU/CSU expresses the opinion of 82% of the population against 17%. After normalization we obtain its representativeness for question 17

$$r_{\text{CDU/CSU},17} = \frac{82}{82+17} \cdot 100\% \approx 83\% \ ,$$

and so on.

The frequency of representing a majority ($\geq 50\%$) is defined to be the *universality* of the party. As one can see, the CDU/CSU represents a majority on 15 questions from 32, that is, with the frequency

$$\mathsf{U}_{\text{CDU/CSU}} = \frac{15}{32} \cdot 100\% \approx 47\% \qquad \text{(Degree of imperfectness } 32 - 17 = 15) \ .$$

A higher universality means that a majority is represented more frequently. For instance *die Linke* represents a majority on 22 of 32 questions, resulting in

$$\mathsf{U}_{\text{Left-Party}} = \frac{22}{32} \cdot 100\% \approx 69\% \qquad \text{(Degree of imperfectness } 32 - 22 = 10) \ .$$

The universality indices and the degree of imperfectness of the parties are shown in the upper section of Table 30.

As for coalitions, it is assumed that the position of a three- or two-party coalition on a given issue is defined by two parties sharing the same opinion. In case of a tie opinion in a two-party coalition, the prevailing public opinion is assumed decisive, as if influencing the internal

Table 29: Data for the model

The Wahl-O-Mat question number and the question shortly formulated (*adjustments based on party public statements, parliamentary voting, etc.)	Party positions and votes received, in %					Survey results, in %	
	CDU/CSU 33.8%	SPD 23.0%	FDP 14.6%	Left-Party 11.9%	Greens 10.7%	Protago-nists	Antago-nists
Labour market							
2. Introduce nation-wide minimal wage	No	Yes	No	Yes	Yes	52	43
17. Relax protection against dismissals	No	No	Yes	No	No	17	82
Economy and taxes							
24. Exclusive governmental owernship of railways	No	Yes	No	Yes	Yes	70	28
10. Equity holding by government in private banks has to be temporary	Yes	Yes	Yes	No	Yes	28	67
5. No state control over top-management salaries*	Yes	Yes	Yes	No	No	30	67
13. Decrease corporate taxes	Yes	No	Yes	No	No	23	59
28. Reintroduce a wealth tax*	No	Yes	No	Yes	Yes	35	65
Environment							
1. Prolong the operation time of nuclear power plants	Yes	No	Yes	No	No	39	55
19. Introduce a general speed limit on Autobahnen (German motorways)	No	Yes	No	Yes	Yes	34	41
9. Unexceptionally ban experiments on animals	No	No	No	Yes	Yes	19	80
26. Authorize production of genetically modified food*	Yes	Yes	Yes	No	No	33	67
Social policy							
36. Increase significantly unemployment benefits (Hartz IV)*	No	No	No	Yes	Yes	48	36
32. If wages decrease, pensions can be reduced*	No	No	Yes	No	Yes	28	68
31. No Praxisgebuehr (quarterly fee for medical visits)	No	No	Yes	Yes	Yes	71	26
15. Compensation to parents who use no public daycares	Yes	No	No	No	No	65	33
Education							
29. Leave the education policy under the authority of the states	Yes	No	Yes	No	No	9	81
34. Leave 3 types of schools with different access to further education	Yes	No	Yes	No	No	63	31

Table 29: Sheet B. Data for the model

The Wahl-O-Mat question number and the question shortly formulated (*adjustments based on party public statements, parliamentary voting, etc.)	Party positions and votes received, in %					Survey results, in %	
	CDU/CSU 33.8%	SPD 23.0%	FDP 14.6%	Left-Party 11.9%	Greens 10.7%	Protagonists	Antagonists
16. Guarantee an apprenticeship training position for every adolescent*	No	Yes	No	Yes	Yes	81	19
7. The first university degree should be free of tuition fees*	No	Yes	No	Yes	Yes	53	47
Gender							
12. There should be a quota for women in leading positions	No	Yes	No	Yes	Yes	34	64
27. Full adoption rights for homosexual couples	No	Yes	Yes	Yes	Yes	51	49
Domestic policy							
6. Prohibit secret online surveillance of private computers	No	No	Yes	Yes	Yes	39	57
25. Retain the compulsory military service	Yes	No	No	No	No	41	53
37. Allow domestic use of German military forces against terrorism	Yes	No	No	No	No	69	28
14. Introduce referenda at the federal level	No	Yes	Yes	Yes	Yes	68	26
23. German politics should follow Christian values*	Yes	No	No	No	No	73	20
38. The German democracy is the best form of government	Yes	Yes	Yes	Yes	Yes	77	11
35. Municipal voting rights for foreign permanent residents	No	Yes	Yes	Yes	Yes	44	42
Foreign policy							
3. Immediate withdrawl of German troops from Afghanistan	No	No	No	Yes	No	57	37
European policy							
4. Germany should leave the European Union	No	No	No	No	No	12	86
18. Turkey should be a full member of the European Union	No	Yes	No	Yes	Yes	37	58
21. Reintriduce the D-Mark	No	No	No	No	No	34	65

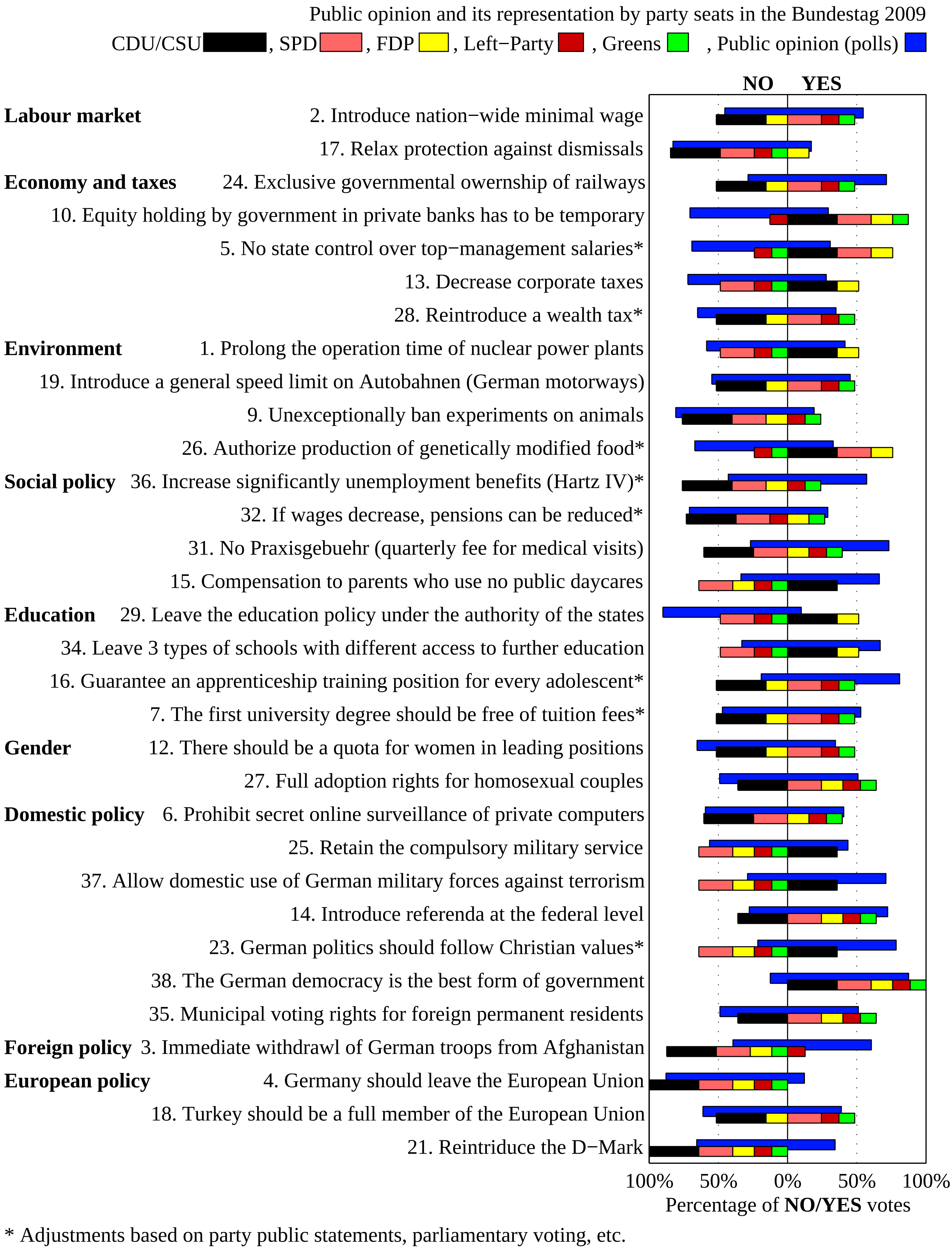


Figure 14: Public opinion and party positions on 32 issues

Table 30: Universality indices of parties and coalitions and statistical significance of their representative capacity. Interpolated significance values are **boldfaced**

Single party or coalition	Universality in %	Ranks	Degree of i-imperfectness	Significance
CDU/CSU	46.9	13	17	.9976
SPD	56.3	10	14	.8299
FDP	43.8	14	18	.9998
Left-Party	68.8	7	10	.1191
Greens	59.4	9	13	.6482
CDU/CSU/SPD	81.3	4	6	**.7293**
CDU/CSU/FDP	62.5	8	12	**.9970**
CDU/CSU/Left-Party	100.0	1	0	.0010
CDU/CSU/Greens	93.8	2	2	.0587
SPD/FDP	75.0	6	8	**.9228**
SPD/Left-Party	75.0	6	8	**.9228**
SPD/Greens	68.8	7	10	**.9816**
FDP/Left-Party	87.5	3	4	.4072
FDP/Greens	78.1	5	7	**.8534**
Left-Party/Greens	68.8	7	10	**.9816**
CDU/CSU/SPD/FDP	50.0	12	16	**.8505**
CDU/CSU/SPD/Left-Party	56.3	10	14	**.6298**
CDU/CSU/SPD/Greens	56.3	10	14	**.6298**
CDU/CSU/FDP/Left-Party	50.0	12	16	**.8505**
CDU/CSU/FDP/Greens	46.9	13	17	**.9139**
CDU/CSU/Left-Party/Greens	62.5	8	12	**.3327**
SPD/FDP/Left-Party	56.3	10	14	**.6298**
SPD/FDP/Greens	53.1	11	15	**.7561**
SPD/Left-Party/Greens	62.5	8	12	**.3327**
FDP/Left-Party/Greens	59.4	9	13	**.4820**

coalition debate. After the positions of coalitions have been determined, their universality and imperfectness indices are defined in the same way as for single parties. These indices for possible two-party coalitions are collected in the middle section of Table 30, and for possible three-party coalitions — in its bottom section (since the CDU/CSU is a party union, the CDU/CSU/SPD is not a three-party but two-party coalition, which applies to other coalitions with CDU/CSU as well).

The statistical significance of the representative capacity of single parties and properly coalitions is shown in the last column of Table 30 (extracted from Tables 31–33). Here, the representative capacity of no party and of no three-party coalition is even 10%-statistically significant, to say nothing of the usual demand for 5%-significance. As for two-party coalitions, the only 5%-significant representative capacity is inherent in the 'impossible' coalition of the CDU/CSU with its extreme political opponent, the Left-Party. Its 100-% universality results from our assumption that in case of tie vote a two-party coalition is influenced by the public opinion. The CDU/CSU and the Left-Party have four unanimous positions, which coincide with the prevailing public opinion, and opposite opinions on the other 28 issues, which, according to our definition, turns into the opinion of majority of the population. Thereby the coalition 'perfectly represents' the majority opinions, which should be regarded rather critically.

Thus, the representative capacity of German parties and their 'possible' coalitions is not statistically significant. Moreover, a relatively high degree of imperfectness of match of their positions to the opinion of the majority of population leaves little hope that new surveys with additional policy issues can change this conclusion.

An indirect confirmation of the low representativeness of German parties and their coalitions is the visible discrepancy between the German electorate and the elected government. In particular, it manifests itself in the unprecedent violent response of population to some rather usual events like starting the construction of a new main railway station in Stuttgart (Protest gegen Stuttgart 21, 2010), or transporting atomic waste (Castor-Transport geht auf schwierigste Etappe 2010). It looks that German voters are not very consistent with their own political profiles, being driven by politicians' personalities rather than by their political positions, or follow traditional images of the parties, even if they are partially outdated.

7 Conclusions: What can be improved

There is a number of improvements which could be made.

The implementation of the Geometric approach enables parallel processing, in particular while finding the graph invariants, so that the computational limits of the method can be somewhat advanced. The graph invariants for larger matrices and higher degrees of imperfectness will be no longer representable by ordinary integers (see the footnote in Section 2.3), and a longer internal computer representation of integers will be required.

Within the Algebraic approach, a meta-model can be developed to count the i-imperfect outcomes of Bernoulli $(m \times n)$-matrices for i close to the height m of the matrix; see Section 3.7. These probabilities will increase in the accuracy of interpolation techniques described in Section 5.3.

Improving the Probabilistic method is imagined due to using more sums in the Inclusion-Exclusion formula; see Section 4.1. Finding isomorphic allocations and thereby drastically reducing their number is another important option; see Section 4.3.

The Interpolations methods can be improved by increasing in the number of reference points due to improvements of Geometric, Algebraic, and Probabilistic methods. Besides, the application of cubic splines in Section 5.2 should be done under 'concavity control' to avoid 'implausible' inflection of certain probability curves. Besides, the non-linear curve fitting in Section 5.3 is actually made with the local optimization techniques, whereas the global optimization would be much better.

Finally, for enlarging the domain of applications, it is would be important to extend the consideration to larger column k-tuples in Bernoulli matrices, at least up to $k = 5$.

8 Annex: Probability tables

This Annex contains tables with probabilities of occurrence of perfect and i-imperfect single columns (containing all 1s or i or fewer 0, respectively), column pairs and column triplets ($k = 1, 2, 3$) in Bernoulli matrices with up to $m = 48$ rows and up to $n = 18, 10$, or 20 columns, depending on k. The paging of these large tables is column-wise: it follows the first nine columns downward, and after having attained the bottom of the table, the next nine columns are followed downward.

Since probabilities increase along the rows of the tables, the rows starting with probability 1 are of little interest. To save space, these rows are omitted. Therefore, if a row for a certain combination of m and i is missing it consists of 1s.

Finally, the probabilities in the tables have different degree of accuracy. The exact probabilities are in roman font style and are 8-decimals long. The probabilities approximated with the first six sums of the Inclusion-Exclusion formula are *italicized* and are also 8-decimals long. The interpolated probabilities are given with 4 decimals and are **boldfaced**.

Table 31: Sheet A. Probabilities of perfect ($i = 0$) and i-imperfect single columns ($k = 1$) in Bernoulli ($m \times n$)-matrices. The rows with all 1's are omitted

k	m	i	Width n of Bernoulli ($m \times n$)-matrix								
			1	2	3	4	5	6	7	8	9
1	1	0	.50000000	.75000000	.87500000	.93750000	.96875000	.98437500	.99218750	.99609375	.99804688
1	2	0	.25000000	.43750000	.57812500	.68359375	.76269531	.82202148	.86651611	.89988708	.92491531
		1	.75000000	.93750000	.98437500	.99609375	.99902344	.99975586	.99993896	.99998474	.99999619
1	3	0	.12500000	.23437500	.33007813	.41381836	.48709106	.55120468	.60730410	.65639108	.69934220
		1	.50000000	.75000000	.87500000	.93750000	.96875000	.98437500	.99218750	.99609375	.99804688
		2	.87500000	.98437500	.99804688	.99975586	.99996948	.99999619	.99999952	.99999994	.99999999
1	4	0	.06250000	.12109375	.17602539	.22752380	.27580357	.32106584	.36349923	.40328053	.44057549
		1	.31250000	.52734375	.67504883	.77659607	.84640980	.89440674	.92740463	.95009068	.96568735
		2	.68750000	.90234375	.96948242	.99046326	.99701977	.99906868	.99970896	.99990905	.99997158
		3	.93750000	.99609375	.99975586	.99998474	.99999905	.99999994	1.00000000	1.00000000	1.00000000
1	5	0	.03125000	.06152344	.09085083	.11926174	.14678481	.17344779	.19927754	.22430012	.24854074
		1	.18750000	.33984375	.46362305	.56419373	.64590740	.71229976	.76624356	.81007289	.84568422
		2	.50000000	.75000000	.87500000	.93750000	.96875000	.98437500	.99218750	.99609375	.99804688
		3	.81250000	.96484375	.99340820	.99876404	.99976826	.99995655	.99999185	.99999847	.99999971
		4	.96875000	.99902344	.99996948	.99999905	.99999997	1.00000000	1.00000000	1.00000000	1.00000000
1	6	0	.01562500	.03100586	.04614639	.06105036	.07572144	.09016330	.10437949	.11837356	.13214898
		1	.10937500	.20678711	.29354477	.37081331	.43963060	.50092101	.55550777	.60412411	.64742303
		2	.34375000	.56933594	.71737671	.81452847	.87828431	.92012408	.94758142	.96560031	.97742520
		3	.65625000	.88183594	.95938110	.98603725	.99520031	.99835011	.99943285	.99980504	.99993298
		4	.89062500	.98803711	.99869156	.99985689	.99998435	.99999829	.99999981	.99999998	1.00000000
		5	.98437500	.99975586	.99999619	.99999994	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
1	7	0	.00781250	.01556396	.02325487	.03088569	.03845690	.04596895	.05342232	.06081746	.06815482
		1	.06250000	.12109375	.17602539	.22752380	.27580357	.32106584	.36349923	.40328053	.44057549
		2	.22656250	.40179443	.53732538	.64215010	.72322547	.78593220	.83443193	.87194345	.90095626
		3	.50000000	.75000000	.87500000	.93750000	.96875000	.98437500	.99218750	.99609375	.99804688
		4	.77343750	.94866943	.98837042	.99736517	.99940305	.99986475	.99996936	.99999306	.99999843
		5	.93750000	.99609375	.99975586	.99998474	.99999905	.99999994	1.00000000	1.00000000	1.00000000
		6	.99218750	.99993896	.99999952	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
1	8	0	.00390625	.00779724	.01167303	.01553369	.01937926	.02320981	.02702539	.03082608	.03461191
		1	.03515625	.06907654	.10180432	.13338151	.16384856	.19324451	.22160701	.24897239	.27537570
		2	.14453125	.26817322	.37394506	.46442956	.54183623	.60805521	.66470348	.71316430	.75462103
		3	.36328125	.59458923	.74186736	.83564211	.89535025	.93336754	.95757386	.97298648	.98279999
		4	.63671875	.86802673	.95205659	.98258306	.99367275	.99770143	.99916497	.99969665	.99988980
		5	.85546875	.97911072	.99698085	.99956364	.99993693	.99999088	.99999868	.99999981	.99999997
		6	.96484375	.99876404	.99995655	.99999847	.99999995	1.00000000	1.00000000	1.00000000	1.00000000
		7	.99609375	.99998474	.99999994	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
1	9	0	.00195313	.00390244	.00584794	.00778964	.00972755	.01166168	.01359203	.01551860	.01744142
		1	.01953125	.03868103	.05745679	.07586584	.09391533	.11161230	.12896362	.14597605	.16265621
		2	.08984375	.17161560	.24604076	.31377929	.37543193	.43154547	.48261755	.52910113	.57140845
		3	.25390625	.44334412	.58468252	.69013423	.76881108	.82751139	.87130733	.90398320	.92836247
		4	.50000000	.75000000	.87500000	.93750000	.96875000	.98437500	.99218750	.99609375	.99804688
		5	.74609375	.93553162	.98363107	.99584383	.99894472	.99973206	.99993197	.99998273	.99999561
		6	.91015625	.99192810	.99927479	.99993484	.99999415	.99999947	.99999995	1.00000000	1.00000000
		7	.98046875	.99961853	.99999255	.99999985	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
		8	.99804688	.99999619	.99999999	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
1	10	0	.00097656	.00195217	.00292683	.00390053	.00487329	.00584509	.00681594	.00778585	.00875481
		1	.01074219	.02136898	.03188162	.04228133	.05256932	.06274680	.07281495	.08277494	.09262795
		2	.05468750	.10638428	.15525389	.20145094	.24512159	.28640400	.32542879	.36231940	.39719256
		3	.17187500	.31420898	.43207932	.52969068	.61052510	.67746610	.73290161	.77880915	.81682632
		4	.37695313	.61181259	.75814105	.84931054	.90611340	.94150425	.96355440	.97729269	.98585228
		5	.62304687	.85790634	.94643735	.97980939	.99238909	.99713104	.99891854	.99959234	.99984633
		6	.82812500	.97045898	.99492264	.99912733	.99985001	.99997422	.99999557	.99999924	.99999987
		7	.94531250	.99700928	.99983644	.99999106	.99999951	.99999997	1.00000000	1.00000000	1.00000000
		8	.98925781	.99988461	.99999876	.99999999	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
		9	.99902344	.99999905	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
1	11	0	.00048828	.00097632	.00146413	.00195169	.00243902	.00292611	.00341297	.00389958	.00438596

Table 31: Sheet B. Probabilities of perfect ($i = 0$) and i-imperfect single columns ($k = 1$) in Bernoulli ($m \times n$)-matrices. The rows with all 1's are omitted

k m i	Width n of Bernoulli ($m \times n$)-matrix								
	1	2	3	4	5	6	7	8	9
1 11 1	.00585938	.01168442	.01747533	.02323231	.02895556	.03464527	.04030165	.04592488	.05151516
2	.03271484	.06435943	.09496876	.12457672	.15321605	.18091846	.20771458	.23363408	.25870562
3	.11328125	.21372986	.30279952	.38177926	.45181208	.51391149	.56897621	.61780312	.66109886
4	.27441406	.47352505	.61799718	.72282412	.79888508	.85407384	.89411803	.92317353	.94425580
5	.50000000	.75000000	.87500000	.93750000	.96875000	.98437500	.99218750	.99609375	.99804687
6	.72558594	.92469692	.97933578	.99432945	.99844392	.99957299	.99988282	.99996784	.99999118
7	.88671875	.98716736	.99854630	.99983532	.99998135	.99999789	.99999976	.99999997	1.00000000
8	.96728516	.99892974	.99996499	.99999885	.99999996	1.00000000	1.00000000	1.00000000	1.00000000
9	.99414063	.99996567	.99999980	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
10	.99951172	.99999976	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
1 12 0	.00024414	.00048822	.00073224	.00097620	.00122011	.00146395	.00170773	.00195146	.00219512
1	.00317383	.00633758	.00949130	.01263500	.01576873	.01889251	.02200638	.02511036	.02820449
2	.01928711	.03820223	.05675253	.07494504	.09278668	.11028420	.12744425	.14427333	.16077782
3	.07299805	.14066738	.20339698	.26154745	.31545304	.36542363	.41174647	.45468783	.49449455
4	.19384766	.35011840	.47609642	.57765390	.65952471	.72552504	.77873137	.82162378	.85620159
5	.38720703	.62448478	.76988691	.85898832	.91358903	.94704797	.96755137	.98011571	.98781504
6	.61279297	.85007071	.94194633	.97752121	.99129605	.99662977	.99869502	.99949470	.99980435
7	.80615234	.96242309	.99271580	.99858798	.99972628	.99994694	.99998971	.99999801	.99999961
8	.92700195	.99467129	.99961101	.99997160	.99999793	.99999985	.99999999	1.00000000	1.00000000
9	.98071289	.99962801	.99999283	.99999986	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
10	.99682617	.99998993	.99999997	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
11	.99975586	.99999994	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
1 13 0	.00012207	.00024413	.00036617	.00048819	.00061020	.00073220	.00085418	.00097615	.00109810
1	.00170898	.00341505	.00511820	.00681843	.00851577	.01021020	.01190173	.01359038	.01527613
2	.01123047	.02233481	.03331445	.04417078	.05490519	.06551905	.07601371	.08639051	.09665077
3	.04614258	.09015602	.13213857	.17218393	.21038150	.24681653	.28157036	.31472055	.34634111
4	.13342285	.24904405	.34923873	.43606515	.51130695	.57650977	.63301304	.68197749	.72440896
5	.29052734	.49664855	.64288591	.74663732	.82024610	.87246953	.90952062	.93580735	.95445707
6	.50000000	.75000000	.87500000	.93750000	.96875000	.98437500	.99218750	.99609375	.99804688
7	.70947266	.91559386	.97547771	.99287560	.99793017	.99939866	.99982529	.99994924	.99998525
8	.86657715	.98219834	.99762485	.99968310	.99995772	.99999436	.99999925	.99999990	.99999999
9	.95385742	.99787086	.99990176	.99999547	.99999979	.99999999	1.00000000	1.00000000	1.00000000
10	.98876953	.99987388	.99999858	.99999998	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
11	.99829102	.99999708	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
12	.99987793	.99999999	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
1 14 0	.00006104	.00012207	.00018309	.00024412	.00030514	.00036616	.00042717	.00048818	.00054918
1	.00091553	.00183022	.00274407	.00365708	.00456926	.00548061	.00639112	.00730079	.00820964
2	.00646973	.01289760	.01928388	.02562884	.03193276	.03819589	.04441850	.05060085	.05674320
3	.02868652	.05655013	.08361443	.10990234	.13543615	.16023748	.18432735	.20772616	.23045374
4	.08978271	.17150449	.24588907	.31359520	.37522248	.43131670	.48237463	.52884845	.57114971
5	.21197510	.37901675	.51064974	.61437981	.69612169	.76053632	.81129666	.85129707	.88281839
6	.39526367	.63429397	.77884428	.86625910	.91912202	.95109015	.97042244	.98211337	.98918331
7	.60473633	.84376663	.93824662	.97559113	.99035206	.99618652	.99849267	.99940421	.99976450
8	.78802490	.95506656	.99047523	.99798099	.99957202	.99990928	.99998077	.99999592	.99999914
9	.91021729	.99193906	.99927627	.99993502	.99999417	.99999948	.99999995	1.00000000	1.00000000
10	.97131348	.99917708	.99997639	.99999932	.99999998	1.00000000	1.00000000	1.00000000	1.00000000
11	.99353027	.99995814	.99999973	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
12	.99908447	.99999916	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
13	.99993896	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
1 15 0	.00003052	.00006103	.00009155	.00012206	.00015258	.00018309	.00021360	.00024411	.00027462
1	.00048828	.00097632	.00146413	.00195169	.00243902	.00292611	.00341297	.00389958	.00438596
2	.00369263	.00737162	.01103702	.01468890	.01832728	.02195223	.02556380	.02916203	.03274697
3	.01757813	.03484726	.05181284	.06848019	.08485456	.10094110	.11674487	.13227084	.14752389
4	.05923462	.11496050	.16738548	.21670508	.26310326	.30675305	.34781727	.38644907	.42279252
5	.15087891	.27899337	.38777806	.48014944	.55858392	.62518430	.68173608	.72975539	.77052960
6	.30361938	.51505404	.66229303	.76482741	.83623037	.88595400	.92058058	.94469386	.96148587

Table 31: Sheet C. Probabilities of perfect ($i = 0$) and i-imperfect single columns ($k = 1$) in Bernoulli ($m \times n$)-matrices. The rows with all 1's are omitted

k m i	Width n of Bernoulli ($m \times n$)-matrix								
	1	2	3	4	5	6	7	8	9
1 15 7	.50000000	.75000000	.87500000	.93750000	.96875000	.98437500	.99218750	.99609375	.99804688
8	.69638062	.90781527	.97201093	.99150198	.99741984	.99921661	.99976215	.99992778	.99997807
9	.84912109	.97723556	.99656533	.99948178	.99992181	.99998820	.99999822	.99999973	.99999996
10	.94076538	.99649126	.99979216	.99998769	.99999927	.99999996	1.00000000	1.00000000	1.00000000
11	.98242188	.99969101	.99999457	.99999990	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
12	.99630737	.99998636	.99999995	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
13	.99951172	.99999976	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
1 16 0	.00001526	.00003052	.00004578	.00006103	.00007629	.00009155	.00010681	.00012206	.00013732
1	.00025940	.00051873	.00077800	.00103719	.00129632	.00155539	.00181438	.00207331	.00233217
2	.00209045	.00417654	.00625826	.00833563	.01040866	.01247736	.01454173	.01660178	.01865753
3	.01063538	.02115764	.03156800	.04186764	.05205773	.06213946	.07211396	.08198237	.09174584
4	.03840637	.07533769	.11085062	.14499962	.17783708	.20941338	.23977694	.26897435	.29705040
5	.10505676	.19907660	.28321902	.35852171	.42591334	.48622503	.54020056	.58850560	.63173587
6	.22724915	.40285612	.53855655	.64341918	.72445187	.78706995	.83545812	.87285012	.90174482
7	.40180969	.64216836	.78594858	.87195651	.92340563	.95418199	.97259211	.98360487	.99019259
8	.59819031	.83854897	.93512741	.97393357	.98952625	.99579155	.99830900	.99932054	.99972699
9	.77275085	.94835783	.98826436	.99733309	.99939395	.99986227	.99996870	.99999289	.99999838
10	.89494324	.98896308	.99884050	.99987819	.99998720	.99999866	.99999986	.99999999	1.00000000
11	.96159363	.99852495	.99994335	.99999782	.99999992	1.00000000	1.00000000	1.00000000	1.00000000
12	.98936462	.99988689	.99999880	.99999999	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
13	.99790955	.99999563	.99999999	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
14	.99974060	.99999993	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
1 17 0	.00000763	.00001526	.00002289	.00003052	.00003815	.00004578	.00005340	.00006103	.00006866
1	.00013733	.00027464	.00041193	.00054920	.00068646	.00082369	.00096091	.00109810	.00123528
2	.00117493	.00234847	.00352064	.00469143	.00586085	.00702889	.00819555	.00936085	.01052478
3	.00636292	.01268534	.01896754	.02520977	.03141228	.03757532	.04369914	.04978401	.05583015
4	.02452087	.04844047	.07177355	.09453447	.11673728	.13839565	.15952294	.18013217	.20023605
5	.07173157	.13831772	.20012754	.25750364	.31076407	.36020404	.40609761	.44869916	.48824483
6	.16615295	.30469910	.42022540	.51655666	.59688220	.66386142	.71971183	.76628254	.80511539
7	.31452942	.53013008	.67791799	.77922226	.84866335	.89626318	.92889146	.95125719	.96658824
8	.50000000	.75000000	.87500000	.93750000	.96875000	.98437500	.99218750	.99609375	.99804687
9	.68547058	.90107124	.96888400	.99021310	.99692173	.99903179	.99969547	.99990422	.99996987
10	.83384705	.97239320	.99541305	.99923786	.99987337	.99997896	.99999650	.99999942	.99999990
11	.92826843	.99485458	.99963091	.99997352	.99999810	.99999986	.99999999	1.00000000	1.00000000
12	.97547913	.99939873	.99998526	.99999964	.99999999	1.00000000	1.00000000	1.00000000	1.00000000
13	.99363708	.99995951	.99999974	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
14	.99882507	.99999862	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
15	.99986267	.99999998	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
1 18 0	.00000381	.00000763	.00001144	.00001526	.00001907	.00002289	.00002670	.00003052	.00003433
1	.00007248	.00014495	.00021742	.00028989	.00036234	.00043480	.00050724	.00057969	.00065212
2	.00065613	.00131183	.00196709	.00262193	.00327634	.00393032	.00458386	.00523699	.00588968
3	.00376892	.00752364	.01126420	.01499067	.01870309	.02240152	.02608601	.02975662	.03341339
4	.01544189	.03064534	.04561401	.06035154	.07486149	.08914738	.10321267	.11706077	.13069502
5	.04812622	.09393631	.13754173	.17904859	.21855788	.25616573	.29196366	.32603878	.35847398
6	.11894226	.22373726	.31606771	.39741616	.46908884	.53223662	.58787345	.63689271	.68008152
7	.24034119	.42291849	.56161494	.66697693	.74701609	.80781854	.85400766	.88909563	.91575052
8	.40726471	.64866488	.79175127	.87656363	.92683491	.95663247	.97429453	.98476346	.99096877
9	.59273529	.83413546	.93244922	.97248895	.98879572	.99543689	.99814161	.99924314	.99969176
10	.75965881	.94223611	.98611696	.99666333	.99919806	.99980726	.99995368	.99998887	.99999732
11	.88105774	.98585274	.99831729	.99979985	.99997619	.99999717	.99999966	.99999996	1.00000000
12	.95187378	.99768387	.99988853	.99999464	.99999974	.99999999	1.00000000	1.00000000	1.00000000
13	.98455811	.99976155	.99999632	.99999994	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
14	.99623108	.99998580	.99999995	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
15	.99934387	.99999957	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
16	.99992752	.99999999	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
1 19 0	.00000191	.00000381	.00000572	.00000763	.00000954	.00001144	.00001335	.00001526	.00001717

Table 31: Sheet D. Probabilities of perfect ($i = 0$) and i-imperfect single columns ($k = 1$) in Bernoulli ($m \times n$)-matrices. The rows with all 1's are omitted

k	m	i	Width n of Bernoulli ($m \times n$)-matrix								
			1	2	3	4	5	6	7	8	9
1	19	1	.00003815	.00007629	.00011444	.00015258	.00019072	.00022886	.00026700	.00030514	.00034327
		2	.00036430	.00072847	.00109251	.00145642	.00182019	.00218383	.00254734	.00291072	.00327396
		3	.00221252	.00442015	.00662290	.00882077	.01101378	.01320193	.01538525	.01756373	.01973740
		4	.00960541	.01911855	.02854032	.03787158	.04711322	.05626609	.06533103	.07430891	.08320055
		5	.03178406	.06255789	.09235360	.12120229	.14913405	.17617802	.20236242	.22771458	.25226095
		6	.08353424	.16009051	.23025171	.29455205	.35348111	.40748758	.45698265	.50234319	.54391458
		7	.17964172	.32701230	.44790897	.54708755	.62844953	.69519549	.74995110	.79487032	.83172017
		8	.32380295	.54275755	.69081400	.79092934	.85862704	.90440402	.93535828	.95628946	.97044306
		9	.50000000	.75000000	.87500000	.93750000	.96875000	.98437500	.99218750	.99609375	.99804688
		10	.67619705	.89515165	.96604980	.98900682	.99644038	.99884738	.99962678	.99987915	.99996087
		11	.82035828	.96772885	.99420276	.99895857	.99981292	.99996639	.99999396	.99999892	.99999981
		12	.91646576	.99302203	.99941710	.99995131	.99999593	.99999966	.99999997	1.00000000	1.00000000
		13	.96821594	.99898977	.99996789	.99999898	.99999997	1.00000000	1.00000000	1.00000000	1.00000000
		14	.99039459	.99990774	.99999911	.99999999	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
		15	.99778748	.99999510	.99999999	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
		16	.99963570	.99999987	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
1	20	0	.00000095	.00000191	.00000286	.00000381	.00000477	.00000572	.00000668	.00000763	.00000858
		1	.00002003	.00004005	.00006008	.00008011	.00010013	.00012016	.00014018	.00016021	.00018023
		2	.00020123	.00040241	.00060355	.00080466	.00100572	.00120674	.00140773	.00160867	.00180957
		3	.00128841	.00257517	.00386026	.00514370	.00642549	.00770563	.00898411	.01026095	.01153614
		4	.00590897	.01178302	.01762236	.02342719	.02919773	.03493417	.04063671	.04630555	.05194090
		5	.02069473	.04096119	.06080825	.08024457	.09927866	.11791885	.13617328	.15404994	.17155665
		6	.05765915	.11199372	.16319541	.21144485	.25691227	.29975807	.34013343	.37818077	.41403434
		7	.13158798	.24586057	.34509625	.43127372	.50611126	.57110108	.62753903	.67655041	.71911249
		8	.25172234	.44008054	.58102477	.68649020	.76540762	.82445976	.86864716	.90171160	.92645299
		9	.41190147	.65414012	.79660032	.88038095	.92965221	.95862857	.97566952	.98569128	.99158506
		10	.58809853	.83033718	.93011563	.97121453	.98814322	.99511618	.99798835	.99917140	.99965870
		11	.74827766	.93663587	.98404983	.99598499	.99898933	.99974559	.99993596	.99998388	.99999594
		12	.86841202	.98268460	.99772150	.99970018	.99996055	.99999481	.99999932	.99999991	.99999999
		13	.94234085	.99667542	.99980831	.99998895	.99999936	.99999996	1.00000000	1.00000000	1.00000000
		14	.97930527	.99957173	.99999114	.99999982	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
		15	.99409103	.99996508	.99999979	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
		16	.99871159	.99999834	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
		17	.99979877	.99999996	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
1	21	0	.00000048	.00000095	.00000143	.00000191	.00000238	.00000286	.00000334	.00000381	.00000429
		1	.00001049	.00002098	.00003147	.00004196	.00005245	.00006294	.00007343	.00008392	.00009441
		2	.00011063	.00022124	.00033184	.00044243	.00055301	.00066357	.00077413	.00088467	.00099520
		3	.00074482	.00148908	.00223280	.00297595	.00371855	.00446060	.00520210	.00594305	.00668344
		4	.00359869	.00718443	.01075726	.01431724	.01786441	.02139881	.02492049	.02842950	.03192588
		5	.01330185	.02642676	.03937708	.05215515	.06476323	.07720361	.08947851	.10159013	.11354064
		6	.03917694	.07681905	.11298645	.14773693	.18112599	.21320697	.24403111	.27364766	.30210393
		7	.09462357	.18029351	.25785706	.32808127	.39166062	.44922386	.50134026	.54852523	.59124538
		8	.19165516	.34657862	.47181020	.57304050	.65486949	.72101553	.77448434	.81770558	.85264325
		9	.33181190	.55352467	.70167050	.80065978	.86680324	.91099951	.94053093	.96026348	.97344853
		10	.50000000	.75000000	.87500000	.93750000	.96875000	.98437500	.99218750	.99609375	.99804687
		11	.66818810	.88990086	.96346779	.98787818	.99597784	.99866540	.99955716	.99985306	.99995124
		12	.80834484	.96326830	.99296018	.99865078	.99974142	.99995044	.99999050	.99999818	.99999965
		13	.90537643	.99104638	.99915278	.99991983	.99999241	.99999928	.99999993	.99999999	1.00000000
		14	.96082306	.99846517	.99993987	.99999764	.99999991	1.00000000	1.00000000	1.00000000	1.00000000
		15	.98669815	.99982306	.99999765	.99999997	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
		16	.99640131	.99998705	.99999995	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
		17	.99925518	.99999945	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
		18	.99988937	.99999999	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
1	22	0	.00000024	.00000048	.00000072	.00000095	.00000119	.00000143	.00000167	.00000191	.00000215
		1	.00000548	.00001097	.00001645	.00002193	.00002742	.00003290	.00003838	.00004387	.00004935
		2	.00006056	.00012111	.00018166	.00024221	.00030275	.00036329	.00042383	.00048436	.00054489

Table 31: Sheet E. Probabilities of perfect ($i = 0$) and i-imperfect single columns ($k = 1$) in Bernoulli ($m \times n$)-matrices. The rows with all 1's are omitted

k m i	Width n of Bernoulli ($m \times n$)-matrix								
	1	2	3	4	5	6	7	8	9
1 22 3	.00042772	.00085526	.00128262	.00170979	.00213679	.00256359	.00299022	.00341667	.00384293
4	.00217175	.00433879	.00650113	.00865876	.01081171	.01295999	.01510359	.01724255	.01937686
5	.00845027	.01682913	.02513719	.03337504	.04154329	.04964250	.05767328	.06563620	.07353182
6	.02623940	.05179028	.07667073	.10089834	.12449022	.14746307	.16983312	.19161620	.21282770
7	.06690025	.12932486	.18757325	.24192481	.29264023	.33996278	.38411943	.42532200	.46376810
8	.14313936	.26578985	.37088422	.46093545	.53809681	.60421334	.66086599	.70940941	.75100436
9	.26173353	.45496262	.59761718	.70293426	.78068632	.83808807	.88046585	.91175194	.93484942
10	.41590595	.65883414	.80072705	.88360586	.93201487	.96029029	.97680580	.98645240	.99208693
11	.58409405	.82702224	.92805752	.97007869	.98755555	.99482428	.99784739	.99910472	.99962765
12	.73826647	.93149556	.98207009	.99530714	.99877172	.99967852	.99991586	.99997798	.99999424
13	.85686064	.97951112	.99706724	.99958021	.99993991	.99999140	.99999877	.99999982	.99999997
14	.93309975	.99552436	.99970058	.99997997	.99999866	.99999991	.99999999	1.00000000	1.00000000
15	.97376060	.99931149	.99998193	.99999953	.99999999	1.00000000	1.00000000	1.00000000	1.00000000
16	.99154973	.99992859	.99999940	.99999999	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
17	.99782825	.99999528	.99999999	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
18	.99957228	.99999982	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
19	.99993944	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
1 23 0	.00000012	.00000024	.00000036	.00000048	.00000060	.00000072	.00000083	.00000095	.00000107
1	.00000286	.00000572	.00000858	.00001144	.00001431	.00001717	.00002003	.00002289	.00002575
2	.00003302	.00006604	.00009906	.00013208	.00016509	.00019811	.00023112	.00026414	.00029715
3	.00024414	.00048822	.00073224	.00097620	.00122011	.00146395	.00170773	.00195146	.00219512
4	.00129974	.00259779	.00389415	.00518883	.00648182	.00777314	.00906277	.01035073	.01163702
5	.00531101	.01059382	.01584857	.02107541	.02627449	.03144596	.03658996	.04170664	.04679615
6	.01734483	.03438882	.05113719	.06759505	.08376746	.09965936	.11527562	.13062101	.14570025
7	.04656982	.09097090	.13330423	.17366610	.21214832	.24883843	.28381990	.31717228	.34897144
8	.10501981	.19901046	.28313022	.35841575	.42579480	.48609772	.54006764	.58836965	.63159899
9	.20243645	.36389238	.49266375	.59536709	.67727954	.74260993	.79471506	.83627221	.86941668
10	.33881974	.56284067	.71095888	.80889172	.87364298	.91645523	.94476185	.96347762	.97585213
11	.50000000	.75000000	.87500000	.93750000	.96875000	.98437500	.99218750	.99609375	.99804688
12	.66118026	.88520118	.96110389	.98682123	.99553477	.99848709	.99948740	.99982632	.99994115
13	.79756355	.95901948	.99170405	.99832060	.99966003	.99993118	.99998607	.99999718	.99999943
14	.89498019	.98897084	.99884172	.99987836	.99998723	.99999866	.99999986	.99999999	1.00000000
15	.95343018	.99783125	.99989900	.99999530	.99999978	.99999999	1.00000000	1.00000000	1.00000000
16	.98265517	.99969916	.99999478	.99999991	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
17	.99468899	.99997179	.99999985	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
18	.99870026	.99999831	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
19	.99975586	.99999994	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
1 24 0	.00000006	.00000012	.00000018	.00000024	.00000030	.00000036	.00000042	.00000048	.00000054
1	.00000149	.00000298	.00000447	.00000596	.00000745	.00000894	.00001043	.00001192	.00001341
2	.00001794	.00003588	.00005382	.00007176	.00008970	.00010764	.00012558	.00014352	.00016146
3	.00013858	.00027714	.00041568	.00055421	.00069271	.00083120	.00096966	.00110811	.00124654
4	.00077194	.00154328	.00231403	.00308419	.00385374	.00462271	.00539108	.00615886	.00692604
5	.00330538	.00659983	.00988339	.01315609	.01641798	.01966909	.02290945	.02613910	.02935808
6	.01132792	.02252752	.03360026	.04454756	.05537085	.06607153	.07665100	.08711063	.09745177
7	.03195733	.06289339	.09284081	.12183119	.14989512	.17706220	.20336110	.22881955	.25346441
8	.07579482	.14584478	.21058532	.27041886	.32571733	.37682446	.42405793	.46771136	.50805608
9	.15372813	.28382392	.39392033	.48709182	.56594023	.63266743	.68913678	.73692520	.77736719
10	.27062809	.46801662	.61198627	.71699369	.79358315	.84944535	.89018967	.91990743	.94158273
11	.41940987	.66291510	.80429184	.88637377	.93402973	.96169831	.97776242	.98708908	.99250405
12	.58059013	.82409536	.92622386	.96905756	.98702243	.99455708	.99771719	.99904257	.99959844
13	.72937191	.92676043	.98017932	.99463597	.99854834	.99960714	.99989368	.99997123	.99999221
14	.84627187	.97636766	.99636705	.99944151	.99991414	.99998680	.99999797	.99999969	.99999995
15	.92420518	.99425515	.99956457	.99996700	.99999750	.99999981	.99999999	1.00000000	1.00000000
16	.96804267	.99897873	.99996736	.99999896	.99999997	1.00000000	1.00000000	1.00000000	1.00000000
17	.98867208	.99987168	.99999855	.99999998	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
18	.99669462	.99998907	.99999996	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000

Table 31: Sheet F. Probabilities of perfect ($i = 0$) and i-imperfect single columns ($k = 1$) in Bernoulli ($m \times n$)-matrices. The rows with all 1's are omitted

k m i	Width n of Bernoulli ($m \times n$)-matrix								
	1	2	3	4	5	6	7	8	9
1 24 19	.99922806	.99999940	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
20	.99986142	.99999998	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
1 25 0	.00000003	.00000006	.00000009	.00000012	.00000015	.00000018	.00000021	.00000024	.00000027
1	.00000077	.00000155	.00000232	.00000310	.00000387	.00000465	.00000542	.00000620	.00000697
2	.00000972	.00001943	.00002915	.00003886	.00004858	.00005829	.00006801	.00007772	.00008744
3	.00007826	.00015652	.00023476	.00031301	.00039124	.00046947	.00054770	.00062592	.00070413
4	.00045526	.00091031	.00136516	.00181980	.00227423	.00272845	.00318247	.00363628	.00408989
5	.00203866	.00407316	.00610351	.00812973	.01015181	.01216977	.01418362	.01619336	.01819901
6	.00731665	.01457976	.02178974	.02894696	.03605181	.04310468	.05010595	.05705599	.06395518
7	.02164263	.04281685	.06353280	.08380041	.10362938	.12302919	.14200914	.16057832	.17874560
8	.05387607	.10484951	.15307671	.19870561	.24187620	.28272093	.32136511	.35792729	.39251965
9	.11476147	.21635275	.30628526	.38589698	.45637235	.51875986	.57398769	.62287749	.66615662
10	.21217811	.37933667	.51102784	.61477703	.69651291	.76090623	.81163670	.85160327	.88308980
11	.34501898	.57099987	.71901306	.81595889	.87945656	.92104634	.94828685	.96612887	.97781505
12	.50000000	.75000000	.87500000	.93750000	.96875000	.98437500	.99218750	.99609375	.99804688
13	.65498102	.88096190	.95892960	.98582993	.99511106	.99831322	.99941803	.99979921	.99993072
14	.78782189	.95498045	.99044784	.99797324	.99956997	.99990876	.99998064	.99999589	.99999913
15	.88523853	.98682980	.99848857	.99982655	.99998009	.99999772	.99999974	.99999997	1.00000000
16	.94612393	.99709737	.99984362	.99999157	.99999955	.99999998	1.00000000	1.00000000	1.00000000
17	.97835737	.99953160	.99998986	.99999978	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
18	.99268335	.99994647	.99999961	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
19	.99796134	.99999584	.99999999	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
20	.99954474	.99999979	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
21	.99992174	.99999999	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
1 26 0	.00000001	.00000003	.00000004	.00000006	.00000007	.00000009	.00000010	.00000012	.00000013
1	.00000040	.00000080	.00000121	.00000161	.00000201	.00000241	.00000282	.00000322	.00000362
2	.00000525	.00001049	.00001574	.00002098	.00002623	.00003147	.00003672	.00004196	.00004721
3	.00004399	.00008797	.00013196	.00017594	.00021992	.00026390	.00030788	.00035185	.00039582
4	.00026676	.00053345	.00080007	.00106662	.00133309	.00159950	.00186583	.00213209	.00239829
5	.00124696	.00249236	.00373621	.00497851	.00621927	.00745847	.00869613	.00993224	.01116682
6	.00467765	.00933343	.01396742	.01857974	.02317048	.02773975	.03228765	.03681427	.04131972
7	.01447964	.02874961	.04281297	.05667269	.07033173	.08379298	.09705933	.11013358	.12301853
8	.03775935	.07409293	.10905458	.14269610	.17506733	.20621625	.23618901	.26503002	.29278200
9	.08431877	.16152789	.23222683	.29696452	.35624361	.41052436	.46022822	.50574111	.54741641
10	.16346979	.30021721	.41461056	.51030405	.59035454	.65731920	.71333716	.76019787	.79939828
11	.27859855	.47957994	.62456822	.72916296	.80461777	.85905097	.89831917	.92664730	.94708326
12	.42250949	.66650471	.80740964	.88878089	.93577202	.96290895	.97858027	.98763031	.99285662
13	.57749051	.82148573	.92457603	.96813266	.98653574	.99431122	.99759644	.99898447	.99957093
14	.72140145	.92238285	.97837597	.99397558	.99832160	.99953240	.99986973	.99996371	.99998989
15	.83653021	.97327763	.99563170	.99928591	.99988327	.99998092	.99999688	.99999949	.99999992
16	.91568123	.99289034	.99940052	.99994945	.99999574	.99999964	.99999997	1.00000000	1.00000000
17	.96224065	.99857423	.99994616	.99999797	.99999992	1.00000000	1.00000000	1.00000000	1.00000000
18	.98552036	.99979034	.99999696	.99999996	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
19	.99532235	.99997812	.99999990	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
20	.99875304	.99999845	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
21	.99973324	.99999993	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
1 27 0	.00000001	.00000001	.00000002	.00000003	.00000004	.00000004	.00000005	.00000006	.00000007
1	.00000021	.00000042	.00000063	.00000083	.00000104	.00000125	.00000146	.00000167	.00000188
2	.00000282	.00000565	.00000847	.00001130	.00001412	.00001694	.00001977	.00002259	.00002541
3	.00002462	.00004923	.00007385	.00009846	.00012308	.00014769	.00017230	.00019692	.00022153
4	.00015537	.00031072	.00046605	.00062135	.00077663	.00093188	.00108711	.00124232	.00139750
5	.00075686	.00151315	.00226886	.00302400	.00377857	.00453257	.00528600	.00603886	.00679115
6	.00296231	.00591584	.00886062	.01179668	.01472404	.01764273	.02055277	.02345419	.02634702
7	.00957865	.01906554	.02846156	.03776759	.04698447	.05611307	.06515422	.07410878	.08297756
8	.02611949	.05155676	.07632961	.10045542	.12395107	.14683302	.16911731	.19081954	.21195493
9	.06103906	.11835235	.17216730	.22269743	.27014325	.31469302	.35652351	.39580071	.43268047

Table 31: Sheet G. Probabilities of perfect ($i = 0$) and i-imperfect single columns ($k = 1$) in Bernoulli ($m \times n$)-matrices. The rows with all 1's are omitted

k m i	Width n of Bernoulli ($m \times n$)-matrix								
	1	2	3	4	5	6	7	8	9
1 27 10	.12389428	.23243877	.32753522	.41084976	.48384210	.54779112	.60381721	.65290199	.69590545
11	.22103417	.39321223	.52733306	.63180861	.71319149	.77658597	.82596810	.86443510	.89439957
12	.35055402	.57821992	.72607662	.82210156	.88446457	.92496598	.95126946	.96835215	.97944643
13	.50000000	.75000000	.87500000	.93750000	.96875000	.98437500	.99218750	.99609375	.99804687
14	.64944598	.87711188	.95692108	.98489851	.99470611	.99814421	.99934944	.99977194	.99992005
15	.77896583	.95114390	.98920113	.99761308	.99947241	.99988338	.99997422	.99999430	.99999874
16	.87610572	.98465021	.99809825	.99976438	.99997081	.99999638	.99999955	.99999994	.99999999
17	.93896094	.99627423	.99977258	.99998612	.99999915	.99999995	1.00000000	1.00000000	1.00000000
18	.97388051	.99931777	.99998218	.99999953	.99999999	1.00000000	1.00000000	1.00000000	1.00000000
19	.99042135	.99990825	.99999912	.99999999	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
20	.99703769	.99999122	.99999997	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
21	.99924314	.99999943	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
22	.99984463	.99999998	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
1 28 0	.00000000	.00000001	.00000001	.00000001	.00000002	.00000002	.00000003	.00000003	.00000003
1	.00000011	.00000022	.00000032	.00000043	.00000054	.00000065	.00000076	.00000086	.00000097
2	.00000152	.00000303	.00000455	.00000606	.00000758	.00000910	.00001061	.00001213	.00001365
3	.00001372	.00002744	.00004116	.00005488	.00006860	.00008232	.00009604	.00010976	.00012348
4	.00009000	.00017998	.00026996	.00035993	.00044990	.00053985	.00062980	.00071974	.00080967
5	.00045612	.00091203	.00136773	.00182322	.00227851	.00273358	.00318845	.00364312	.00409757
6	.00185958	.00371571	.00556838	.00741761	.00926340	.01110576	.01294469	.01478020	.01661230
7	.00627048	.01250163	.01869372	.02484697	.03096165	.03703798	.04307621	.04907658	.05503932
8	.01784907	.03537955	.05259713	.06950739	.08611581	.10242779	.11844862	.13418349	.14963751
9	.04357928	.08525940	.12512313	.16324963	.19971461	.23459047	.26794646	.29984883	.33036091
10	.09246667	.17638326	.25254036	.32165546	.38437972	.44130408	.49296483	.53984869	.58239735
11	.17246423	.31518454	.43329071	.53102779	.61190872	.67884058	.73422909	.78006506	.81799597
12	.28579409	.48990992	.63569066	.73980811	.81416942	.86727870	.90520966	.93230018	.95164839
13	.42527701	.66969348	.81016525	.89089761	.93729635	.96396277	.97928857	.98809667	.99315888
14	.57472299	.81913947	.92308417	.96728947	.98608896	.99408396	.99748404	.99893002	.99954496
15	.71420591	.91832174	.97665683	.99332866	.99809337	.99945510	.99984427	.99995549	.99998728
16	.82753577	.97025609	.99487024	.99911530	.99984742	.99997369	.99999546	.99999922	.99999987
17	.90753333	.99144991	.99920940	.99992690	.99999324	.99999937	.99999994	.99999999	1.00000000
18	.95642072	.99810085	.99991724	.99999639	.99999984	.99999999	1.00000000	1.00000000	1.00000000
19	.98215093	.99968141	.99999431	.99999990	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
20	.99372952	.99996068	.99999975	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
21	.99814042	.99999654	.99999999	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
22	.99954388	.99999979	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
23	.99991000	.99999999	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
1 29 0	.00000000	.00000000	.00000001	.00000001	.00000001	.00000001	.00000001	.00000001	.00000002
1	.00000006	.00000011	.00000017	.00000022	.00000028	.00000034	.00000039	.00000045	.00000050
2	.00000081	.00000162	.00000244	.00000325	.00000406	.00000487	.00000568	.00000650	.00000731
3	.00000762	.00001524	.00002285	.00003047	.00003809	.00004571	.00005333	.00006094	.00006856
4	.00005186	.00010371	.00015557	.00020742	.00025926	.00031111	.00036295	.00041479	.00046662
5	.00027306	.00054604	.00081895	.00109178	.00136454	.00163722	.00190983	.00218236	.00245482
6	.00115785	.00231436	.00346953	.00462336	.00577586	.00692702	.00807685	.00922535	.01037252
7	.00406503	.00811353	.01214558	.01616124	.02016057	.02414365	.02811053	.03206129	.03599599
8	.01205977	.02397411	.03574476	.04737346	.05886191	.07021183	.08142486	.09250267	.10344688
9	.03071417	.06048499	.08934141	.11731154	.14442258	.17070094	.19617217	.22086108	.24479169
10	.06802297	.13141882	.19050230	.24556674	.29688553	.34471347	.38928801	.43083045	.46954706
11	.13246545	.24738380	.34707944	.43356886	.50860141	.57369475	.63016546	.67915576	.72165654
12	.22912916	.40575815	.54191628	.64687662	.72778748	.79015931	.83823993	.87530388	.90387540
13	.35553555	.58466558	.73233173	.82749732	.88882815	.92835370	.95382650	.97024282	.98082256
14	.50000000	.75000000	.87500000	.93750000	.96875000	.98437500	.99218750	.99609375	.99804688
15	.64446445	.87359447	.95505834	.98402164	.99431913	.99798025	.99928191	.99974469	.99990923
16	.77087084	.94749983	.98797068	.99724373	.99936846	.99985530	.99996684	.99999240	.99999826
17	.86753455	.98245291	.99767562	.99969210	.99995921	.99999460	.99999928	.99999991	.99999999
18	.93197703	.99537288	.99968525	.99997859	.99999854	.99999990	.99999999	1.00000000	1.00000000

Table 31: Sheet H. Probabilities of perfect ($i = 0$) and i-imperfect single columns ($k = 1$) in Bernoulli ($m \times n$)-matrices. The rows with all 1's are omitted

k m i	Width n of Bernoulli ($m \times n$)-matrix								
	1	2	3	4	5	6	7	8	9
1 29 19	.96928583	.99905664	.99997103	.99999911	.99999997	1.00000000	1.00000000	1.00000000	1.00000000
20	.98794023	.99985456	.99999825	.99999998	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
21	.99593497	.99998348	.99999993	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
22	.99884215	.99999866	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
23	.99972694	.99999993	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
24	.99994814	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
1 30 0	.00000000	.00000000	.00000000	.00000000	.00000000	.00000001	.00000001	.00000001	.00000001
1	.00000003	.00000006	.00000009	.00000012	.00000014	.00000017	.00000020	.00000023	.00000026
2	.00000043	.00000087	.00000130	.00000174	.00000217	.00000260	.00000304	.00000347	.00000391
3	.00000422	.00000843	.00001265	.00001686	.00002108	.00002529	.00002951	.00003372	.00003794
4	.00002974	.00005948	.00008921	.00011895	.00014868	.00017842	.00020815	.00023788	.00026761
5	.00016246	.00032489	.00048729	.00064967	.00081202	.00097435	.00113665	.00129892	.00146116
6	.00071545	.00143039	.00214482	.00285874	.00357215	.00428505	.00499744	.00570931	.00642068
7	.00261144	.00521606	.00781388	.01040491	.01298918	.01556670	.01813749	.02070156	.02325894
8	.00806240	.01605980	.02399272	.03186168	.03966720	.04740979	.05508995	.06270820	.07026502
9	.02138697	.04231654	.06279849	.08284240	.10245762	.12165333	.14043851	.15882193	.17681218
10	.04936857	.09629989	.14091428	.18332611	.22364414	.26197172	.29840712	.33304376	.36597044
11	.10024421	.19043952	.27159327	.34461183	.41031070	.46942364	.52261085	.57046635	.61352461
12	.18079730	.32890694	.45023876	.54963411	.63105905	.69776258	.75240629	.79717056	.83384158
13	.29233236	.49920651	.64560465	.74920588	.82252111	.87440393	.91111973	.93710231	.95548934
14	.42776778	.67255028	.81262272	.89277668	.93864336	.96488975	.97990879	.98850316	.99342114
15	.57223222	.81701473	.92172480	.96651639	.98567679	.99387299	.99737906	.99887885	.99952041
16	.70766764	.91454179	.97501780	.99269689	.99786507	.99937589	.99981755	.99994666	.99998441
17	.81920270	.96731233	.99409016	.99893152	.99980682	.99996507	.99999369	.99999886	.99999979
18	.89975579	.98995110	.99899266	.99989902	.99998988	.99999899	.99999990	.99999999	1.00000000
19	.95063143	.99756274	.99987968	.99999406	.99999971	.99999999	1.00000000	1.00000000	1.00000000
20	.97861303	.99954260	.99999022	.99999979	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
21	.99193760	.99993500	.99999948	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
22	.99738856	.99999318	.99999998	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
23	.99928455	.99999949	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
24	.99983754	.99999997	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
1 31 0	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000
1	.00000001	.00000003	.00000004	.00000006	.00000007	.00000009	.00000010	.00000012	.00000013
2	.00000023	.00000046	.00000069	.00000093	.00000116	.00000139	.00000162	.00000185	.00000208
3	.00000232	.00000465	.00000697	.00000930	.00001162	.00001395	.00001627	.00001860	.00002092
4	.00001698	.00003395	.00005093	.00006790	.00008488	.00010186	.00011883	.00013580	.00015278
5	.00009610	.00019219	.00028827	.00038433	.00048040	.00057645	.00067249	.00076852	.00086455
6	.00043896	.00087772	.00131629	.00175466	.00219285	.00263084	.00306864	.00350625	.00394367
7	.00166345	.00332413	.00498204	.00663720	.00828961	.00993926	.01158618	.01323035	.01487179
8	.00533692	.01064536	.01592546	.02117739	.02640129	.03159731	.03676560	.04190630	.04701957
9	.01472469	.02923256	.04352680	.05761057	.07148696	.08515902	.09862977	.11190217	.12497913
10	.03537777	.06950396	.10242284	.13417712	.16480800	.19435524	.22285715	.25035074	.27687166
11	.07480639	.14401679	.20804980	.26729274	.32210393	.37281489	.41973234	.46314007	.50330063
12	.14052076	.26129543	.36509876	.45431556	.53099555	.59690041	.65354427	.70222849	.74407157
13	.23656483	.41716674	.55504459	.66030539	.74066519	.80201468	.84885105	.88460757	.91190536
14	.36005007	.59046408	.73791752	.83228033	.89266781	.93131277	.95604361	.97187011	.98199828
15	.50000000	.75000000	.87500000	.93750000	.96875000	.98437500	.99218750	.99609375	.99804688
16	.63994993	.87036395	.95332453	.98319449	.99394918	.99782140	.99921560	.99971757	.99989831
17	.76343517	.94403708	.98676114	.99686815	.99925911	.99982473	.99995854	.99999019	.99999768
18	.85947924	.98025392	.99722527	.99961009	.99994521	.99999230	.99999892	.99999985	.99999998
19	.92519361	.99440400	.99958138	.99996868	.99999766	.99999982	.99999999	1.00000000	1.00000000
20	.96462223	.99874841	.99995572	.99999843	.99999994	1.00000000	1.00000000	1.00000000	1.00000000
21	.98527531	.99978318	.99999681	.99999995	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
22	.99466308	.99997152	.99999985	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
23	.99833655	.99999723	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
24	.99956104	.99999981	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000

Table 31: Sheet I. Probabilities of perfect ($i = 0$) and i-imperfect single columns ($k = 1$) in Bernoulli ($m \times n$)-matrices. The rows with all 1's are omitted

k m i	Width n of Bernoulli ($m \times n$)-matrix								
	1	2	3	4	5	6	7	8	9
1 31 25	.99990390	.99999999	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
1 32 0	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000
1	.00000001	.00000002	.00000002	.00000003	.00000004	.00000005	.00000005	.00000006	.00000007
2	.00000012	.00000025	.00000037	.00000049	.00000062	.00000074	.00000086	.00000099	.00000111
3	.00000128	.00000256	.00000383	.00000511	.00000639	.00000767	.00000895	.00001022	.00001150
4	.00000965	.00001930	.00002895	.00003860	.00004825	.00005790	.00006755	.00007720	.00008685
5	.00005654	.00011307	.00016960	.00022613	.00028265	.00033917	.00039569	.00045221	.00050872
6	.00026753	.00053498	.00080236	.00106968	.00133692	.00160409	.00187118	.00213821	.00240516
7	.00105120	.00210130	.00315029	.00419818	.00524497	.00629065	.00733524	.00837873	.00942112
8	.00350018	.00698812	.01046384	.01392740	.01737883	.02081819	.02424550	.02766082	.03106419
9	.01003080	.01996099	.02979157	.03952354	.04915789	.05869560	.06813764	.07748497	.08673853
10	.02505123	.04947490	.07328672	.09650203	.11913576	.14120249	.16271643	.18369141	.20414094
11	.05509208	.10714903	.15633805	.20281714	.24673561	.28823451	.32744715	.36449949	.39951054
12	.10766357	.20373570	.28946436	.36596317	.43422584	.49513911	.54949424	.59799730	.64127835
13	.18854279	.34153720	.46568562	.56642674	.64817386	.71450814	.76833557	.81201423	.84745759
14	.29830745	.50762756	.65450593	.75756938	.82988824	.88063385	.91624166	.94122740	.95875970
15	.43002503	.67512854	.81483140	.89445853	.93984401	.96571259	.98045703	.98886100	.99365105
16	.56997497	.81507847	.92047911	.96580403	.98529488	.99367643	.99728071	.99883064	.99949714
17	.70169255	.91101267	.97345442	.99208125	.99763778	.99929533	.99978979	.99993729	.99998129
18	.81145721	.96445161	.99329761	.99873631	.99976174	.99995508	.99999153	.99999840	.99999970
19	.89233643	.98840855	.99875202	.99986564	.99998553	.99999844	.99999983	.99999998	1.00000000
20	.94490792	.99696486	.99983279	.99999079	.99999949	.99999997	1.00000000	1.00000000	1.00000000
21	.97494877	.99937244	.99998428	.99999961	.99999999	1.00000000	1.00000000	1.00000000	1.00000000
22	.98996920	.99989938	.99999899	.99999999	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
23	.99649982	.99998775	.99999996	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
24	.99894880	.99999889	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
25	.99973247	.99999993	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
26	.99994346	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
1 33 0	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000
1	.00000000	.00000001	.00000001	.00000002	.00000002	.00000002	.00000003	.00000003	.00000004
2	.00000007	.00000013	.00000020	.00000026	.00000033	.00000039	.00000046	.00000052	.00000059
3	.00000070	.00000140	.00000210	.00000280	.00000350	.00000420	.00000490	.00000560	.00000631
4	.00000546	.00001093	.00001639	.00002186	.00002732	.00003279	.00003825	.00004371	.00004918
5	.00003309	.00006619	.00009928	.00013237	.00016546	.00019855	.00023163	.00026472	.00029781
6	.00016203	.00032404	.00048602	.00064797	.00080990	.00097180	.00113367	.00129552	.00145734
7	.00065936	.00131829	.00197679	.00263485	.00329247	.00394967	.00460643	.00526275	.00591864
8	.00227569	.00454621	.00681155	.00907174	.01132679	.01357671	.01582150	.01806119	.02029578
9	.00676549	.01348521	.02015947	.02678858	.03337283	.03991254	.04640801	.05285953	.05926740
10	.01754102	.03477435	.05170539	.06833944	.08468171	.10073732	.11651131	.13200860	.14723405
11	.04007166	.07853757	.11546210	.15090700	.18493156	.21759270	.24894506	.27904108	.30793109
12	.08137783	.15613331	.22480535	.28788900	.34583905	.39907325	.44797536	.49289793	.53416479
13	.14810318	.27427182	.38175447	.47331860	.55132179	.61777247	.67438158	.72260671	.76368954
14	.24342512	.42759445	.56693234	.67235189	.75210967	.81245240	.85810620	.89264672	.91877920
15	.36416624	.59571543	.74294222	.83655399	.89607551	.93392130	.95798493	.97328540	.98301396
16	.50000000	.75000000	.87500000	.93750000	.96875000	.98437500	.99218750	.99609375	.99804688
17	.63583376	.86738295	.95170535	.98241272	.99359531	.99766763	.99915063	.99969069	.99988736
18	.75657488	.94074421	.98557565	.99648875	.99914527	.99979194	.99994935	.99998767	.99999700
19	.85189682	.97806545	.99675142	.99951888	.99992874	.99998945	.99999844	.99999977	.99999997
20	.91862217	.99337765	.99946109	.99995614	.99999643	.99999971	.99999998	1.00000000	1.00000000
21	.95992834	.99839426	.99993566	.99999742	.99999990	1.00000000	1.00000000	1.00000000	1.00000000
22	.98245898	.99969231	.99999460	.99999991	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
23	.99323451	.99995423	.99999969	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
24	.99772431	.99999482	.99999999	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
25	.99934064	.99999957	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
26	.99983797	.99999997	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
1 34 0	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000

Table 31: Sheet J. Probabilities of perfect ($i = 0$) and i-imperfect single columns ($k = 1$) in Bernoulli ($m \times n$)-matrices. The rows with all 1's are omitted

k m i	Width n of Bernoulli ($m \times n$)-matrix								
	1	2	3	4	5	6	7	8	9
1 34 1	.00000000	.00000000	.00000001	.00000001	.00000001	.00000001	.00000001	.00000002	.00000002
2	.00000003	.00000007	.00000010	.00000014	.00000017	.00000021	.00000024	.00000028	.00000031
3	.00000038	.00000077	.00000115	.00000153	.00000192	.00000230	.00000268	.00000306	.00000345
4	.00000308	.00000616	.00000925	.00001233	.00001541	.00001849	.00002158	.00002466	.00002774
5	.00001928	.00003856	.00005784	.00007711	.00009639	.00011567	.00013495	.00015422	.00017350
6	.00009756	.00019512	.00029266	.00039019	.00048772	.00058523	.00068274	.00078024	.00087772
7	.00041070	.00082123	.00123159	.00164178	.00205180	.00246166	.00287134	.00328086	.00369021
8	.00146753	.00293290	.00439613	.00585720	.00731613	.00877293	.01022758	.01168010	.01313048
9	.00452059	.00902075	.01350056	.01796013	.02239953	.02681886	.03121822	.03559768	.03995736
10	.01215326	.02415881	.03601846	.04773397	.05930710	.07073958	.08203312	.09318941	.10421011
11	.02880634	.05678287	.08395350	.11034144	.13596925	.16085881	.18503139	.20850765	.23130764
12	.06072474	.11776199	.17133567	.22165609	.26892083	.31331542	.35501417	.39418077	.43096898
13	.11474051	.21631563	.30623597	.38583881	.45630797	.51869147	.57391706	.62280603	.66608546
14	.19576415	.35320470	.47982404	.58165584	.66355263	.72941697	.78238742	.82498817	.85924921
15	.30379568	.51529955	.66254945	.76506547	.83643757	.88612713	.92072121	.94480577	.96157354
16	.43208312	.67747042	.81683001	.89597467	.94092226	.96644875	.98094568	.98917873	.99385442
17	.56791688	.81330418	.91933189	.96514467	.98493960	.99349266	.99718829	.99878511	.99947506
18	.69620432	.90770818	.97196215	.99148222	.99741234	.99921388	.99976118	.99992745	.99997796
19	.80423585	.96167640	.99249761	.99853130	.99971248	.99994371	.99998898	.99999784	.99999958
20	.88525949	.98683462	.99848940	.99982667	.99998011	.99999772	.99999974	.99999997	1.00000000
21	.93927526	.99631251	.99977608	.99998640	.99999917	.99999995	1.00000000	1.00000000	1.00000000
22	.97119366	.99917019	.99997610	.99999931	.99999998	1.00000000	1.00000000	1.00000000	1.00000000
23	.98784674	.99985230	.99999820	.99999998	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
24	.99547941	.99997956	.99999991	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
25	.99853247	.99999785	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
26	.99958930	.99999983	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
27	.99990244	.99999999	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
1 35 0	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000
1	.00000000	.00000000	.00000000	.00000000	.00000001	.00000001	.00000001	.00000001	.00000001
2	.00000002	.00000004	.00000006	.00000007	.00000009	.00000011	.00000013	.00000015	.00000017
3	.00000021	.00000042	.00000063	.00000084	.00000104	.00000125	.00000146	.00000167	.00000188
4	.00000173	.00000347	.00000520	.00000693	.00000866	.00001040	.00001213	.00001386	.00001559
5	.00001118	.00002236	.00003354	.00004472	.00005590	.00006708	.00007826	.00008944	.00010062
6	.00005842	.00011684	.00017525	.00023366	.00029207	.00035047	.00040887	.00046727	.00052567
7	.00025413	.00050820	.00076220	.00101613	.00127001	.00152381	.00177756	.00203123	.00228485
8	.00093911	.00187734	.00281469	.00375116	.00468675	.00562146	.00655530	.00748825	.00842033
9	.00299406	.00597916	.00895531	.01192256	.01488093	.01783043	.02077111	.02370298	.02662607
10	.00833692	.01660434	.02480284	.03293298	.04099535	.04899050	.05691899	.06478138	.07257823
11	.02047980	.04054017	.06018971	.07943683	.09828978	.11675662	.13484526	.15256346	.16991878
12	.04476554	.08752713	.12837447	.16739325	.20466534	.24026893	.27427870	.30676601	.33779900
13	.08773262	.16776824	.24078211	.30739029	.36815476	.42358820	.47415832	.52029179	.56237785
14	.15525233	.28640137	.39718922	.49077700	.56983506	.63661917	.69303489	.74069194	.78095012
15	.24977992	.43716983	.57775350	.68322220	.76234693	.82170789	.86624168	.89965182	.92471678
16	.36793940	.60049940	.74749141	.84039927	.89912267	.93623941	.95969944	.97452761	.98389990
17	.50000000	.75000000	.87500000	.93750000	.96875000	.98437500	.99218750	.99609375	.99804688
18	.63206060	.86462060	.95018858	.98167242	.99325656	.99751882	.99908708	.99966410	.99987641
19	.75022008	.93760999	.98441623	.99610749	.99902773	.99975715	.99993934	.99998485	.99999622
20	.84474767	.97589671	.99625791	.99941903	.99990980	.99998600	.99999783	.99999966	.99999995
21	.91226738	.99230299	.99932472	.99994076	.99999480	.99999954	.99999996	1.00000000	1.00000000
22	.95523446	.99799605	.99991029	.99999598	.99999982	.99999999	1.00000000	1.00000000	1.00000000
23	.97952020	.99958058	.99999141	.99999982	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
24	.99166308	.99993050	.99999942	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
25	.99700594	.99999104	.99999997	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
26	.99906089	.99999912	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
27	.99974587	.99999994	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
28	.99994158	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000

Table 31: Sheet K. Probabilities of perfect ($i = 0$) and i-imperfect single columns ($k = 1$) in Bernoulli ($m \times n$)-matrices. The rows with all 1's are omitted

k m i	Width n of Bernoulli ($m \times n$)-matrix								
	1	2	3	4	5	6	7	8	9
1 36 0	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000
1	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000
2	.00000001	.00000002	.00000003	.00000004	.00000005	.00000006	.00000007	.00000008	.00000009
3	.00000011	.00000023	.00000034	.00000045	.00000057	.00000068	.00000080	.00000091	.00000102
4	.00000097	.00000194	.00000291	.00000388	.00000485	.00000582	.00000680	.00000777	.00000874
5	.00000646	.00001291	.00001937	.00002583	.00003228	.00003874	.00004520	.00005165	.00005811
6	.00003480	.00006960	.00010440	.00013920	.00017399	.00020879	.00024358	.00027837	.00031316
7	.00015628	.00031253	.00046875	.00062496	.00078113	.00093729	.00109342	.00124952	.00140560
8	.00059662	.00119289	.00178880	.00238435	.00297955	.00357439	.00416888	.00476302	.00535680
9	.00196659	.00392931	.00588816	.00784317	.00979433	.01174166	.01368515	.01562483	.01756069
10	.00566549	.01129889	.01690036	.02247011	.02800830	.03351511	.03899072	.04443531	.04984905
11	.01440836	.02860912	.04260527	.05639976	.06999549	.08339533	.09660210	.10961858	.12244752
12	.03262267	.06418110	.09471001	.12424298	.15281251	.18045003	.20718593	.23304964	.25806961
13	.06624908	.12810922	.18587119	.23980647	.29016860	.33719428	.38110455	.42210580	.46039076
14	.12149248	.22822453	.32198945	.40436263	.47672809	.54030169	.59615157	.64521612	.68831969
15	.20251612	.36401947	.49281578	.59552876	.67744071	.74276417	.79485857	.83640302	.86953404
16	.30885966	.52232503	.66985956	.77182662	.84230017	.89100729	.92467074	.94793691	.96401710
17	.43396970	.67960970	.81864938	.89735006	.94189702	.96711195	.98138437	.98946299	.99403573
18	.56603030	.81167030	.91827062	.96453192	.98460793	.99332031	.99710122	.99874202	.99945407
19	.69114034	.90460571	.97053655	.99089993	.99718936	.99913191	.99973188	.99991719	.99997442
20	.79748388	.95898722	.99169425	.99831795	.99965936	.99993101	.99998603	.99999717	.99999943
21	.87850752	.98523958	.99820672	.99978213	.99997353	.99999678	.99999961	.99999995	.99999999
22	.93375092	.99561106	.99970924	.99998074	.99999872	.99999992	.99999999	1.00000000	1.00000000
23	.96737733	.99893576	.99996528	.99999887	.99999996	1.00000000	1.00000000	1.00000000	1.00000000
24	.98559164	.99979240	.99999701	.99999996	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
25	.99433451	.99996790	.99999982	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
26	.99803341	.99999613	.99999999	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
27	.99940338	.99999964	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
28	.99984372	.99999998	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
1 37 0	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000
1	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000
2	.00000001	.00000001	.00000002	.00000002	.00000003	.00000003	.00000004	.00000004	.00000005
3	.00000006	.00000012	.00000018	.00000025	.00000031	.00000037	.00000043	.00000049	.00000055
4	.00000054	.00000108	.00000163	.00000217	.00000271	.00000325	.00000380	.00000434	.00000488
5	.00000371	.00000743	.00001114	.00001485	.00001857	.00002228	.00002600	.00002971	.00003342
6	.00002063	.00004126	.00006189	.00008251	.00010314	.00012377	.00014439	.00016502	.00018564
7	.00009554	.00019107	.00028659	.00038210	.00047760	.00057309	.00066858	.00076405	.00085952
8	.00037645	.00075276	.00112892	.00150494	.00188083	.00225657	.00263217	.00300762	.00338294
9	.00128160	.00256157	.00383989	.00511657	.00639162	.00766503	.00893681	.01020696	.01147548
10	.00381604	.00761752	.01140449	.01517701	.01893513	.02267891	.02640841	.03012367	.03382476
11	.01003693	.01997311	.02980957	.03954730	.04918729	.05873053	.06817798	.07753061	.08678937
12	.02351551	.04647805	.06890061	.09079589	.11217629	.13305392	.15344060	.17334788	.19278703
13	.04943587	.09642784	.14109672	.18355736	.22391891	.26228516	.29875474	.33342141	.36637431
14	.09387078	.17892984	.25600433	.32584379	.38912735	.44647045	.49843070	.54551340	.58817641
15	.16200430	.29776321	.41152859	.50686349	.58675372	.65370140	.70980326	.75681638	.79621317
16	.25568789	.44599948	.58765071	.69308343	.77155828	.82996806	.87344317	.90580222	.92988745
17	.37141468	.60488049	.75163368	.84388058	.90186562	.93831417	.96122519	.97562673	.98467932
18	.50000000	.75000000	.87500000	.93750000	.96875000	.98437500	.99218750	.99609375	.99804687
19	.62858532	.86205114	.94876377	.98097011	.99293202	.99737485	.99902498	.99963786	.99986550
20	.74431211	.93462370	.98328407	.99572594	.99890717	.99972058	.99992856	.99998173	.99999533
21	.83799570	.97375461	.99574813	.99931118	.99988841	.99998192	.99999707	.99999953	.99999992
22	.90612922	.99118828	.99917284	.99992235	.99999271	.99999932	.99999994	.99999999	1.00000000
23	.95056413	.99755609	.99987918	.99999403	.99999970	.99999999	1.00000000	1.00000000	1.00000000
24	.97648449	.99944702	.99998700	.99999969	.99999999	1.00000000	1.00000000	1.00000000	1.00000000
25	.98996307	.99989926	.99999899	.99999999	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
26	.99618396	.99998544	.99999994	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000

Table 31: Sheet L. Probabilities of perfect ($i = 0$) and i-imperfect single columns ($k = 1$) in Bernoulli ($m \times n$)-matrices. The rows with all 1's are omitted

k m i	Width n of Bernoulli ($m \times n$)-matrix								
	1	2	3	4	5	6	7	8	9
1 37 27	.99871840	.99999836	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
28	.99962355	.99999986	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
29	.99990446	.99999999	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
1 38 0	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000
1	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000
2	.00000000	.00000001	.00000001	.00000001	.00000001	.00000002	.00000002	.00000002	.00000002
3	.00000003	.00000007	.00000010	.00000013	.00000017	.00000020	.00000023	.00000027	.00000030
4	.00000030	.00000060	.00000091	.00000121	.00000151	.00000181	.00000211	.00000242	.00000272
5	.00000213	.00000426	.00000638	.00000851	.00001064	.00001277	.00001490	.00001702	.00001915
6	.00001217	.00002434	.00003651	.00004868	.00006085	.00007303	.00008520	.00009737	.00010954
7	.00005808	.00011616	.00017424	.00023231	.00029038	.00034845	.00040651	.00046457	.00052263
8	.00023599	.00047193	.00070781	.00094364	.00117941	.00141513	.00165078	.00188639	.00212194
9	.00082903	.00165737	.00248502	.00331198	.00413826	.00496386	.00578877	.00661300	.00743654
10	.00254882	.00509115	.00762699	.01015637	.01267931	.01519581	.01770590	.02020960	.02270691
11	.00692648	.01380499	.02063585	.02741940	.03415596	.04084587	.04748943	.05408698	.06063883
12	.01677622	.03327100	.04948906	.06543504	.08111350	.09652895	.11168578	.12658833	.14124088
13	.03647569	.07162091	.10548418	.13811227	.16955022	.19984145	.22902779	.25714954	.28424553
14	.07165333	.13817246	.19992527	.25725328	.31047356	.35988042	.40574712	.44832731	.48785650
15	.12793754	.23950707	.33680266	.42165050	.49564311	.56016929	.61644015	.66551185	.70830544
16	.20884610	.37407550	.50479739	.60821852	.69004055	.75477437	.80598879	.84650727	.87856363
17	.31355129	.52878816	.67653724	.77795940	.84758052	.89537184	.92817814	.95069797	.96615669
18	.43570734	.68157379	.82031443	.89860475	.94278341	.96771310	.98178074	.98971900	.99419851
19	.56429266	.81015911	.91728493	.96396044	.98429730	.99315822	.99701899	.99870115	.99943408
20	.68644871	.90168559	.96917339	.99033428	.99696930	.99904972	.99970204	.99990657	.99997071
21	.79115390	.95638331	.99089082	.99809758	.99960269	.99991702	.99998267	.99999638	.99999924
22	.87206246	.98363199	.99790592	.99973209	.99996572	.99999561	.99999944	.99999993	.99999999
23	.92834667	.99486580	.99963212	.99997364	.99999811	.99999986	.99999999	1.00000000	1.00000000
24	.96352431	.99866952	.99995147	.99999823	.99999994	1.00000000	1.00000000	1.00000000	1.00000000
25	.98322378	.99971856	.99999528	.99999992	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
26	.99307352	.99995202	.99999967	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
27	.99745118	.99999350	.99999998	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
28	.99917097	.99999931	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
29	.99976401	.99999994	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
30	.99994192	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
1 39 0	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000
1	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000
2	.00000000	.00000000	.00000000	.00000001	.00000001	.00000001	.00000001	.00000001	.00000001
3	.00000002	.00000004	.00000005	.00000007	.00000009	.00000011	.00000013	.00000014	.00000016
4	.00000017	.00000034	.00000050	.00000067	.00000084	.00000101	.00000117	.00000134	.00000151
5	.00000121	.00000243	.00000364	.00000486	.00000607	.00000729	.00000850	.00000972	.00001093
6	.00000715	.00001430	.00002145	.00002860	.00003575	.00004290	.00005005	.00005720	.00006434
7	.00003513	.00007025	.00010538	.00014050	.00017562	.00021075	.00024587	.00028098	.00031610
8	.00014704	.00029406	.00044105	.00058802	.00073498	.00088191	.00102882	.00117570	.00132257
9	.00053251	.00106474	.00159668	.00212834	.00265971	.00319081	.00372162	.00425215	.00478239
10	.00168892	.00337500	.00505822	.00673860	.00841614	.01009085	.01176273	.01343179	.01509803
11	.00473765	.00945286	.01414573	.01881636	.02346487	.02809135	.03269592	.03727867	.04183971
12	.01185135	.02356225	.03513435	.04656932	.05786876	.06903429	.08006749	.09096993	.10174317
13	.02662596	.05254297	.07776992	.10232518	.12622663	.14949168	.17213728	.19417992	.21563565
14	.05406451	.10520605	.15358265	.19934379	.24263087	.28357766	.32231069	.35894963	.39360770
15	.09979543	.18963174	.27050279	.34330328	.40883861	.46783382	.52094158	.56874942	.61178626
16	.16839182	.30842783	.42488293	.52172794	.60226504	.66924035	.72493757	.77125583	.80977448
17	.26119869	.45417262	.59674202	.70207248	.77989076	.83738300	.87985835	.91123919	.93442340
18	.37462931	.60891150	.75542472	.84704979	.90434942	.94018293	.96259216	.97660623	.98537022
19	.50000000	.75000000	.87500000	.93750000	.96875000	.98437500	.99218750	.99609375	.99804687
20	.62537069	.85965288	.94742185	.98030269	.99262081	.99723554	.99896435	.99961202	.99985465
21	.73880131	.93177524	.98217978	.99534538	.99878422	.99968244	.99991705	.99997833	.99999434

Table 31: Sheet M. Probabilities of perfect ($i = 0$) and i-imperfect single columns ($k = 1$) in Bernoulli ($m \times n$)-matrices. The rows with all 1's are omitted

k m i	Width n of Bernoulli ($m \times n$)-matrix								
	1	2	3	4	5	6	7	8	9
1 39 22	.83160818	.97164420	.99522511	.99919595	.99986460	.99997720	.99999616	.99999935	.99999989
23	.90020457	.99004087	.99900612	.99990082	.99999010	.99999901	.99999990	.99999999	1.00000000
24	.94593549	.99707703	.99984197	.99999146	.99999954	.99999998	1.00000000	1.00000000	1.00000000
25	.97337404	.99929106	.99998112	.99999950	.99999999	1.00000000	1.00000000	1.00000000	1.00000000
26	.98814865	.99985955	.99999834	.99999998	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
27	.99526235	.99997755	.99999989	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
28	.99831108	.99999715	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
29	.99946749	.99999972	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
30	.99985296	.99999998	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
1 40 0	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000
1	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000
2	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000001	.00000001	.00000001
3	.00000001	.00000002	.00000003	.00000004	.00000005	.00000006	.00000007	.00000008	.00000009
4	.00000009	.00000019	.00000028	.00000037	.00000046	.00000056	.00000065	.00000074	.00000084
5	.00000069	.00000138	.00000207	.00000277	.00000346	.00000415	.00000484	.00000553	.00000622
6	.00000418	.00000836	.00001255	.00001673	.00002091	.00002509	.00002928	.00003346	.00003764
7	.00002114	.00004228	.00006341	.00008455	.00010569	.00012682	.00014796	.00016910	.00019023
8	.00009108	.00018216	.00027322	.00036428	.00045533	.00054637	.00063741	.00072843	.00081945
9	.00033977	.00067943	.00101898	.00135840	.00169772	.00203691	.00237600	.00271496	.00305381
10	.00111072	.00222020	.00332845	.00443547	.00554126	.00664582	.00774916	.00885127	.00995215
11	.00321329	.00641625	.00960892	.01279133	.01596352	.01912551	.02227734	.02541905	.02855066
12	.00829450	.01652020	.02467768	.03276749	.04079020	.04874637	.05663655	.06446128	.07222110
13	.01923865	.03810718	.05661271	.07476221	.09256254	.11002041	.12714242	.14393503	.16040457
14	.04034523	.07906273	.11621816	.15187454	.18609236	.21892966	.25044212	.28068321	.30970422
15	.07692997	.14794172	.21349054	.27399669	.32984811	.38140287	.42899153	.47291920	.51346751
16	.13409363	.25020615	.35074873	.43780918	.51319539	.57847278	.63499690	.68394149	.72632292
17	.21479525	.38345351	.51588477	.61987042	.70152045	.76563244	.81597348	.85550150	.88653909
18	.31791400	.53475869	.68266542	.78355052	.85236284	.89929876	.93131310	.95314962	.96804401
19	.43731466	.68338520	.82184549	.89975507	.94359365	.96826097	.98214091	.98995095	.99434555
20	.56268534	.80875589	.91636615	.96342569	.98400552	.99300538	.99694115	.99866232	.99941501
21	.68208600	.89893069	.96786865	.98978499	.99675251	.99896758	.99967178	.99989565	.99996683
22	.78520475	.95386300	.99008999	.99787138	.99954278	.99990179	.99997891	.99999547	.99999903
23	.86590637	.98201890	.99758885	.99967668	.99995664	.99999419	.99999922	.99999990	.99999999
24	.92307003	.99408178	.99954471	.99996497	.99999731	.99999979	.99999998	1.00000000	1.00000000
25	.95965477	.99837226	.99993433	.99999735	.99999989	1.00000000	1.00000000	1.00000000	1.00000000
26	.98076135	.99962987	.99999288	.99999986	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
27	.99170550	.99993120	.99999943	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
28	.99678671	.99998967	.99999997	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
29	.99888928	.99999877	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
30	.99966023	.99999988	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
31	.99990892	.99999999	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
1 41 0	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000
1	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000
2	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000
3	.00000001	.00000001	.00000002	.00000002	.00000003	.00000003	.00000004	.00000004	.00000005
4	.00000005	.00000010	.00000015	.00000021	.00000026	.00000031	.00000036	.00000041	.00000046
5	.00000039	.00000078	.00000118	.00000157	.00000196	.00000235	.00000274	.00000314	.00000353
6	.00000244	.00000487	.00000731	.00000975	.00001218	.00001462	.00001706	.00001949	.00002193
7	.00001266	.00002532	.00003798	.00005064	.00006330	.00007596	.00008862	.00010128	.00011394
8	.00005611	.00011222	.00016832	.00022442	.00028052	.00033662	.00039271	.00044880	.00050488
9	.00021543	.00043081	.00064615	.00086144	.00107668	.00129188	.00150703	.00172213	.00193719
10	.00072525	.00144997	.00217416	.00289783	.00362097	.00434359	.00506569	.00578726	.00650831
11	.00216200	.00431933	.00647199	.00862000	.01076337	.01290210	.01503621	.01716571	.01929060
12	.00575389	.01147468	.01716255	.02281770	.02844030	.03403055	.03958864	.04511475	.05060906
13	.01376658	.02734364	.04073379	.05393960	.06696361	.07980833	.09247622	.10496972	.11729122
14	.02979194	.05869633	.08673959	.11394740	.14034463	.16595543	.19080324	.21491079	.23830012

Table 31: Sheet N. Probabilities of perfect ($i = 0$) and i-imperfect single columns ($k = 1$) in Bernoulli ($m \times n$)-matrices. The rows with all 1's are omitted

k m i	Width n of Bernoulli ($m \times n$)-matrix								
	1	2	3	4	5	6	7	8	9
1 41 15	.05863760	.11383684	.16579932	.21471485	.26076209	.30410923	.34491459	.38332723	.41948745
16	.10551180	.19989086	.28431181	.35982536	.42737134	.48779042	.54183457	.59017643	.63341765
17	.17444444	.31845802	.43734923	.53550053	.61652988	.68342411	.73864901	.78424024	.82187833
18	.26635463	.46176447	.60512599	.71030251	.78746478	.84407452	.88560599	.91607537	.93842908
19	.37761433	.61263608	.75891024	.84994919	.90661053	.94187573	.96382429	.97748475	.98598683
20	.50000000	.75000000	.87500000	.93750000	.96875000	.98437500	.99218750	.99609375	.99804687
21	.62238567	.85740742	.94615500	.97966736	.99232210	.99710072	.99890519	.99958658	.99984389
22	.73364537	.92905521	.98110353	.99496684	.99865939	.99964292	.99990489	.99997467	.99999325
23	.82555556	.96956914	.99469151	.99907396	.99983846	.99997182	.99999508	.99999914	.99999985
24	.89448820	.98886726	.99882536	.99987606	.99998692	.99999862	.99999985	.99999998	1.00000000
25	.94136240	.99656163	.99979838	.99998818	.99999931	.99999996	1.00000000	1.00000000	1.00000000
26	.97020806	.99911244	.99997356	.99999921	.99999998	1.00000000	1.00000000	1.00000000	1.00000000
27	.98623342	.99981048	.99999739	.99999996	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
28	.99424611	.99996689	.99999981	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
29	.99783800	.99999533	.99999999	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
30	.99927475	.99999947	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
31	.99978457	.99999995	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
32	.99994389	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
1 42 0	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000
1	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000
2	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000
3	.00000000	.00000001	.00000001	.00000001	.00000001	.00000002	.00000002	.00000002	.00000003
4	.00000003	.00000006	.00000008	.00000011	.00000014	.00000017	.00000020	.00000023	.00000025
5	.00000022	.00000044	.00000067	.00000089	.00000111	.00000133	.00000155	.00000177	.00000200
6	.00000141	.00000283	.00000424	.00000566	.00000707	.00000849	.00000990	.00001132	.00001273
7	.00000755	.00001510	.00002265	.00003019	.00003774	.00004529	.00005284	.00006039	.00006794
8	.00003439	.00006877	.00010315	.00013754	.00017192	.00020630	.00024067	.00027505	.00030943
9	.00013577	.00027152	.00040725	.00054297	.00067866	.00081434	.00095000	.00108564	.00122126
10	.00047034	.00094045	.00141035	.00188002	.00234947	.00281871	.00328772	.00375651	.00422508
11	.00144362	.00288516	.00432462	.00576200	.00719731	.00863054	.01006171	.01149081	.01291784
12	.00395795	.00790023	.01182691	.01573805	.01963371	.02351395	.02737883	.03122841	.03506276
13	.00976024	.01942521	.02899585	.03847308	.04785781	.05715095	.06635337	.07546599	.08448966
14	.02177926	.04308419	.06392510	.08431212	.10425513	.12376379	.14284757	.16151571	.17977728
15	.04421477	.08647460	.12686592	.16547135	.20236984	.23763688	.27134459	.30356192	.33435478
16	.08207470	.15741315	.22656821	.29004739	.34831654	.40180327	.45090008	.49596729	.53733563
17	.13997812	.26036236	.36389545	.45293617	.52951313	.59537100	.65201021	.70072116	.74261365
18	.22039953	.39222311	.52617686	.63060726	.71202124	.77549163	.82497317	.86354900	.89362274
19	.32198448	.54029495	.68831284	.78867127	.85671584	.90285112	.93413155	.95534017	.96971994
20	.43880716	.68506260	.82325939	.90081443	.94433777	.96876276	.98246988	.99016222	.99447911
21	.56119284	.80744827	.91550692	.96292383	.98373071	.99286092	.99686732	.99862536	.99939680
22	.67801552	.89632600	.96661858	.98925170	.99653921	.99888568	.99964121	.99988447	.99996280
23	.77960047	.95142405	.98929388	.99764038	.99947994	.99988538	.99997474	.99999443	.99999877
24	.86002188	.98040613	.99725729	.99961608	.99994626	.99999248	.99999895	.99999985	.99999998
25	.91792530	.99326374	.99944712	.99995462	.99999628	.99999969	.99999997	1.00000000	1.00000000
26	.95578523	.99804505	.99991356	.99999618	.99999983	.99999999	1.00000000	1.00000000	1.00000000
27	.97822074	.99952566	.99998967	.99999978	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
28	.99023976	.99990474	.99999907	.99999999	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
29	.99604205	.99998433	.99999994	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
30	.99855638	.99999792	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
31	.99952966	.99999978	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
32	.99986423	.99999998	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
1 43 0	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000
1	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000
2	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000
3	.00000000	.00000000	.00000000	.00000001	.00000001	.00000001	.00000001	.00000001	.00000001
4	.00000002	.00000003	.00000005	.00000006	.00000008	.00000009	.00000011	.00000012	.00000014

Table 31: Sheet O. Probabilities of perfect ($i = 0$) and i-imperfect single columns ($k = 1$) in Bernoulli ($m \times n$)-matrices. The rows with all 1's are omitted

k m i	Width n of Bernoulli ($m \times n$)-matrix								
	1	2	3	4	5	6	7	8	9
1 43 5	.00000012	.00000025	.00000037	.00000050	.00000062	.00000075	.00000087	.00000100	.00000112
6	.00000082	.00000164	.00000245	.00000327	.00000409	.00000491	.00000573	.00000654	.00000736
7	.00000448	.00000896	.00001344	.00001793	.00002241	.00002689	.00003137	.00003585	.00004033
8	.00002097	.00004193	.00006290	.00008387	.00010483	.00012580	.00014676	.00016772	.00018869
9	.00008508	.00017015	.00025521	.00034027	.00042532	.00051036	.00059539	.00068042	.00076544
10	.00030305	.00060601	.00090888	.00121166	.00151435	.00181694	.00211945	.00242186	.00272418
11	.00095698	.00191305	.00286819	.00382243	.00477575	.00572816	.00667966	.00763025	.00857993
12	.00270079	.00539428	.00808050	.01075946	.01343119	.01609570	.01875301	.02140315	.02404613
13	.00685909	.01367114	.02043646	.02715538	.03382821	.04045527	.04703687	.05357334	.06006497
14	.01576975	.03129081	.04656711	.06160251	.07640080	.09096573	.10530097	.11941015	.13329683
15	.03299702	.06490523	.09576057	.12559777	.15445044	.18235105	.20933103	.23542075	.26064958
16	.06314474	.12230222	.17772421	.22964660	.27829036	.32386253	.36655705	.40655564	.44402853
17	.11102641	.20972596	.29746725	.37546694	.44480660	.50644773	.56124507	.60995845	.65326336
18	.18018883	.32790964	.44901281	.54829455	.62968682	.69641312	.75111608	.79596218	.83272752
19	.27119201	.46883891	.61288555	.71786789	.79437987	.85014240	.89078258	.92040147	.94198796
20	.38039582	.61609066	.76212817	.85261362	.90867878	.94341699	.96494093	.97827725	.98654050
21	.50000000	.75000000	.87500000	.93750000	.96875000	.98437500	.99218750	.99609375	.99804688
22	.61960418	.85529902	.94495635	.97906163	.99203513	.99697020	.99884748	.99956158	.99983323
23	.72880799	.92645490	.98005516	.99459112	.99853315	.99960220	.99989212	.99997074	.99999207
24	.81981117	.96753199	.99414963	.99894583	.99981005	.99996577	.99999383	.99999889	.99999980
25	.88897359	.98767314	.99863139	.99984805	.99998313	.99999813	.99999979	.99999998	1.00000000
26	.93685526	.99601274	.99974823	.99998410	.99999900	.99999994	1.00000000	1.00000000	1.00000000
27	.96700298	.99891120	.99996407	.99999881	.99999996	1.00000000	1.00000000	1.00000000	1.00000000
28	.98423025	.99975132	.99999608	.99999994	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
29	.99314091	.99995295	.99999968	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
30	.99729921	.99999271	.99999998	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
31	.99904302	.99999908	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
32	.99969695	.99999991	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
33	.99991492	.99999999	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
1 44 0	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000
1	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000
2	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000
3	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000001	.00000001	.00000001
4	.00000001	.00000002	.00000003	.00000003	.00000004	.00000005	.00000006	.00000007	.00000008
5	.00000007	.00000014	.00000021	.00000028	.00000035	.00000042	.00000049	.00000056	.00000063
6	.00000047	.00000094	.00000141	.00000189	.00000236	.00000283	.00000330	.00000377	.00000424
7	.00000265	.00000530	.00000795	.00001060	.00001325	.00001590	.00001855	.00002120	.00002385
8	.00001272	.00002545	.00003817	.00005090	.00006362	.00007634	.00008907	.00010179	.00011451
9	.00005302	.00010604	.00015906	.00021207	.00026508	.00031809	.00037110	.00042410	.00047710
10	.00019407	.00038809	.00058208	.00077604	.00096995	.00116383	.00135767	.00155147	.00174523
11	.00063002	.00125964	.00188886	.00251769	.00314612	.00377415	.00440179	.00502904	.00565588
12	.00182888	.00365442	.00547662	.00729549	.00911103	.01092325	.01273216	.01453775	.01634005
13	.00477994	.00953703	.01427138	.01898311	.02367231	.02833910	.03298358	.03760586	.04220604
14	.01131442	.02250083	.03356066	.04449536	.05530634	.06599501	.07656273	.08701089	.09734083
15	.02438338	.04817222	.07138100	.09402387	.11611463	.13766675	.15869335	.17920725	.19922096
16	.04807088	.09383095	.13739129	.17885764	.21833068	.25590621	.29167545	.32572523	.35813821
17	.08708557	.16658725	.23916548	.30542319	.36591081	.42113083	.47154198	.51756305	.55957635
18	.14560762	.27001366	.37630523	.46711994	.54471134	.61100483	.66764549	.71603884	.75738575
19	.22569042	.40044467	.53575856	.64053340	.72166157	.78447989	.83312071	.87078377	.89994663
20	.32579391	.54544615	.69353703	.79338080	.86069608	.90608045	.93667887	.95730851	.97121714
21	.44019791	.68662162	.82457013	.90179399	.94502407	.96922436	.98277173	.99035558	.99460103
22	.55980209	.80622580	.91470100	.96245156	.98347125	.99272408	.99679716	.99859011	.99937937
23	.67420609	.89385833	.96541969	.98873395	.99632959	.99880420	.99961042	.99987308	.99995865
24	.77430958	.94906384	.98850420	.99740551	.99941445	.99986785	.99997017	.99999327	.99999848
25	.85439238	.97879842	.99691289	.99955049	.99993455	.99999047	.99999861	.99999980	.99999997
26	.91291443	.99241610	.99933955	.99994248	.99999499	.99999956	.99999996	1.00000000	1.00000000

Table 31: Sheet P. Probabilities of perfect ($i = 0$) and i-imperfect single columns ($k = 1$) in Bernoulli ($m \times n$)-matrices. The rows with all 1's are omitted

k	m	i	Width n of Bernoulli ($m \times n$)-matrix								
			1	2	3	4	5	6	7	8	9
1	44	27	.95192912	.99768919	.99988892	.99999466	.99999974	.99999999	1.00000000	1.00000000	1.00000000
		28	.97561662	.99940545	.99998550	.99999965	.99999999	1.00000000	1.00000000	1.00000000	1.00000000
		29	.98868558	.99987198	.99999855	.99999998	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
		30	.99522006	.99997715	.99999989	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
		31	.99817112	.99999666	.99999999	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
		32	.99936998	.99999960	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
		33	.99980593	.99999996	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
		34	.99994698	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
1	45	0	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000
		1	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000
		2	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000
		3	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000
		4	.00000000	.00000001	.00000001	.00000002	.00000002	.00000003	.00000003	.00000004	.00000004
		5	.00000004	.00000008	.00000012	.00000016	.00000020	.00000024	.00000028	.00000032	.00000035
		6	.00000027	.00000054	.00000081	.00000108	.00000135	.00000163	.00000190	.00000217	.00000244
		7	.00000156	.00000312	.00000468	.00000624	.00000780	.00000936	.00001092	.00001249	.00001405
		8	.00000769	.00001537	.00002306	.00003075	.00003843	.00004612	.00005381	.00006149	.00006918
		9	.00003287	.00006575	.00009862	.00013149	.00016436	.00019722	.00023009	.00026296	.00029582
		10	.00012354	.00024707	.00037059	.00049408	.00061757	.00074103	.00086449	.00098792	.00111135
		11	.00041204	.00082391	.00123561	.00164715	.00205851	.00246970	.00288073	.00329158	.00370226
		12	.00122945	.00245739	.00368382	.00490874	.00613215	.00735406	.00857447	.00979338	.01101079
		13	.00330441	.00659790	.00988051	.01315228	.01641323	.01966340	.02290284	.02613157	.02934963
		14	.00804718	.01602960	.02394779	.03180226	.03959352	.04732208	.05498845	.06259313	.07013661
		15	.01784890	.03537922	.05259664	.06950675	.08611503	.10242688	.11844757	.13418231	.14963621
		16	.03622713	.07114186	.10479172	.13722255	.16847850	.19860214	.22763448	.25561507	.28258200
		17	.06757823	.13058963	.18934284	.24412562	.29520627	.34283498	.38724502	.42865392	.46726447
		18	.11634660	.21915666	.31000513	.39028368	.46122210	.52390707	.57929887	.62824601	.67149832
		19	.18564902	.33683248	.45994888	.56020884	.64185563	.70834478	.76249029	.80658373	.84249127
		20	.27574216	.47545059	.62009098	.72484791	.80071895	.85566914	.89546724	.92429133	.94516740
		21	.38299591	.61930596	.76511022	.85507204	.91057886	.94482679	.96595790	.97899589	.98704038
		22	.50000000	.75000000	.87500000	.93750000	.96875000	.98437500	.99218750	.99609375	.99804687
		23	.61700409	.85331413	.94381991	.97848326	.99175917	.99684380	.99879119	.99953703	.99982268
		24	.72425784	.92396626	.97903429	.99421887	.99840590	.99956044	.99987879	.99996658	.99999078
		25	.81435098	.96553444	.99360150	.99881213	.99977947	.99995906	.99999240	.99999859	.99999974
		26	.88365340	.98646347	.99842507	.99981676	.99997868	.99999752	.99999971	.99999997	1.00000000
		27	.93242177	.99543318	.99969138	.99997914	.99999859	.99999990	.99999999	1.00000000	1.00000000
		28	.96377287	.99868760	.99995246	.99999828	.99999994	1.00000000	1.00000000	1.00000000	1.00000000
		29	.98215110	.99968142	.99999431	.99999990	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
		30	.99195282	.99993524	.99999948	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
		31	.99669559	.99998908	.99999996	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
		32	.99877055	.99999849	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
		33	.99958796	.99999983	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
		34	.99987646	.99999998	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
1	46	0	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000
		1	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000
		2	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000
		3	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000
		4	.00000000	.00000001	.00000001	.00000001	.00000001	.00000002	.00000002	.00000002	.00000002
		5	.00000002	.00000004	.00000007	.00000009	.00000011	.00000013	.00000015	.00000018	.00000020
		6	.00000016	.00000031	.00000047	.00000062	.00000078	.00000093	.00000109	.00000124	.00000140
		7	.00000092	.00000183	.00000275	.00000366	.00000458	.00000549	.00000641	.00000733	.00000824
		8	.00000462	.00000925	.00001387	.00001850	.00002312	.00002774	.00003237	.00003699	.00004161
		9	.00002028	.00004056	.00006084	.00008112	.00010140	.00012167	.00014195	.00016223	.00018251
		10	.00007821	.00015641	.00023461	.00031280	.00039098	.00046916	.00054733	.00062550	.00070366
		11	.00026779	.00053551	.00080316	.00107074	.00133825	.00160568	.00187304	.00214033	.00240755
		12	.00082075	.00164082	.00246022	.00327894	.00409700	.00491438	.00573109	.00654713	.00736251

Table 31: Sheet Q. Probabilities of perfect ($i = 0$) and i-imperfect single columns ($k = 1$) in Bernoulli ($m \times n$)-matrices. The rows with all 1's are omitted

k m i	Width n of Bernoulli $(m \times n)$-matrix								
	1	2	3	4	5	6	7	8	9
1 46 13	.00226693	.00452872	.00678539	.00903694	.01128338	.01352473	.01576100	.01799221	.02021835
14	.00567580	.01131938	.01693093	.02251063	.02805866	.03357520	.03906043	.04451452	.04993766
15	.01294804	.02572843	.03834334	.05079491	.06308525	.07521646	.08719060	.09900969	.11067575
16	.02703802	.05334498	.07894065	.10384427	.12807454	.15164968	.17458739	.19690490	.21861900
17	.05190268	.10111147	.14776619	.19199941	.23393680	.27369753	.31139457	.34713504	.38102048
18	.09196241	.17546774	.25129371	.32014655	.38266751	.43943889	.49098944	.53779928	.58030437
19	.15099781	.27919528	.38803521	.48044055	.55889289	.62549909	.68204791	.73005798	.77081863
20	.23069559	.40817073	.54470313	.64973811	.73054198	.79270476	.84052686	.87731661	.90561913
21	.32936904	.55025411	.69838648	.79772864	.86435056	.90902929	.93899222	.95908630	.97256200
22	.44149796	.68807547	.82578951	.90270309	.94565947	.96965071	.98304986	.99053331	.99471283
23	.55850204	.80507955	.91394302	.96200602	.98322574	.99259420	.99673035	.99855646	.99936268
24	.67063096	.89151604	.96426874	.98823123	.99612373	.99872328	.99957949	.99986150	.99995438
25	.76930441	.94677954	.98772228	.99716758	.99934657	.99984926	.99996522	.99999198	.99999815
26	.84900219	.97719966	.99655720	.99948014	.99992150	.99998815	.99999821	.99999973	.99999996
27	.90803759	.99154292	.99922227	.99992848	.99999342	.99999940	.99999994	.99999999	1.00000000
28	.94809732	.99730611	.99986018	.99999274	.99999962	.99999998	1.00000000	1.00000000	1.00000000
29	.97296198	.99926895	.99998023	.99999947	.99999999	1.00000000	1.00000000	1.00000000	1.00000000
30	.98705196	.99983235	.99999783	.99999997	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
31	.99432420	.99996779	.99999982	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
32	.99773307	.99999486	.99999999	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
33	.99917925	.99999933	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
34	.99973221	.99999993	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
35	.99992179	.99999999	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
1 47 0	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000
1	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000
2	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000
3	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000
4	.00000000	.00000000	.00000000	.00000001	.00000001	.00000001	.00000001	.00000001	.00000001
5	.00000001	.00000002	.00000004	.00000005	.00000006	.00000007	.00000009	.00000010	.00000011
6	.00000009	.00000018	.00000027	.00000035	.00000044	.00000053	.00000062	.00000071	.00000080
7	.00000054	.00000107	.00000161	.00000214	.00000268	.00000321	.00000375	.00000428	.00000482
8	.00000277	.00000554	.00000831	.00001108	.00001385	.00001662	.00001939	.00002216	.00002493
9	.00001245	.00002490	.00003736	.00004981	.00006226	.00007471	.00008716	.00009961	.00011206
10	.00004924	.00009849	.00014773	.00019696	.00024620	.00029543	.00034466	.00039389	.00044311
11	.00017300	.00034597	.00051891	.00069182	.00086470	.00103755	.00121038	.00138317	.00155593
12	.00054427	.00108824	.00163192	.00217530	.00271838	.00326117	.00380367	.00434587	.00488777
13	.00154384	.00308529	.00462437	.00616107	.00769539	.00922735	.01075694	.01228418	.01380905
14	.00397136	.00792695	.01186684	.01579107	.01969972	.02359285	.02747052	.03133279	.03517972
15	.00931192	.01853712	.02767643	.03673062	.04570051	.05458687	.06339048	.07211211	.08075253
16	.01999303	.03958634	.05878791	.07760559	.09604705	.11411981	.13183124	.14918856	.16619886
17	.03947035	.07738279	.11379881	.14877748	.18237552	.21464745	.24564558	.27542021	.30401963
18	.07193254	.13869080	.20064696	.25814646	.31150987	.36103472	.40699712	.44965332	.48924116
19	.12148011	.22820280	.32196081	.40432908	.47669125	.54026285	.59611177	.64517616	.68828020
20	.19084670	.34527094	.47022382	.57132985	.65314013	.71933719	.77290076	.81624190	.85131153
21	.28003231	.48164653	.62680225	.73130968	.80655165	.86072344	.89972538	.92780551	.94802230
22	.38543350	.62230801	.76788316	.85734876	.91233133	.94612177	.96688825	.97965062	.98749396
23	.50000000	.75000000	.87500000	.93750000	.96875000	.98437500	.99218750	.99609375	.99804688
24	.61456650	.85144102	.94274039	.97793023	.99149357	.99672134	.99873629	.99951293	.99981227
25	.71996769	.92158190	.97804040	.99385060	.99827797	.99951778	.99986496	.99996218	.99998941
26	.80915330	.96357754	.99304889	.99867340	.99974682	.99995168	.99999078	.99999824	.99999966
27	.87851989	.98524258	.99820727	.99978222	.99997354	.99999679	.99999961	.99999995	.99999999
28	.92806746	.99482571	.99962780	.99997323	.99999807	.99999986	.99999999	1.00000000	1.00000000
29	.96052965	.99844209	.99993851	.99999757	.99999990	1.00000000	1.00000000	1.00000000	1.00000000
30	.98000697	.99960028	.99999201	.99999984	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
31	.99068808	.99991329	.99999919	.99999999	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
32	.99602864	.99998423	.99999994	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000

Table 31: Sheet R. Probabilities of perfect ($i = 0$) and i-imperfect single columns ($k = 1$) in Bernoulli ($m \times n$)-matrices. The rows with all 1's are omitted

k m i	Width n of Bernoulli ($m \times n$)-matrix								
	1	2	3	4	5	6	7	8	9
1 47 33	.99845616	.99999762	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
34	.99945573	.99999970	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
35	.99982700	.99999997	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
1 48 0	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000
1	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000
2	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000
3	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000
4	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000001	.00000001	.00000001
5	.00000001	.00000001	.00000002	.00000003	.00000003	.00000004	.00000005	.00000005	.00000006
6	.00000005	.00000010	.00000015	.00000020	.00000025	.00000030	.00000035	.00000040	.00000045
7	.00000031	.00000062	.00000094	.00000125	.00000156	.00000187	.00000218	.00000250	.00000281
8	.00000165	.00000331	.00000496	.00000661	.00000826	.00000992	.00001157	.00001322	.00001487
9	.00000761	.00001522	.00002283	.00003044	.00003805	.00004566	.00005328	.00006089	.00006850
10	.00003085	.00006170	.00009254	.00012339	.00015423	.00018507	.00021592	.00024676	.00027760
11	.00011112	.00022223	.00033333	.00044442	.00055549	.00066655	.00077760	.00088863	.00099966
12	.00035863	.00071714	.00107552	.00143377	.00179189	.00214988	.00250774	.00286548	.00322309
13	.00104405	.00208702	.00312889	.00416968	.00520938	.00624799	.00728552	.00832197	.00935734
14	.00275760	.00550760	.00825001	.01098486	.01371217	.01643196	.01914425	.02184905	.02454640
15	.00664164	.01323917	.01979288	.02630306	.03277001	.03919400	.04557533	.05191428	.05821112
16	.01465247	.02909025	.04331648	.05733426	.07114665	.08475664	.09816722	.11138130	.12440177
17	.02973169	.05857940	.08656943	.11372726	.14007764	.16564458	.19045138	.21452063	.23787425
18	.05570145	.10830024	.15796921	.20487154	.24916134	.29098414	.33047735	.36777073	.40298681
19	.09670633	.18406054	.26296705	.33424280	.39862573	.45678243	.50931500	.55676735	.59963075
20	.15616340	.28793980	.39913754	.49297027	.57214976	.63896431	.69534487	.74292085	.78306721
21	.23543951	.41544725	.55307406	.65829809	.73874822	.80025721	.84728455	.88323980	.91072976
22	.33273291	.55475463	.70290241	.80175656	.86771867	.91173302	.94110235	.96069954	.97377609
23	.44271675	.68943538	.82692754	.90354962	.94624982	.97004592	.98330709	.99069732	.99481577
24	.55728325	.80400188	.91322835	.96158474	.98299292	.99247068	.99666664	.99852427	.99934667
25	.66726709	.88928881	.96316275	.98774303	.99592170	.99864302	.99954849	.99984977	.99995001
26	.76456049	.94456824	.98694917	.99692732	.99927657	.99982968	.99995990	.99999056	.99999778
27	.84383660	.97561299	.99619164	.99940527	.99990713	.99998550	.99999774	.99999965	.99999994
28	.90329367	.99064789	.99909559	.99991254	.99999154	.99999918	.99999992	.99999999	1.00000000
29	.94429855	.99689735	.99982718	.99999037	.99999946	.99999997	1.00000000	1.00000000	1.00000000
30	.97026831	.99911603	.99997372	.99999922	.99999998	1.00000000	1.00000000	1.00000000	1.00000000
31	.98534753	.99978531	.99999685	.99999995	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
32	.99335836	.99995589	.99999971	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
33	.99724240	.99999240	.99999998	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
34	.99895595	.99999891	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
35	.99964137	.99999987	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
36	.99988888	.99999999	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000

Table 31: Sheet S. Probabilities of perfect ($i = 0$) and i-imperfect single columns ($k = 1$) in Bernoulli ($m \times n$)-matrices. The rows with all 1's are omitted

k m i	Width n of Bernoulli ($m \times n$)-matrix								
	10	11	12	13	14	15	16	17	18
1 1 0	.99902344	.99951172	.99975586	.99987793	.99993896	.99996948	.99998474	.99999237	.99999619
1 2 0	.94368649	.95776486	.96832365	.97624274	.98218205	.98663654	.98997740	.99248305	.99436229
1	.99999905	.99999976	.99999994	.99999999	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
1 3 0	.73692442	.76980887	.79858276	.82375992	.84578993	.86506619	.88193291	.89669130	.90960489
1	.99902344	.99951172	.99975586	.99987793	.99993896	.99996948	.99998474	.99999237	.99999619
2	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
1 4 0	.47553952	.50831830	.53904841	.56785788	.59486677	.62018759	.64392587	.66618050	.68704422
1	.97641005	.98378191	.98885006	.99233442	.99472991	.99637681	.99750906	.99828748	.99882264
2	.99999112	.99999722	.99999913	.99999973	.99999992	.99999997	.99999999	1.00000000	1.00000000
3	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
1 5 0	.27202384	.29477310	.31681144	.33816108	.35884355	.37887969	.39828970	.41709314	.43530898
1	.87461843	.89812748	.91722857	.93274822	.94535793	.95560331	.96392769	.97069125	.97618664
2	.99902344	.99951172	.99975586	.99987793	.99993896	.99996948	.99998474	.99999237	.99999619
3	.99999995	.99999999	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
4	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
1 6 0	.14570915	.15905744	.17219717	.18513159	.19786391	.21039729	.22273483	.23487960	.24683460
1	.68598614	.72033141	.75092016	.77816327	.80242666	.82403624	.84328228	.86042328	.87568948
2	.98518529	.99027785	.99361984	.99581302	.99725229	.99819682	.99881666	.99922343	.99949038
3	.99997696	.99999208	.99999728	.99999906	.99999968	.99999989	.99999996	.99999999	1.00000000
4	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
5	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
1 7 0	.07543486	.08265803	.08982476	.09693551	.10399070	.11099077	.11793616	.12482728	.13166457
1	.47553952	.50831830	.53904841	.56785788	.59486677	.62018759	.64392587	.66618050	.68704422
2	.92339586	.94075148	.95417498	.96455721	.97258722	.97879792	.98360152	.98731680	.99019034
3	.99902344	.99951172	.99975586	.99987793	.99993896	.99996948	.99998474	.99999237	.99999619
4	.99999964	.99999992	.99999998	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
5	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
6	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
1 8 0	.03838296	.04213927	.04588092	.04960795	.05332042	.05701838	.06070190	.06437104	.06802584
1	.30085078	.32543024	.34914558	.37202719	.39410435	.41540537	.43595753	.45578715	.47491963
2	.79008596	.82042510	.84637928	.86858228	.88757624	.90382499	.91772528	.92961655	.93978916
3	.98904843	.99302693	.99556012	.99717304	.99820002	.99885392	.99927027	.99953537	.99970416
4	.99995997	.99998546	.99999472	.99999808	.99999930	.99999975	.99999991	.99999997	.99999999
5	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
6	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
7	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
1 9 0	.01936048	.02127579	.02318736	.02509520	.02699931	.02889970	.03079638	.03268936	.03457864
1	.17901058	.19504553	.21076729	.22618200	.24129563	.25611407	.27064310	.28488835	.29885537
2	.60991472	.64496145	.67685944	.70589160	.73231540	.75636519	.77825426	.79817673	.81630929
3	.94655168	.96012255	.97024768	.97780198	.98343820	.98764334	.99078077	.99312159	.99486806
4	.99902344	.99951172	.99975586	.99987793	.99993896	.99996948	.99998474	.99999237	.99999619
5	.99999889	.99999972	.99999993	.99999998	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
6	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
7	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
8	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
1 10 0	.00972282	.01068989	.01165601	.01262119	.01358543	.01454872	.01551108	.01647249	.01743297
1	.10237511	.11201756	.12155644	.13099284	.14032788	.14956264	.15869820	.16773562	.17667596
2	.43015859	.46132179	.49078076	.51862868	.54495368	.56983902	.59336345	.61560139	.63662319
3	.84830930	.87438114	.89597188	.91385171	.92865845	.94092028	.95107461	.95948366	.96644740
4	.99118531	.99450803	.99657825	.99786809	.99867172	.99917242	.99948438	.99967874	.99979984
5	.99994207	.99997816	.99999177	.99999690	.99999883	.99999956	.99999983	.99999994	.99999998
6	.99999998	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
7	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
8	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
9	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
1 11 0	.00487210	.00535800	.00584366	.00632909	.00681428	.00729924	.00778395	.00826844	.00875268

Table 31: Sheet T. Probabilities of perfect ($i = 0$) and i-imperfect single columns ($k = 1$) in Bernoulli ($m \times n$)-matrices. The rows with all 1's are omitted

k m i	Width n of Bernoulli ($m \times n$)-matrix								
	10	11	12	13	14	15	16	17	18
1 11 1	.05707269	.06259766	.06809025	.07355066	.07897907	.08437568	.08974066	.09507422	.10037651
2	.28295695	.30641490	.32910543	.35105364	.37228382	.39281945	.41268327	.43189725	.45048264
3	.69949001	.73353215	.76371797	.79048429	.81421849	.83526405	.85392555	.87047304	.88514602
4	.95955279	.97065207	.97870556	.98454905	.98878901	.99186546	.99409769	.99571737	.99689258
5	.99902344	.99951172	.99975586	.99987793	.99993896	.99996948	.99998474	.99999237	.99999619
6	.99999758	.99999934	.99999982	.99999995	.99999999	1.00000000	1.00000000	1.00000000	1.00000000
7	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
8	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
9	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
10	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
1 12 0	.00243873	.00268227	.00292576	.00316918	.00341255	.00365586	.00389911	.00414229	.00438542
1	.03128880	.03436333	.03742809	.04048313	.04352847	.04656415	.04959019	.05260663	.05561349
2	.17696399	.19283797	.20840580	.22367336	.23864646	.25333077	.26773186	.28185519	.29570613
3	.53139546	.56560268	.59731283	.62670821	.65395778	.67921819	.70263463	.72434172	.74446424
4	.88407657	.90654806	.92466350	.93926730	.95104019	.96053094	.96818192	.97434978	.97932202
5	.99253314	.99542436	.99719608	.99828178	.99894709	.99935478	.99960461	.99975771	.99985153
6	.99992424	.99997067	.99998864	.99999560	.99999830	.99999934	.99999974	.99999990	.99999996
7	.99999993	.99999999	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
8	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
9	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
10	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
11	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
1 13 0	.00122003	.00134195	.00146386	.00158575	.00170763	.00182949	.00195134	.00207317	.00219499
1	.01695901	.01863901	.02031614	.02199041	.02366181	.02533036	.02699605	.02865890	.03031891
2	.10679581	.11682691	.12674536	.13655242	.14624934	.15583736	.16531770	.17469157	.18396017
3	.37650262	.40527240	.43271466	.45889067	.48385885	.50767493	.53039208	.55206100	.57273006
4	.76117910	.79304327	.82065603	.84458461	.86532057	.88328989	.89886168	.91235585	.92404958
5	.96768854	.97707590	.98373598	.98846112	.99181348	.99419189	.99587930	.99707648	.99792584
6	.99902344	.99951172	.99975586	.99987793	.99993896	.99996948	.99998474	.99999237	.99999619
7	.99999572	.99999876	.99999964	.99999989	.99999997	.99999999	1.00000000	1.00000000	1.00000000
8	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
9	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
10	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
11	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
12	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
1 14 0	.00061018	.00067118	.00073218	.00079317	.00085415	.00091514	.00097612	.00103709	.00109806
1	.00911765	.01002483	.01093118	.01183670	.01274139	.01364525	.01454828	.01545049	.01635187
2	.06284582	.06890895	.07493285	.08091778	.08686399	.09277173	.09864125	.10447280	.11026661
3	.25252935	.27397168	.29479891	.31502868	.33467813	.35376390	.37230216	.39030863	.40779856
4	.60965305	.64469946	.67659931	.70563510	.73206398	.75612000	.77801621	.79794652	.81608743
5	.90765797	.92723218	.94265715	.95481240	.96439105	.97193926	.97788744	.98257475	.98626847
6	.99345875	.99604427	.99760783	.99855337	.99912517	.99947096	.99968007	.99980653	.99988300
7	.99990692	.99996321	.99998546	.99999425	.99999773	.99999910	.99999965	.99999986	.99999994
8	.99999982	.99999996	.99999999	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
9	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
10	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
11	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
12	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
13	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
1 15 0	.00030513	.00033564	.00036615	.00039666	.00042716	.00045767	.00048817	.00051867	.00054917
1	.00487210	.00535800	.00584366	.00632909	.00681428	.00729924	.00778395	.00826844	.00875268
2	.03631868	.03987719	.04342257	.04695485	.05047409	.05398034	.05747363	.06095403	.06442158
3	.16250882	.17723035	.19169309	.20590161	.21986038	.23357377	.24704610	.26028162	.27328448
4	.45698319	.48914858	.51940867	.54787632	.57465769	.59985268	.62355525	.64585382	.66683153
5	.80515185	.83455032	.85951319	.88070968	.89870808	.91399089	.92696785	.93798686	.94734334
6	.97317951	.98132273	.98699351	.99094253	.99369256	.99560762	.99694123	.99786993	.99851666

Table 31: Sheet U. Probabilities of perfect ($i = 0$) and i-imperfect single columns ($k = 1$) in Bernoulli ($m \times n$)-matrices. The rows with all 1's are omitted

k m i	Width n of Bernoulli ($m \times n$)-matrix								
	10	11	12	13	14	15	16	17	18
1 15 7	.99902344	.99951172	.99975586	.99987793	.99993896	.99996948	.99998474	.99999237	.99999619
8	.99999334	.99999798	.99999939	.99999981	.99999994	.99999998	.99999999	1.00000000	1.00000000
9	.99999999	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
10	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
11	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
12	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
13	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
1 16 0	.00015258	.00016783	.00018309	.00019835	.00021360	.00022886	.00024411	.00025937	.00027462
1	.00259097	.00284970	.00310836	.00336695	.00362547	.00388393	.00414233	.00440065	.00465891
2	.02070898	.02275615	.02479903	.02683764	.02887199	.03090209	.03292795	.03494957	.03696696
3	.10140546	.11096235	.12041760	.12977229	.13902749	.14818425	.15724364	.16620667	.17507437
4	.32404814	.35000900	.37497279	.39897782	.42206090	.44425745	.46560150	.48612581	.50586185
5	.67042451	.70504864	.73603528	.76376656	.78858448	.81079511	.83067236	.84846138	.86438153
6	.92407323	.94132752	.95466079	.96496409	.97292597	.97907852	.98383291	.98750687	.99034592
7	.99413330	.99649060	.99790071	.99874422	.99924881	.99955064	.99973120	.99983921	.99990381
8	.99989030	.99995592	.99998229	.99999288	.99999714	.99999885	.99999954	.99999981	.99999993
9	.99999963	.99999992	.99999998	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
10	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
11	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
12	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
13	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
14	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
1 17 0	.00007629	.00008392	.00009155	.00009918	.00010681	.00011443	.00012206	.00012969	.00013732
1	.00137244	.00150958	.00164671	.00178381	.00192089	.00205796	.00219500	.00233203	.00246904
2	.01168734	.01284854	.01400837	.01516684	.01632394	.01747969	.01863408	.01978711	.02093879
3	.06183782	.06780727	.07373873	.07963245	.08548867	.09130763	.09708956	.10283471	.10854329
4	.21984696	.23897700	.25763794	.27584131	.29359831	.31091990	.32781675	.34429927	.36037762
5	.52495383	.55902964	.59066113	.62002365	.64727995	.67258111	.69606738	.71786895	.73810665
6	.83749604	.86449655	.88701085	.90578433	.92143854	.93449176	.94537615	.95445206	.96201999
7	.97709722	.98430082	.98923867	.99262343	.99494358	.99653397	.99762414	.99837142	.99888365
8	.99902344	.99951172	.99975586	.99987793	.99993896	.99996948	.99998474	.99999237	.99999619
9	.99999052	.99999702	.99999906	.99999971	.99999991	.99999997	.99999999	1.00000000	1.00000000
10	.99999998	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
11	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
12	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
13	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
14	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
15	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
1 18 0	.00003815	.00004196	.00004578	.00004959	.00005340	.00005722	.00006103	.00006485	.00006866
1	.00072456	.00079698	.00086940	.00094182	.00101423	.00108664	.00115904	.00123143	.00130382
2	.00654194	.00719378	.00784518	.00849616	.00914672	.00979684	.01044654	.01109582	.01174467
3	.03705638	.04068564	.04430122	.04790317	.05149155	.05506640	.05862778	.06217574	.06571032
4	.14411874	.15733516	.17034751	.18315891	.19577249	.20819128	.22041831	.23245653	.24430885
5	.38934821	.41873657	.44671058	.47333831	.49868455	.52281096	.54577627	.56763634	.58844437
6	.71813334	.75165920	.78119742	.80722229	.83015171	.85035385	.86815310	.88383527	.89765216
7	.93599914	.95138118	.96306629	.97194298	.97868624	.98380881	.98770022	.99065636	.99290203
8	.99464687	.99682701	.99811926	.99888522	.99933923	.99960834	.99976785	.99986240	.99991844
9	.99987446	.99994887	.99997918	.99999152	.99999655	.99999859	.99999943	.99999977	.99999990
10	.99999936	.99999985	.99999996	.99999999	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
11	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
12	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
13	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
14	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
15	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
16	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
1 19 0	.00001907	.00002098	.00002289	.00002480	.00002670	.00002861	.00003052	.00003242	.00003433

Table 31: Sheet V. Probabilities of perfect ($i = 0$) and i-imperfect single columns ($k = 1$) in Bernoulli ($m \times n$)-matrices. The rows with all 1's are omitted

k m i	Width n of Bernoulli ($m \times n$)-matrix								
	10	11	12	13	14	15	16	17	18
1 19 1	.00038140	.00041954	.00045767	.00049580	.00053393	.00057205	.00061018	.00064830	.00068642
2	.00363707	.00400005	.00436289	.00472561	.00508819	.00545064	.00581296	.00617514	.00653720
3	.02190625	.02407031	.02622958	.02838407	.03053379	.03267876	.03481898	.03695447	.03908523
4	.09200678	.10072843	.10936630	.11792120	.12639393	.13478527	.14309601	.15132692	.15947877
5	.27602713	.29903792	.32131734	.34288863	.36377430	.38399613	.40357523	.42253203	.44088631
6	.58201333	.61692953	.64892903	.67825547	.70513216	.72976372	.75233770	.77302598	.79198609
7	.86195025	.88674974	.90709421	.92378397	.93747555	.94870755	.95792181	.96548081	.97168190
8	.98001368	.98648531	.99086141	.99382051	.99582145	.99717448	.99808939	.99870805	.99912639
9	.99902344	.99951172	.99975586	.99987793	.99993896	.99996948	.99998474	.99999237	.99999619
10	.99998733	.99999590	.99999867	.99999957	.99999986	.99999995	.99999999	1.00000000	1.00000000
11	.99999996	.99999999	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
12	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
13	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
14	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
15	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
16	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
1 20 0	.00000954	.00001049	.00001144	.00001240	.00001335	.00001431	.00001526	.00001621	.00001717
1	.00020025	.00022028	.00024030	.00026032	.00028034	.00030037	.00032039	.00034041	.00036043
2	.00201043	.00221125	.00241203	.00261277	.00281347	.00301413	.00321475	.00341533	.00361587
3	.01280970	.01408161	.01535188	.01662051	.01788751	.01915288	.02041662	.02167872	.02293921
4	.05754295	.06311190	.06864794	.07415126	.07962207	.08506056	.09046690	.09584130	.10118394
5	.18870107	.20549068	.22193284	.23803473	.25380340	.26924574	.28436850	.29917831	.31368162
6	.44782062	.47965881	.50966124	.53793376	.56457611	.58968228	.61334085	.63563529	.65664425
7	.75607391	.78817165	.81604572	.84025189	.86127282	.87952765	.89538037	.90914705	.92110221
8	.94496641	.95881960	.96918562	.97694229	.98274643	.98708954	.99033939	.99277118	.99459084
9	.99505119	.99708961	.99828840	.99899341	.99940803	.99965186	.99979526	.99987959	.99992919
10	.99985942	.99994209	.99997615	.99999018	.99999595	.99999833	.99999931	.99999972	.99999988
11	.99999898	.99999974	.99999994	.99999998	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
12	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
13	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
14	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
15	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
16	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
17	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
1 21 0	.00000477	.00000525	.00000572	.00000620	.00000668	.00000715	.00000763	.00000811	.00000858
1	.00010490	.00011539	.00012588	.00013637	.00014686	.00015734	.00016783	.00017832	.00018881
2	.00110571	.00121622	.00132671	.00143719	.00154765	.00165811	.00176855	.00187898	.00198940
3	.00742328	.00816257	.00890131	.00963950	.01037714	.01111423	.01185077	.01258677	.01332221
4	.03540968	.03888094	.04233971	.04578604	.04921996	.05264152	.05605077	.05944775	.06283251
5	.12533219	.13696689	.14844683	.15977406	.17095062	.18197851	.19285971	.20359617	.21418981
6	.32944536	.35571564	.38095673	.40520895	.42851104	.45090023	.47241228	.49308156	.51294107
7	.62992320	.66494118	.69664564	.72535012	.75133847	.77486771	.79617053	.81545760	.83291966
8	.88088493	.90371395	.92216767	.93708463	.94914269	.95888976	.96676875	.97313769	.97828599
9	.98225862	.98814542	.99207891	.99470722	.99646343	.99763691	.99842101	.99894494	.99929502
10	.99902344	.99951172	.99975586	.99987793	.99993896	.99996948	.99998474	.99999237	.99999619
11	.99998382	.99999463	.99999822	.99999941	.99999980	.99999993	.99999998	.99999999	1.00000000
12	.99999993	.99999999	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
13	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
14	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
15	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
16	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
17	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
18	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
1 22 0	.00000238	.00000262	.00000286	.00000310	.00000334	.00000358	.00000381	.00000405	.00000429
1	.00005483	.00006032	.00006580	.00007128	.00007677	.00008225	.00008773	.00009322	.00009870
2	.00060542	.00066594	.00072646	.00078697	.00084748	.00090799	.00096849	.00102899	.00108949

Table 31: Sheet W. Probabilities of perfect ($i = 0$) and i-imperfect single columns ($k = 1$) in Bernoulli ($m \times n$)-matrices. The rows with all 1's are omitted

k m i	Width n of Bernoulli ($m \times n$)-matrix								
	10	11	12	13	14	15	16	17	18
1 22 3	.00426901	.00469490	.00512062	.00554615	.00597150	.00639667	.00682166	.00724646	.00767109
4	.02150653	.02363158	.02575201	.02786784	.02997907	.03208572	.03418779	.03628530	.03837825
5	.08136073	.08912348	.09682063	.10445274	.11202036	.11952402	.12696428	.13434167	.14165672
6	.23348262	.25359558	.27318078	.29225207	.31082295	.32890654	.34651563	.36366266	.38035977
7	.49964215	.53311622	.56435086	.59349590	.62069113	.64606699	.66974520	.69183933	.71245535
8	.78664544	.81718488	.84335292	.86577528	.88498812	.90145085	.91555711	.92764421	.93800117
9	.95190151	.96449050	.97378453	.98064599	.98571159	.98945134	.99221228	.99425059	.99575540
10	.99537802	.99730033	.99842314	.99907896	.99946203	.99968577	.99981646	.99989280	.99993738
11	.99984514	.99993559	.99997321	.99998886	.99999537	.99999807	.99999920	.99999967	.99999986
12	.99999849	.99999961	.99999990	.99999997	.99999999	1.00000000	1.00000000	1.00000000	1.00000000
13	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
14	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
15	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
16	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
17	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
18	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
19	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
1 23 0	.00000119	.00000131	.00000143	.00000155	.00000167	.00000179	.00000191	.00000203	.00000215
1	.00002861	.00003147	.00003433	.00003719	.00004005	.00004291	.00004578	.00004864	.00005150
2	.00033016	.00036317	.00039618	.00042919	.00046219	.00049520	.00052820	.00056121	.00059421
3	.00243873	.00268227	.00292576	.00316918	.00341255	.00365586	.00389911	.00414229	.00438542
4	.01292163	.01420458	.01548585	.01676546	.01804341	.01931970	.02059433	.02186730	.02313862
5	.05185863	.05689422	.06190306	.06688531	.07184109	.07677055	.08167384	.08655108	.09140242
6	.16051793	.17507861	.18938673	.20344668	.21726277	.23083921	.24418018	.25728975	.27017193
7	.37928973	.40819610	.43575630	.46203303	.48708606	.51097237	.53374630	.55545965	.57616182
8	.67028839	.70491464	.73590445	.76363971	.78846223	.81067788	.83056045	.84835496	.86428070
9	.89585151	.91693496	.93375035	.94716169	.95785809	.96638915	.97319321	.97861988	.98294800
10	.98403390	.98944353	.99302027	.99538514	.99694875	.99798257	.99866612	.99911806	.99941688
11	.99902344	.99951172	.99975586	.99987793	.99993896	.99996948	.99998474	.99999237	.99999619
12	.99998006	.99999324	.99999771	.99999922	.99999974	.99999991	.99999997	.99999999	1.00000000
13	.99999988	.99999998	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
14	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
15	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
16	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
17	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
18	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
19	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
1 24 0	.00000060	.00000066	.00000072	.00000077	.00000083	.00000089	.00000095	.00000101	.00000107
1	.00001490	.00001639	.00001788	.00001937	.00002086	.00002235	.00002384	.00002533	.00002682
2	.00017940	.00019733	.00021527	.00023321	.00025114	.00026908	.00028702	.00030495	.00032289
3	.00138494	.00152333	.00166170	.00180005	.00193838	.00207670	.00221499	.00235326	.00249152
4	.00769264	.00845864	.00922405	.00998887	.01075310	.01151674	.01227979	.01304225	.01380412
5	.03256642	.03576415	.03895131	.04212794	.04529406	.04844972	.05159496	.05472979	.05785426
6	.10767576	.11778394	.12777762	.13765809	.14742663	.15708451	.16663299	.17607331	.18540669
7	.27732170	.30041657	.32277338	.34441574	.36536646	.38564765	.40528071	.42428635	.44268462
8	.54534287	.57980353	.61165224	.64108699	.66829073	.69343258	.71666880	.73814383	.75799117
9	.81159212	.84055571	.86506678	.88580981	.90336406	.91821972	.93079165	.94143092	.95043463
10	.95739208	.96892298	.97733330	.98346754	.98794169	.99120501	.99358518	.99532121	.99658742
11	.99564792	.99747323	.99853298	.99914826	.99950549	.99971289	.99983331	.99990322	.99994381
12	.99983158	.99992936	.99997037	.99998757	.99999479	.99999781	.99999908	.99999962	.99999984
13	.99999789	.99999943	.99999985	.99999996	.99999999	1.00000000	1.00000000	1.00000000	1.00000000
14	.99999999	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
15	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
16	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
17	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
18	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000

Table 31: Sheet X. Probabilities of perfect ($i = 0$) and i-imperfect single columns ($k = 1$) in Bernoulli ($m \times n$)-matrices. The rows with all 1's are omitted

k m i	Width n of Bernoulli ($m \times n$)-matrix								
	10	11	12	13	14	15	16	17	18
1 24 19	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
20	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
1 25 0	.00000030	.00000033	.00000036	.00000039	.00000042	.00000045	.00000048	.00000051	.00000054
1	.00000775	.00000852	.00000930	.00001007	.00001085	.00001162	.00001240	.00001317	.00001395
2	.00009715	.00010687	.00011658	.00012629	.00013601	.00014572	.00015544	.00016515	.00017487
3	.00078233	.00086053	.00093873	.00101691	.00109510	.00117327	.00125144	.00132960	.00140776
4	.00454329	.00499648	.00544946	.00590224	.00635482	.00680718	.00725935	.00771130	.00816305
5	.02020056	.02219804	.02419144	.02618078	.02816607	.03014730	.03212450	.03409767	.03606681
6	.07080389	.07760250	.08435135	.09105083	.09770130	.10430310	.11085660	.11736215	.12382010
7	.19651971	.21390913	.23092220	.24756706	.26385169	.27978387	.29537124	.31062125	.32554122
8	.42524830	.45621367	.48551074	.51322940	.53945469	.56426706	.58774264	.60995345	.63096762
9	.70446898	.73838455	.76840793	.79498577	.81851351	.83934117	.85777861	.87410015	.88854860
10	.90789559	.92743813	.94283417	.95496351	.96451927	.97204750	.97797841	.98265091	.98633201
11	.98546928	.99048265	.99376632	.99591706	.99732575	.99824842	.99885275	.99924857	.99950783
12	.99902344	.99951172	.99975586	.99987793	.99993896	.99996948	.99998474	.99999237	.99999619
13	.99997610	.99999175	.99999715	.99999902	.99999966	.99999988	.99999996	.99999999	1.00000000
14	.99999982	.99999996	.99999999	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
15	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
16	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
17	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
18	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
19	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
20	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
21	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
1 26 0	.00000015	.00000016	.00000018	.00000019	.00000021	.00000022	.00000024	.00000025	.00000027
1	.00000402	.00000443	.00000483	.00000523	.00000563	.00000603	.00000644	.00000684	.00000724
2	.00005245	.00005770	.00006294	.00006819	.00007343	.00007868	.00008392	.00008916	.00009441
3	.00043980	.00048376	.00052773	.00057170	.00061566	.00065962	.00070358	.00074754	.00079149
4	.00266441	.00293046	.00319643	.00346234	.00372818	.00399395	.00425964	.00452526	.00479082
5	.01239985	.01363135	.01486131	.01608974	.01731663	.01854200	.01976584	.02098815	.02220894
6	.04580410	.05026749	.05471001	.05913175	.06353281	.06791328	.07227325	.07661284	.08093212
7	.13571690	.14823141	.16056471	.17271942	.18469815	.19650342	.20813776	.21960364	.23090349
8	.31948610	.34518186	.36990736	.39369925	.41659277	.43862185	.45981912	.48021600	.49984271
9	.58557771	.62052129	.65251846	.68181768	.70864642	.73321300	.75570815	.77630654	.79516810
10	.83219060	.85962237	.88256987	.90176615	.91782442	.93125764	.94249494	.95189528	.95975895
11	.96182578	.97246107	.98013337	.98566819	.98966101	.99254144	.99461938	.99611841	.99719982
12	.99587477	.99761772	.99862425	.99920552	.99954120	.99973504	.99984699	.99991164	.99994897
13	.99981871	.99992340	.99996764	.99998633	.99999422	.99999756	.99999897	.99999956	.99999982
14	.99999718	.99999922	.99999978	.99999994	.99999998	1.00000000	1.00000000	1.00000000	1.00000000
15	.99999999	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
16	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
17	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
18	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
19	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
20	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
21	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
1 27 0	.00000007	.00000008	.00000009	.00000010	.00000010	.00000011	.00000012	.00000013	.00000013
1	.00000209	.00000229	.00000250	.00000271	.00000292	.00000313	.00000334	.00000355	.00000376
2	.00002824	.00003106	.00003388	.00003671	.00003953	.00004236	.00004518	.00004800	.00005083
3	.00024614	.00027075	.00029536	.00031997	.00034458	.00036919	.00039379	.00041840	.00044301
4	.00155266	.00170779	.00186290	.00201799	.00217305	.00232808	.00248310	.00263808	.00279305
5	.00754287	.00829402	.00904461	.00979462	.01054407	.01129295	.01204126	.01278900	.01353618
6	.02923128	.03210699	.03497419	.03783289	.04068312	.04352491	.04635829	.04918326	.05199987
7	.09176140	.10046109	.10907746	.11761129	.12606337	.13443450	.14272545	.15093698	.15906985
8	.23253826	.25258398	.27210610	.29111832	.30963395	.32766596	.34522699	.36232933	.37898496
9	.46730912	.49982407	.53035434	.55902107	.58593801	.61121196	.63494322	.65722594	.67814855

Table 31: Sheet Y. Probabilities of perfect ($i = 0$) and i-imperfect single columns ($k = 1$) in Bernoulli ($m \times n$)-matrices. The rows with all 1's are omitted

k m i	Width n of Bernoulli ($m \times n$)-matrix								
	10	11	12	13	14	15	16	17	18
1 27 10	.73358103	.76658881	.79550713	.82084262	.84303920	.86248574	.87952297	.89444939	.90752651
11	.91774088	.93592295	.95008617	.96111883	.96971290	.97640738	.98162216	.98568429	.98884855
12	.98665157	.99133091	.99436990	.99634355	.99762533	.99845778	.99899841	.99934952	.99957755
13	.99902344	.99951172	.99975586	.99987793	.99993896	.99996948	.99998474	.99999237	.99999619
14	.99997197	.99999018	.99999656	.99999879	.99999958	.99999985	.99999995	.99999998	.99999999
15	.99999972	.99999994	.99999999	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
16	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
17	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
18	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
19	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
20	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
21	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
22	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
1 28 0	.00000004	.00000004	.00000004	.00000005	.00000005	.00000006	.00000006	.00000006	.00000007
1	.00000108	.00000119	.00000130	.00000140	.00000151	.00000162	.00000173	.00000184	.00000194
2	.00001516	.00001668	.00001819	.00001971	.00002123	.00002274	.00002426	.00002577	.00002729
3	.00013719	.00015091	.00016463	.00017835	.00019207	.00020578	.00021950	.00023322	.00024694
4	.00089959	.00098951	.00107941	.00116931	.00125920	.00134908	.00143896	.00152882	.00161868
5	.00455182	.00500586	.00545970	.00591332	.00636674	.00681996	.00727296	.00772576	.00817835
6	.01844099	.02026628	.02208817	.02390668	.02572181	.02753356	.02934194	.03114696	.03294862
7	.06096467	.06685287	.07270415	.07851873	.08429686	.09003875	.09574464	.10141475	.10704931
8	.16481569	.17972295	.19436414	.20874399	.22286717	.23673827	.25036178	.26374212	.27688364
9	.35954329	.38745393	.41414825	.43967924	.46409762	.48745186	.50978833	.53115140	.55158349
10	.62101167	.65605546	.68785887	.71672152	.74291534	.76668710	.78826077	.80783959	.82560802
11	.84938516	.87536083	.89685663	.91464517	.92936582	.94154769	.95162862	.95997096	.96687453
12	.96546700	.97533632	.98238506	.98741930	.99101479	.99358271	.99541673	.99672660	.99766212
13	.99606825	.99774033	.99870132	.99925362	.99957104	.99975347	.99985831	.99991857	.99995320
14	.99980648	.99991770	.99996500	.99998512	.99999367	.99999731	.99999886	.99999951	.99999979
15	.99999636	.99999896	.99999970	.99999992	.99999998	.99999999	1.00000000	1.00000000	1.00000000
16	.99999998	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
17	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
18	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
19	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
20	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
21	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
22	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
23	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
1 29 0	.00000002	.00000002	.00000002	.00000002	.00000003	.00000003	.00000003	.00000003	.00000003
1	.00000056	.00000061	.00000067	.00000073	.00000078	.00000084	.00000089	.00000095	.00000101
2	.00000812	.00000893	.00000975	.00001056	.00001137	.00001218	.00001299	.00001381	.00001462
3	.00007618	.00008380	.00009141	.00009903	.00010665	.00011427	.00012188	.00012950	.00013712
4	.00051846	.00057029	.00062212	.00067394	.00072577	.00077759	.00082940	.00088122	.00093303
5	.00272721	.00299952	.00327176	.00354392	.00381601	.00408803	.00435997	.00463183	.00490362
6	.01151836	.01266287	.01380606	.01494792	.01608847	.01722769	.01836559	.01950218	.02063745
7	.03991470	.04381747	.04770438	.05157549	.05543086	.05927057	.06309466	.06690321	.07069627
8	.11425910	.12494094	.13549395	.14591970	.15621971	.16639551	.17644859	.18638043	.19619250
9	.26798728	.29047045	.31226306	.33338633	.35386082	.37370645	.39294254	.41158781	.42966040
10	.50563005	.53925856	.57059956	.59980866	.62703086	.65240133	.67604603	.69808234	.71861968
11	.75852743	.79051420	.81826383	.84233759	.86322242	.88134072	.89705897	.91069510	.92252492
12	.92590035	.94287874	.95596688	.96605616	.97383368	.97982915	.98445088	.98801363	.99076006
13	.98764082	.99203495	.99486681	.99669184	.99786801	.99862601	.99911451	.99942933	.99963223
14	.99902344	.99951172	.99975586	.99987793	.99993896	.99996948	.99998474	.99999237	.99999619
15	.99996773	.99998853	.99999592	.99999855	.99999948	.99999982	.99999993	.99999998	.99999999
16	.99999960	.99999991	.99999998	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
17	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
18	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000

Table 31: Sheet Z. Probabilities of perfect ($i = 0$) and i-imperfect single columns ($k = 1$) in Bernoulli ($m \times n$)-matrices. The rows with all 1's are omitted

k m i	Width n of Bernoulli ($m \times n$)-matrix								
	10	11	12	13	14	15	16	17	18
1 29 19	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
20	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
21	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
22	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
23	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
24	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
1 30 0	.00000001	.00000001	.00000001	.00000001	.00000001	.00000001	.00000001	.00000002	.00000002
1	.00000029	.00000032	.00000035	.00000038	.00000040	.00000043	.00000046	.00000049	.00000052
2	.00000434	.00000477	.00000521	.00000564	.00000608	.00000651	.00000694	.00000738	.00000781
3	.00004215	.00004637	.00005058	.00005480	.00005901	.00006323	.00006744	.00007166	.00007587
4	.00029734	.00032707	.00035680	.00038653	.00041625	.00044598	.00047570	.00050543	.00053515
5	.00162338	.00178558	.00194774	.00210989	.00227200	.00243409	.00259615	.00275818	.00292019
6	.00713154	.00784189	.00855173	.00926107	.00996990	.01067822	.01138603	.01209334	.01280014
7	.02580964	.02835368	.03089108	.03342185	.03594601	.03846358	.04097457	.04347901	.04597690
8	.07776091	.08519638	.09257189	.09988794	.10714500	.11434356	.12148408	.12856702	.13559286
9	.19441768	.21164664	.22850714	.24500703	.26115405	.27695572	.29241945	.30755246	.32236181
10	.39727157	.42702742	.45531426	.48220461	.50776743	.53206825	.55516938	.57713003	.59800652
11	.65226653	.68712480	.71848872	.74670860	.77209960	.79494529	.81550084	.83399581	.85063677
12	.86388257	.88849224	.90865254	.92516791	.93869735	.94978071	.95886022	.96629818	.97239138
13	.96850124	.97770935	.98422563	.98883699	.99210030	.99440964	.99604388	.99720038	.99801880
14	.99623536	.99784575	.99876727	.99929459	.99959634	.99976901	.99986782	.99992436	.99995672
15	.99979485	.99991224	.99996246	.99998394	.99999313	.99999706	.99999874	.99999946	.99999977
16	.99999544	.99999867	.99999961	.99999989	.99999997	.99999999	1.00000000	1.00000000	1.00000000
17	.99999996	.99999999	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
18	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
19	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
20	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
21	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
22	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
23	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
24	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
1 31 0	.00000000	.00000001	.00000001	.00000001	.00000001	.00000001	.00000001	.00000001	.00000001
1	.00000015	.00000016	.00000018	.00000019	.00000021	.00000022	.00000024	.00000025	.00000027
2	.00000231	.00000255	.00000278	.00000301	.00000324	.00000347	.00000370	.00000393	.00000417
3	.00002325	.00002557	.00002789	.00003022	.00003254	.00003487	.00003719	.00003952	.00004184
4	.00016975	.00018673	.00020370	.00022067	.00023765	.00025462	.00027159	.00028856	.00030553
5	.00096056	.00105657	.00115256	.00124855	.00134453	.00144049	.00153645	.00163240	.00172834
6	.00438089	.00481792	.00525476	.00569141	.00612787	.00656413	.00700021	.00743609	.00787178
7	.01651050	.01814648	.01977974	.02141028	.02303812	.02466324	.02628566	.02790538	.02952241
8	.05210555	.05716439	.06219623	.06720121	.07217948	.07713119	.08205647	.08695546	.09182830
9	.13786354	.15055823	.16306599	.17538958	.18753171	.19949505	.21128224	.22289586	.23433847
10	.30245433	.32713194	.35093651	.37389894	.39604900	.41741544	.43802598	.45790737	.47708540
11	.54045691	.57483368	.60663883	.63606476	.66328945	.68847755	.71178142	.73334201	.75328973
12	.78003483	.81094450	.83751072	.86034384	.87996843	.89683536	.91133213	.92379181	.93450064
13	.93274546	.94865552	.96080181	.97007473	.97715399	.98255856	.98668459	.98983455	.99223933
14	.98847980	.99262765	.99528206	.99698076	.99806784	.99876351	.99920871	.99949361	.99967594
15	.99902344	.99951172	.99975586	.99987793	.99993896	.99996948	.99998474	.99999237	.99999619
16	.99996339	.99998682	.99999525	.99999829	.99999938	.99999978	.99999992	.99999997	.99999999
17	.99999945	.99999987	.99999997	.99999999	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
18	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
19	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
20	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
21	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
22	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
23	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
24	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000

Table 31: Sheet Z1. Probabilities of perfect ($i = 0$) and i-imperfect single columns ($k = 1$) in Bernoulli ($m \times n$)-matrices. The rows with all 1's are omitted

k m i	Width n of Bernoulli ($m \times n$)-matrix								
	10	11	12	13	14	15	16	17	18
1 31 25	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
1 32 0	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000
1	.00000008	.00000008	.00000009	.00000010	.00000011	.00000012	.00000012	.00000013	.00000014
2	.00000123	.00000135	.00000148	.00000160	.00000172	.00000185	.00000197	.00000209	.00000222
3	.00001278	.00001406	.00001534	.00001661	.00001789	.00001917	.00002045	.00002173	.00002300
4	.00009650	.00010615	.00011580	.00012545	.00013510	.00014475	.00015440	.00016405	.00017370
5	.00056523	.00062173	.00067823	.00073473	.00079123	.00084772	.00090421	.00096070	.00101718
6	.00267205	.00293886	.00320560	.00347227	.00373886	.00400539	.00427184	.00453823	.00480454
7	.01046242	.01150262	.01254173	.01357975	.01461668	.01565251	.01668726	.01772092	.01875349
8	.03445564	.03783522	.04120298	.04455894	.04790316	.05123567	.05455652	.05786575	.06116339
9	.09589928	.10496814	.11394603	.12283386	.13163254	.14034296	.14896601	.15750257	.16595350
10	.22407819	.24351599	.26246684	.28094296	.29895622	.31651823	.33364029	.35033342	.36660836
11	.43259275	.46385240	.49338989	.52130009	.54767267	.57259232	.59613910	.61838864	.63941240
12	.67989960	.71436276	.74511548	.77255726	.79704456	.81889547	.83839383	.85579293	.87131878
13	.87621837	.89955650	.91849440	.93386169	.94633159	.95645038	.96466135	.97132420	.97673081
14	.97106199	.97969441	.98575172	.99000209	.99298454	.99507730	.99654578	.99757620	.99829924
15	.99638126	.99793741	.99882437	.99932992	.99961807	.99978231	.99987592	.99992928	.99995969
16	.99978376	.99990701	.99996001	.99998280	.99999261	.99999682	.99999863	.99999941	.99999975
17	.99999442	.99999834	.99999950	.99999985	.99999996	.99999999	1.00000000	1.00000000	1.00000000
18	.99999994	.99999999	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
19	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
20	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
21	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
22	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
23	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
24	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
25	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
26	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
1 33 0	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000
1	.00000004	.00000004	.00000005	.00000005	.00000006	.00000006	.00000006	.00000007	.00000007
2	.00000065	.00000072	.00000079	.00000085	.00000092	.00000098	.00000105	.00000111	.00000118
3	.00000701	.00000771	.00000841	.00000911	.00000981	.00001051	.00001121	.00001191	.00001261
4	.00005464	.00006011	.00006557	.00007103	.00007650	.00008196	.00008743	.00009289	.00009835
5	.00033089	.00036397	.00039705	.00043013	.00046321	.00049629	.00052937	.00056245	.00059552
6	.00161914	.00178091	.00194265	.00210437	.00226606	.00242772	.00258936	.00275097	.00291256
7	.00657411	.00722913	.00788373	.00853790	.00919163	.00984493	.01049781	.01115025	.01180226
8	.02252528	.02474972	.02696909	.02918340	.03139268	.03359694	.03579617	.03799040	.04017964
9	.06563192	.07195338	.07823208	.08446829	.09066232	.09681443	.10292493	.10899409	.11502218
10	.16219243	.17688843	.19132664	.20551159	.21944773	.23313941	.24659092	.25980648	.27279023
11	.33566344	.36228451	.38783882	.41236914	.43591648	.45852024	.48021823	.50104675	.52104063
12	.57207345	.60689719	.63888704	.66827363	.69526880	.72006716	.74284749	.76377400	.78299756
13	.79868787	.82850284	.85390211	.87553967	.89397264	.90967563	.92305296	.93444906	.94415736
14	.93855038	.95350876	.96482590	.97338816	.97986615	.98476723	.98847527	.99128068	.99340318
15	.98919970	.99313280	.99563361	.99722370	.99823473	.99887758	.99928633	.99954622	.99971147
16	.99902344	.99951172	.99975586	.99987793	.99993896	.99996948	.99998474	.99999237	.99999619
17	.99995898	.99998506	.99999456	.99999802	.99999928	.99999974	.99999990	.99999997	.99999999
18	.99999927	.99999982	.99999996	.99999999	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
19	.99999999	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
20	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
21	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
22	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
23	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
24	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
25	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
26	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
1 34 0	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000

Table 31: Sheet Z2. Probabilities of perfect ($i = 0$) and i-imperfect single columns ($k = 1$) in Bernoulli ($m \times n$)-matrices. The rows with all 1's are omitted

k m i	Width n of Bernoulli ($m \times n$)-matrix								
	10	11	12	13	14	15	16	17	18
1 34 1	.00000002	.00000002	.00000002	.00000003	.00000003	.00000003	.00000003	.00000003	.00000004
2	.00000035	.00000038	.00000042	.00000045	.00000049	.00000052	.00000056	.00000059	.00000062
3	.00000383	.00000421	.00000460	.00000498	.00000536	.00000575	.00000613	.00000651	.00000689
4	.00003082	.00003391	.00003699	.00004007	.00004315	.00004624	.00004932	.00005240	.00005548
5	.00019277	.00021205	.00023132	.00025060	.00026987	.00028915	.00030842	.00032769	.00034697
6	.00097520	.00107267	.00117013	.00126757	.00136501	.00146244	.00155986	.00165727	.00175467
7	.00409939	.00450841	.00491725	.00532593	.00573444	.00614279	.00655096	.00695897	.00736681
8	.01457874	.01602488	.01746889	.01891078	.02035055	.02178822	.02322377	.02465722	.02608856
9	.04429732	.04861766	.05291847	.05719984	.06146186	.06570461	.06992817	.07413265	.07831812
10	.11509687	.12585132	.13647507	.14696971	.15733681	.16757791	.17769455	.18768823	.19756047
11	.25345086	.27495620	.29584206	.31612627	.33582616	.35495858	.37353986	.39158588	.40911206
12	.46552324	.49797921	.52846429	.55709818	.58399327	.60925518	.63298305	.65527006	.67620370
13	.70439898	.73831639	.76834210	.79492265	.81845332	.83928408	.85772471	.87404945	.88850108
14	.88680317	.90896305	.92678482	.94111773	.95264477	.96191522	.96937086	.97536695	.98018921
15	.97324733	.98137468	.98703297	.99097230	.99371487	.99562427	.99695360	.99787908	.99852341
16	.99650982	.99801787	.99887431	.99936070	.99963693	.99979381	.99988290	.99993350	.99996223
17	.99977318	.99990200	.99995765	.99998170	.99999209	.99999658	.99999852	.99999936	.99999972
18	.99999330	.99999797	.99999938	.99999981	.99999994	.99999998	.99999999	1.00000000	1.00000000
19	.99999992	.99999998	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
20	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
21	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
22	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
23	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
24	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
25	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
26	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
27	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
1 35 0	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000
1	.00000001	.00000001	.00000001	.00000001	.00000001	.00000002	.00000002	.00000002	.00000002
2	.00000018	.00000020	.00000022	.00000024	.00000026	.00000028	.00000029	.00000031	.00000033
3	.00000209	.00000230	.00000251	.00000272	.00000292	.00000313	.00000334	.00000355	.00000376
4	.00001733	.00001906	.00002079	.00002253	.00002426	.00002599	.00002772	.00002946	.00003119
5	.00011180	.00012298	.00013416	.00014534	.00015652	.00016770	.00017888	.00019006	.00020123
6	.00058406	.00064244	.00070083	.00075921	.00081758	.00087596	.00093433	.00099269	.00105105
7	.00253840	.00279188	.00304530	.00329866	.00355195	.00380518	.00405834	.00431144	.00456448
8	.00935154	.01028187	.01121133	.01213991	.01306762	.01399446	.01492043	.01584554	.01676977
9	.02954041	.03244602	.03534294	.03823118	.04111077	.04398175	.04684412	.04969793	.05254319
10	.08031008	.08797746	.09558092	.10312100	.11059821	.11801308	.12536614	.13265790	.13988886
11	.18691868	.20357042	.21988113	.23585781	.25150728	.26683626	.28185131	.29655884	.31096517
12	.36744278	.39575955	.42280870	.44864698	.47332859	.49690532	.51942663	.54093975	.56148983
13	.60077159	.63579695	.66774944	.69689865	.72349053	.74774943	.76988003	.79006906	.80848685
14	.81495812	.84368630	.86795437	.88845476	.90577242	.92040147	.93275933	.94319860	.95201715
15	.94352102	.95762833	.96821192	.97615195	.98210871	.98657760	.98993024	.99244547	.99433244
16	.98982376	.99356800	.99593459	.99743041	.99837587	.99897345	.99935116	.99958989	.99974079
17	.99902344	.99951172	.99975586	.99987793	.99993896	.99996948	.99998474	.99999237	.99999619
18	.99995453	.99998327	.99999384	.99999773	.99999917	.99999969	.99999989	.99999996	.99999998
19	.99999905	.99999976	.99999994	.99999999	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
20	.99999999	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
21	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
22	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
23	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
24	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
25	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
26	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
27	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
28	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000

Table 31: Sheet Z3. Probabilities of perfect ($i = 0$) and i-imperfect single columns ($k = 1$) in Bernoulli ($m \times n$)-matrices. The rows with all 1's are omitted

k m i	Width n of Bernoulli ($m \times n$)-matrix								
	10	11	12	13	14	15	16	17	18
1 36 0	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000
1	.00000001	.00000001	.00000001	.00000001	.00000001	.00000001	.00000001	.00000001	.00000001
2	.00000010	.00000011	.00000012	.00000013	.00000014	.00000015	.00000016	.00000017	.00000017
3	.00000114	.00000125	.00000136	.00000148	.00000159	.00000170	.00000182	.00000193	.00000204
4	.00000971	.00001068	.00001165	.00001262	.00001359	.00001456	.00001553	.00001650	.00001747
5	.00006457	.00007102	.00007748	.00008393	.00009039	.00009685	.00010330	.00010976	.00011621
6	.00034795	.00038274	.00041753	.00045232	.00048710	.00052189	.00055667	.00059145	.00062623
7	.00156166	.00171769	.00187370	.00202968	.00218564	.00234157	.00249748	.00265337	.00280923
8	.00595022	.00654329	.00713601	.00772838	.00832039	.00891204	.00950335	.01009430	.01068490
9	.01949274	.02142099	.02334545	.02526613	.02718303	.02909615	.03100552	.03291113	.03481300
10	.05523213	.06058470	.06590695	.07119905	.07646116	.08169347	.08689612	.09206931	.09721318
11	.13509161	.14755352	.15983587	.17194126	.18387223	.19563129	.20722093	.21864357	.22990164
12	.28227336	.30568752	.32833784	.35024925	.37144585	.39195097	.41178715	.43097622	.44953930
13	.49613938	.52951968	.56068857	.58979255	.61696842	.64234391	.66603830	.68816295	.70882187
14	.72618650	.75945278	.78867746	.81435156	.83690645	.85672109	.87412840	.88942085	.90285539
15	.89595550	.91702619	.93382973	.94723027	.95791699	.96643948	.97323603	.97865616	.98297863
16	.97513076	.98281187	.98812059	.99178966	.99432550	.99607813	.99728943	.99812662	.99870523
17	.99662404	.99808911	.99891838	.99938777	.99965346	.99980385	.99988897	.99993715	.99996443
18	.99976308	.99989719	.99995538	.99998064	.99999160	.99999635	.99999842	.99999931	.99999970
19	.99999210	.99999756	.99999925	.99999977	.99999993	.99999998	.99999999	1.00000000	1.00000000
20	.99999988	.99999998	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
21	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
22	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
23	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
24	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
25	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
26	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
27	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
28	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
1 37 0	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000
1	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000
2	.00000005	.00000006	.00000006	.00000007	.00000007	.00000008	.00000008	.00000009	.00000009
3	.00000062	.00000068	.00000074	.00000080	.00000086	.00000092	.00000099	.00000105	.00000111
4	.00000542	.00000596	.00000651	.00000705	.00000759	.00000813	.00000868	.00000922	.00000976
5	.00003714	.00004085	.00004456	.00004828	.00005199	.00005571	.00005942	.00006313	.00006685
6	.00020627	.00022689	.00024752	.00026814	.00028876	.00030939	.00033001	.00035063	.00037125
7	.00095497	.00105042	.00114586	.00124129	.00133670	.00143212	.00152752	.00162291	.00171829
8	.00375811	.00413315	.00450804	.00488279	.00525740	.00563187	.00600620	.00638039	.00675443
9	.01274238	.01400765	.01527130	.01653334	.01779375	.01905255	.02030974	.02156531	.02281928
10	.03751172	.04118461	.04484349	.04848841	.05211941	.05573656	.05933991	.06292950	.06650540
11	.09595519	.10502902	.11401178	.12290438	.13170772	.14042271	.14905022	.15759114	.16604634
12	.21176906	.23030471	.24840449	.26607865	.28333719	.30018988	.31664627	.33271569	.34840722
13	.39769815	.42747347	.45577682	.48268097	.50825509	.53256493	.55567299	.57763868	.59851848
14	.62683461	.66186394	.69360503	.72236657	.74842823	.77204347	.79344193	.81283170	.83040133
15	.82922751	.85689339	.88007728	.89950527	.91578585	.92942891	.94086173	.95044238	.95847093
16	.94781438	.96115761	.97108914	.97848130	.98398337	.98807863	.99112678	.99339555	.99508423
17	.99036964	.99394650	.99619486	.99760814	.99849651	.99905493	.99940594	.99962658	.99976528
18	.99902344	.99951172	.99975586	.99987793	.99993896	.99996948	.99998474	.99999237	.99999619
19	.99995004	.99998145	.99999311	.99999744	.99999905	.99999965	.99999987	.99999995	.99999998
20	.99999881	.99999969	.99999992	.99999998	.99999999	1.00000000	1.00000000	1.00000000	1.00000000
21	.99999999	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
22	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
23	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
24	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
25	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
26	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000

Table 31: Sheet Z4. Probabilities of perfect ($i = 0$) and i-imperfect single columns ($k = 1$) in Bernoulli ($m \times n$)-matrices. The rows with all 1's are omitted

k m i	Width n of Bernoulli ($m \times n$)-matrix								
	10	11	12	13	14	15	16	17	18
1 37 27	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
28	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
29	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
1 38 0	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000
1	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000
2	.00000003	.00000003	.00000003	.00000004	.00000004	.00000004	.00000004	.00000005	.00000005
3	.00000033	.00000037	.00000040	.00000043	.00000047	.00000050	.00000053	.00000057	.00000060
4	.00000302	.00000332	.00000362	.00000393	.00000423	.00000453	.00000483	.00000513	.00000543
5	.00002128	.00002341	.00002554	.00002766	.00002979	.00003192	.00003405	.00003618	.00003830
6	.00012171	.00013388	.00014605	.00015822	.00017038	.00018255	.00019472	.00020689	.00021906
7	.00058068	.00063873	.00069678	.00075482	.00081286	.00087090	.00092893	.00098696	.00104499
8	.00235743	.00259287	.00282825	.00306357	.00329884	.00353406	.00376922	.00400432	.00423937
9	.00825940	.00908158	.00990308	.01072390	.01154403	.01236349	.01318226	.01400036	.01481778
10	.02519785	.02768245	.03016071	.03263266	.03509831	.03755767	.04001076	.04245761	.04489821
11	.06714530	.07360670	.08002335	.08639555	.09272361	.09900785	.10524855	.11144603	.11760059
12	.15564761	.16981265	.18374005	.19743381	.21089784	.22413599	.23715205	.24994976	.26253277
13	.31035317	.33550851	.35974630	.38310000	.40560186	.42728294	.44817319	.46830146	.48769553
14	.52455328	.55862062	.59024692	.61960709	.64686351	.67216692	.69565725	.71746442	.73770903
15	.74562413	.77816835	.80654895	.83129860	.85288184	.87170378	.88811768	.90243163	.91491429
16	.90392514	.92399000	.93986439	.95242348	.96235965	.97022069	.97643998	.98136040	.98525321
17	.97676830	.98405263	.98905295	.99248541	.99484162	.99645904	.99756931	.99833146	.99885463
18	.99672626	.99815265	.99895756	.99941176	.99966806	.99981269	.99989430	.99994035	.99996634
19	.99975343	.99989257	.99995319	.99997960	.99999111	.99999613	.99999831	.99999926	.99999968
20	.99999081	.99999712	.99999910	.99999972	.99999991	.99999997	.99999999	1.00000000	1.00000000
21	.99999984	.99999997	.99999999	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
22	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
23	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
24	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
25	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
26	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
27	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
28	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
29	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
30	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
1 39 0	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000
1	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000
2	.00000001	.00000002	.00000002	.00000002	.00000002	.00000002	.00000002	.00000002	.00000003
3	.00000018	.00000020	.00000022	.00000023	.00000025	.00000027	.00000029	.00000031	.00000032
4	.00000168	.00000184	.00000201	.00000218	.00000235	.00000251	.00000268	.00000285	.00000302
5	.00001215	.00001336	.00001458	.00001579	.00001701	.00001822	.00001944	.00002065	.00002187
6	.00007149	.00007864	.00008579	.00009294	.00010009	.00010724	.00011439	.00012154	.00012869
7	.00035122	.00038633	.00042145	.00045656	.00049167	.00052678	.00056189	.00059700	.00063210
8	.00146941	.00161623	.00176304	.00190981	.00205657	.00220331	.00235002	.00249672	.00264339
9	.00531236	.00584204	.00637144	.00690055	.00742939	.00795794	.00848621	.00901420	.00954191
10	.01676146	.01842207	.02007988	.02173489	.02338711	.02503653	.02668317	.02832703	.02996811
11	.04637914	.05089706	.05539358	.05986880	.06432281	.06875572	.07316764	.07755864	.08192885
12	.11238872	.12290812	.13330284	.14357437	.15372417	.16375369	.17366433	.18345753	.19313466
13	.23652010	.25684848	.27663561	.29589587	.31464332	.33289160	.35065400	.36794346	.38477257
14	.42639201	.45740384	.48673904	.51448824	.54073720	.56556701	.58905442	.61127199	.63228838
15	.65052822	.68540390	.71679916	.74506131	.77050303	.79340578	.81402294	.83258260	.84929009
16	.84180690	.86844532	.89059805	.90902045	.92434066	.93708107	.94767611	.95648702	.96381425
17	.95155192	.96420650	.97355571	.98046293	.98556598	.98933613	.99212152	.99417937	.99569971
18	.99085097	.99427846	.99642192	.99776237	.99860065	.99912489	.99945273	.99965775	.99978597
19	.99902344	.99951172	.99975586	.99987793	.99993896	.99996948	.99998474	.99999237	.99999619
20	.99994555	.99997960	.99999236	.99999714	.99999893	.99999960	.99999985	.99999994	.99999998
21	.99999852	.99999961	.99999990	.99999997	.99999999	1.00000000	1.00000000	1.00000000	1.00000000

Table 31: Sheet Z5. Probabilities of perfect ($i = 0$) and i-imperfect single columns ($k = 1$) in Bernoulli ($m \times n$)-matrices. The rows with all 1's are omitted

k m i	Width n of Bernoulli ($m \times n$)-matrix								
	10	11	12	13	14	15	16	17	18
1 39 22	.99999998	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
23	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
24	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
25	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
26	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
27	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
28	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
29	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
30	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
1 40 0	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000
1	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000
2	.00000001	.00000001	.00000001	.00000001	.00000001	.00000001	.00000001	.00000001	.00000001
3	.00000010	.00000011	.00000012	.00000013	.00000014	.00000015	.00000016	.00000017	.00000018
4	.00000093	.00000102	.00000111	.00000121	.00000130	.00000139	.00000149	.00000158	.00000167
5	.00000691	.00000760	.00000830	.00000899	.00000968	.00001037	.00001106	.00001175	.00001244
6	.00004182	.00004600	.00005019	.00005437	.00005855	.00006273	.00006691	.00007110	.00007528
7	.00021137	.00023250	.00025363	.00027477	.00029590	.00031703	.00033816	.00035929	.00038042
8	.00091046	.00100146	.00109245	.00118343	.00127441	.00136537	.00145633	.00154728	.00163822
9	.00339255	.00373117	.00406968	.00440807	.00474635	.00508451	.00542255	.00576049	.00609830
10	.01105182	.01215026	.01324748	.01434348	.01543827	.01653184	.01762419	.01871533	.01980526
11	.03167220	.03478372	.03788524	.04097679	.04405841	.04713012	.05019197	.05324398	.05628618
12	.07991657	.08754820	.09511653	.10262209	.11006539	.11744696	.12476730	.13202691	.13922632
13	.17655725	.19239918	.20793633	.22317457	.23811965	.25277720	.26715276	.28125176	.29507951
14	.33755436	.36428088	.38992912	.41454257	.43816299	.46083044	.48258336	.50345865	.52349173
15	.55089644	.58544597	.61733760	.64677580	.67394933	.69903240	.72218583	.74355807	.76328614
16	.76302127	.79479861	.82231481	.84614126	.86677273	.88463766	.90010702	.91350203	.92510085
17	.91090996	.93004608	.94507185	.95687015	.96613424	.97340844	.97912018	.98360507	.98712662
18	.97820327	.98513276	.98985926	.99308314	.99528211	.99678199	.99780504	.99850285	.99897882
19	.99681832	.99820972	.99899263	.99943317	.99968105	.99982053	.99989902	.99994318	.99996803
20	.99974418	.99988812	.99995108	.99997860	.99999064	.99999591	.99999821	.99999922	.99999966
21	.99998945	.99999665	.99999893	.99999966	.99999989	.99999997	.99999999	1.00000000	1.00000000
22	.99999979	.99999996	.99999999	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
23	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
24	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
25	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
26	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
27	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
28	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
29	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
30	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
31	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
1 41 0	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000
1	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000
2	.00000000	.00000000	.00000000	.00000001	.00000001	.00000001	.00000001	.00000001	.00000001
3	.00000005	.00000006	.00000006	.00000007	.00000007	.00000008	.00000008	.00000009	.00000009
4	.00000051	.00000056	.00000062	.00000067	.00000072	.00000077	.00000082	.00000087	.00000092
5	.00000392	.00000431	.00000470	.00000510	.00000549	.00000588	.00000627	.00000667	.00000706
6	.00002437	.00002680	.00002924	.00003168	.00003411	.00003655	.00003899	.00004142	.00004386
7	.00012660	.00013926	.00015191	.00016457	.00017723	.00018989	.00020255	.00021521	.00022786
8	.00056097	.00061704	.00067312	.00072919	.00078526	.00084133	.00089739	.00095345	.00100951
9	.00215220	.00236716	.00258208	.00279695	.00301178	.00322656	.00344129	.00365598	.00387062
10	.00722883	.00794883	.00866832	.00938727	.01010571	.01082363	.01154102	.01225790	.01297425
11	.02141089	.02352660	.02563774	.02774431	.02984633	.03194381	.03403675	.03612516	.03820906
12	.05607175	.06150301	.06690303	.07227197	.07761002	.08291735	.08819415	.09344059	.09865683
13	.12944310	.14142769	.15324729	.16490418	.17640059	.18773874	.19892080	.20994892	.22082522
14	.26099264	.28300911	.30436966	.32509384	.34520060	.36470835	.38363492	.40199764	.41981329

Table 31: Sheet Z6. Probabilities of perfect ($i = 0$) and i-imperfect single columns ($k = 1$) in Bernoulli ($m \times n$)-matrices. The rows with all 1's are omitted

k m i	Width n of Bernoulli ($m \times n$)-matrix								
	10	11	12	13	14	15	16	17	18
1 41 15	.45352731	.48557116	.51573603	.54413211	.57086311	.59602667	.61971470	.64201372	.66300517
16	.67209642	.70669411	.73764134	.76532328	.79008444	.81223301	.83204464	.84976591	.86561738
17	.85295066	.87860260	.89977970	.91726258	.93169566	.94361097	.95344773	.96156851	.96827267
18	.95482878	.96686034	.97568724	.98216306	.98691401	.99039952	.99295666	.99483268	.99620902
19	.99127841	.99457180	.99662157	.99789731	.99869132	.99918549	.99949306	.99968449	.99980363
20	.99902344	.99951172	.99975586	.99987793	.99993896	.99996948	.99998474	.99999237	.99999619
21	.99994105	.99997774	.99999159	.99999683	.99999880	.99999955	.99999983	.99999994	.99999998
22	.99999820	.99999952	.99999987	.99999997	.99999999	1.00000000	1.00000000	1.00000000	1.00000000
23	.99999997	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
24	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
25	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
26	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
27	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
28	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
29	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
30	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
31	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
32	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
1 42 0	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000
1	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000
2	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000
3	.00000003	.00000003	.00000003	.00000004	.00000004	.00000004	.00000005	.00000005	.00000005
4	.00000028	.00000031	.00000034	.00000037	.00000040	.00000042	.00000045	.00000048	.00000051
5	.00000222	.00000244	.00000266	.00000288	.00000310	.00000333	.00000355	.00000377	.00000399
6	.00001414	.00001556	.00001697	.00001839	.00001980	.00002122	.00002263	.00002405	.00002546
7	.00007548	.00008303	.00009058	.00009813	.00010568	.00011322	.00012077	.00012832	.00013587
8	.00034380	.00037818	.00041255	.00044692	.00048129	.00051566	.00055003	.00058439	.00061876
9	.00135687	.00149245	.00162802	.00176357	.00189910	.00203461	.00217010	.00230558	.00244103
10	.00469343	.00516156	.00562947	.00609716	.00656463	.00703187	.00749890	.00796571	.00843230
11	.01434282	.01576574	.01718660	.01860541	.02002218	.02143690	.02284957	.02426021	.02566881
12	.03888193	.04268599	.04647499	.05024899	.05400806	.05775225	.06148161	.06519622	.06889613
13	.09342525	.10227364	.11103566	.11971216	.12830398	.13681194	.14523686	.15357955	.16184081
14	.19764113	.21511591	.23221011	.24893200	.26528971	.28129116	.29694410	.31225614	.32723469
15	.36378613	.39191618	.41880247	.44449999	.46906129	.49253663	.51497401	.53641932	.55691644
16	.57530867	.61016508	.64216067	.67153022	.69848928	.72323568	.74595103	.76680203	.78594168
17	.77864211	.80962737	.83627537	.85919324	.87890310	.89585402	.91043218	.92296971	.93375227
18	.91706824	.93534636	.94959599	.96070501	.96936561	.97611741	.98138112	.98548472	.98868388
19	.97946965	.98608010	.99056209	.99360095	.99566135	.99705833	.99800550	.99864770	.99908312
20	.99690172	.99826127	.99902423	.99945241	.99969269	.99982754	.99990322	.99994569	.99996952
21	.99973531	.99988385	.99994903	.99997764	.99999019	.99999569	.99999811	.99999917	.99999964
22	.99998802	.99999614	.99999876	.99999960	.99999987	.99999996	.99999999	1.00000000	1.00000000
23	.99999973	.99999994	.99999999	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
24	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
25	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
26	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
27	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
28	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
29	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
30	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
31	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
32	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
1 43 0	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000
1	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000
2	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000
3	.00000002	.00000002	.00000002	.00000002	.00000002	.00000002	.00000002	.00000003	.00000003
4	.00000016	.00000017	.00000019	.00000020	.00000022	.00000023	.00000025	.00000026	.00000028

Table 31: Sheet Z7. Probabilities of perfect ($i = 0$) and i-imperfect single columns ($k = 1$) in Bernoulli ($m \times n$)-matrices. The rows with all 1's are omitted

k m i	Width n of Bernoulli ($m \times n$)-matrix								
	10	11	12	13	14	15	16	17	18
1 43 5	.00000125	.00000137	.00000150	.00000162	.00000175	.00000187	.00000200	.00000212	.00000225
6	.00000818	.00000900	.00000982	.00001063	.00001145	.00001227	.00001309	.00001391	.00001473
7	.00004481	.00004930	.00005378	.00005826	.00006274	.00006722	.00007170	.00007618	.00008066
8	.00020965	.00023061	.00025158	.00027254	.00029350	.00031446	.00033542	.00035638	.00037734
9	.00085045	.00093546	.00102045	.00110544	.00119043	.00127540	.00136037	.00144534	.00153029
10	.00302640	.00332854	.00363058	.00393254	.00423440	.00453617	.00483785	.00513943	.00544093
11	.00952870	.01047656	.01142351	.01236956	.01331471	.01425894	.01520228	.01614471	.01708624
12	.02668198	.02931070	.03193232	.03454687	.03715435	.03975479	.04234821	.04493462	.04751405
13	.06651207	.07291495	.07927391	.08558925	.09186128	.09809029	.10427657	.11042042	.11652213
14	.14696452	.16041668	.17365670	.18668792	.19951365	.21213712	.22456152	.23678999	.24882562
15	.28504594	.30863729	.33145020	.35351035	.37484258	.39547091	.41541856	.43470801	.45336096
16	.47913520	.51202507	.54283812	.57170549	.59875003	.62408686	.64782379	.67006187	.69089572
17	.69176029	.72598304	.75640616	.78345151	.80749411	.82886735	.84786759	.86475831	.87977371
18	.86286815	.88757778	.90783501	.92444211	.93805680	.94921827	.95836857	.96587009	.97201992
19	.95772036	.96918626	.97754270	.98363294	.98807156	.99130645	.99366407	.99538233	.99663460
20	.99166044	.99483277	.99679836	.99801625	.99877086	.99923842	.99952812	.99970762	.99981884
21	.99902344	.99951172	.99975586	.99987793	.99993896	.99996948	.99998474	.99999237	.99999619
22	.99993656	.99997587	.99999082	.99999651	.99999867	.99999949	.99999981	.99999993	.99999997
23	.99999785	.99999942	.99999984	.99999996	.99999999	1.00000000	1.00000000	1.00000000	1.00000000
24	.99999996	.99999999	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
25	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
26	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
27	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
28	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
29	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
30	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
31	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
32	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
33	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
1 44 0	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000
1	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000
2	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000
3	.00000001	.00000001	.00000001	.00000001	.00000001	.00000001	.00000001	.00000001	.00000001
4	.00000009	.00000009	.00000010	.00000011	.00000012	.00000013	.00000014	.00000014	.00000015
5	.00000070	.00000077	.00000084	.00000091	.00000098	.00000105	.00000112	.00000119	.00000126
6	.00000472	.00000519	.00000566	.00000613	.00000660	.00000707	.00000754	.00000802	.00000849
7	.00002650	.00002915	.00003180	.00003445	.00003710	.00003975	.00004240	.00004505	.00004770
8	.00012724	.00013996	.00015268	.00016540	.00017813	.00019085	.00020357	.00021629	.00022901
9	.00053010	.00058309	.00063608	.00068907	.00074206	.00079504	.00084802	.00090100	.00095397
10	.00193896	.00213265	.00232630	.00251992	.00271349	.00290703	.00310053	.00329400	.00348742
11	.00628234	.00690840	.00753406	.00815933	.00878421	.00940869	.01003278	.01065648	.01127978
12	.01813905	.01993476	.02172718	.02351633	.02530221	.02708481	.02886416	.03064026	.03241310
13	.04678424	.05134055	.05587509	.06038795	.06487924	.06934906	.07379751	.07822470	.08263073
14	.10755390	.11765141	.12763467	.13750498	.14726361	.15691183	.16645088	.17588201	.18520643
15	.21874666	.23779626	.25638136	.27451330	.29220312	.30946160	.32629927	.34272637	.35875292
16	.38899307	.41836471	.44632443	.47294010	.49827633	.52239463	.54535354	.56720879	.58801344
17	.59793090	.63294532	.66491048	.69409195	.72073212	.74505233	.76725459	.78752336	.80602701
18	.79271223	.82289491	.84868276	.87071570	.88954048	.90562423	.91936606	.93110698	.94113833
19	.92252772	.94001247	.95355108	.96403416	.97215130	.97843649	.98330317	.98707148	.98998932
20	.98059442	.98691664	.99117912	.99405291	.99599043	.99729673	.99817744	.99877122	.99917155
21	.99697765	.99830808	.99905286	.99946979	.99970319	.99983384	.99990699	.99994793	.99997085
22	.99972680	.99987974	.99994706	.99997670	.99998974	.99999548	.99999801	.99999912	.99999961
23	.99998653	.99999561	.99999857	.99999953	.99999985	.99999995	.99999998	.99999999	1.00000000
24	.99999966	.99999992	.99999998	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
25	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
26	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000

Table 31: Sheet Z8. Probabilities of perfect ($i = 0$) and i-imperfect single columns ($k = 1$) in Bernoulli ($m \times n$)-matrices. The rows with all 1's are omitted

k m i	Width n of Bernoulli ($m \times n$)-matrix								
	10	11	12	13	14	15	16	17	18
1 44 27	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
28	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
29	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
30	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
31	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
32	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
33	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
34	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
1 45 0	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000
1	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000
2	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000
3	.00000000	.00000000	.00000001	.00000001	.00000001	.00000001	.00000001	.00000001	.00000001
4	.00000005	.00000005	.00000006	.00000006	.00000007	.00000007	.00000007	.00000008	.00000008
5	.00000039	.00000043	.00000047	.00000051	.00000055	.00000059	.00000063	.00000067	.00000071
6	.00000271	.00000298	.00000325	.00000352	.00000379	.00000406	.00000433	.00000461	.00000488
7	.00001561	.00001717	.00001873	.00002029	.00002185	.00002341	.00002497	.00002653	.00002809
8	.00007687	.00008455	.00009224	.00009993	.00010761	.00011530	.00012299	.00013067	.00013836
9	.00032868	.00036155	.00039441	.00042727	.00046013	.00049299	.00052584	.00055870	.00059155
10	.00123475	.00135814	.00148152	.00160488	.00172823	.00185156	.00197487	.00209817	.00222146
11	.00411278	.00452313	.00493330	.00534331	.00575315	.00616282	.00657232	.00698166	.00739082
12	.01222670	.01344112	.01465405	.01586548	.01707543	.01828388	.01949085	.02069634	.02190035
13	.03255706	.03575389	.03894015	.04211589	.04528113	.04843592	.05158028	.05471425	.05783786
14	.07761939	.08504195	.09240479	.09970837	.10695318	.11413969	.12126836	.12833968	.13535408
15	.16481427	.17972142	.19436249	.20874223	.22286532	.23673632	.25035973	.26373999	.27688142
16	.30857199	.33362044	.35776146	.38102792	.40345150	.42506274	.44589107	.46596485	.48531141
17	.50326579	.53683421	.56813413	.59731886	.62453134	.64990484	.67356365	.69562364	.71619286
18	.70971837	.74349165	.77333553	.79970717	.82301056	.84360267	.86179897	.87787819	.89208665
19	.87173261	.89554533	.91493723	.93072905	.94358914	.95406176	.96259015	.96953525	.97519100
20	.96028706	.97123759	.97916860	.98491270	.98907290	.99208596	.99426820	.99584870	.99699339
21	.99200386	.99506635	.99695592	.99812179	.99884114	.99928498	.99955883	.99972779	.99983205
22	.99902344	.99951172	.99975586	.99987793	.99993896	.99996948	.99998474	.99999237	.99999619
23	.99993209	.99997399	.99999004	.99999618	.99999854	.99999944	.99999979	.99999992	.99999997
24	.99999746	.99999930	.99999981	.99999995	.99999999	1.00000000	1.00000000	1.00000000	1.00000000
25	.99999995	.99999999	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
26	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
27	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
28	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
29	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
30	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
31	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
32	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
33	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
34	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
1 46 0	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000
1	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000
2	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000
3	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000
4	.00000003	.00000003	.00000003	.00000003	.00000004	.00000004	.00000004	.00000004	.00000005
5	.00000022	.00000024	.00000026	.00000029	.00000031	.00000033	.00000035	.00000037	.00000040
6	.00000155	.00000171	.00000186	.00000202	.00000217	.00000233	.00000248	.00000264	.00000279
7	.00000916	.00001007	.00001099	.00001190	.00001282	.00001374	.00001465	.00001557	.00001648
8	.00004624	.00005086	.00005548	.00006011	.00006473	.00006936	.00007398	.00007860	.00008323
9	.00020278	.00022306	.00024334	.00026361	.00028389	.00030416	.00032443	.00034471	.00036498
10	.00078181	.00085996	.00093810	.00101623	.00109436	.00117249	.00125060	.00132871	.00140682
11	.00267470	.00294178	.00320878	.00347571	.00374258	.00400937	.00427609	.00454273	.00480931
12	.00817721	.00899124	.00980461	.01061731	.01142934	.01224071	.01305140	.01386144	.01467081

Table 31: Sheet Z9. Probabilities of perfect ($i = 0$) and i-imperfect single columns ($k = 1$) in Bernoulli ($m \times n$)-matrices. The rows with all 1's are omitted

k m i	Width n of Bernoulli ($m \times n$)-matrix								
	10	11	12	13	14	15	16	17	18
1 46 13	.02243945	.02465551	.02686655	.02907257	.03127360	.03346963	.03566069	.03784678	.04002792
14	.05533002	.06069178	.06602310	.07132416	.07659513	.08183619	.08704750	.09222923	.09738156
15	.12219076	.13355667	.14477541	.15584890	.16677900	.17756758	.18821647	.19872747	.20910238
16	.23974599	.26030175	.28030173	.29976094	.31869402	.33711518	.35503827	.37247675	.38944374
17	.41314717	.44360641	.47248473	.49986418	.52582257	.55043365	.57376734	.59588996	.61686435
18	.61890060	.65394742	.68577125	.71466848	.74090825	.76473496	.78637050	.80601638	.82385558
19	.80542451	.83480499	.85974907	.88092665	.89890647	.91417137	.92713130	.93813432	.94747590
20	.92739238	.94414264	.95702868	.96694198	.97456832	.98043529	.98494879	.98842103	.99109225
21	.98159923	.98765987	.99172433	.99445008	.99627805	.99750395	.99832607	.99887741	.99924716
22	.99704711	.99835080	.99907892	.99948558	.99971269	.99983954	.99991038	.99994995	.99997205
23	.99971862	.99987577	.99994515	.99997579	.99998931	.99999528	.99999792	.99999908	.99999959
24	.99998497	.99999505	.99999837	.99999946	.99999982	.99999994	.99999998	.99999999	1.00000000
25	.99999957	.99999990	.99999998	.99999999	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
26	.99999999	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
27	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
28	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
29	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
30	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
31	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
32	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
33	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
34	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
35	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
1 47 0	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000
1	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000
2	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000
3	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000
4	.00000001	.00000002	.00000002	.00000002	.00000002	.00000002	.00000002	.00000002	.00000003
5	.00000012	.00000014	.00000015	.00000016	.00000017	.00000018	.00000020	.00000021	.00000022
6	.00000089	.00000097	.00000106	.00000115	.00000124	.00000133	.00000142	.00000151	.00000159
7	.00000535	.00000589	.00000643	.00000696	.00000750	.00000803	.00000857	.00000910	.00000964
8	.00002770	.00003047	.00003324	.00003601	.00003878	.00004155	.00004432	.00004709	.00004986
9	.00012451	.00013696	.00014941	.00016186	.00017431	.00018676	.00019921	.00021166	.00022411
10	.00049233	.00054155	.00059077	.00063999	.00068920	.00073841	.00078762	.00083682	.00088603
11	.00172866	.00190136	.00207403	.00224667	.00241929	.00259187	.00276442	.00293694	.00310944
12	.00542938	.00597069	.00651171	.00705244	.00759287	.00813301	.00867285	.00921240	.00975165
13	.01533157	.01685174	.01836956	.01988504	.02139818	.02290898	.02441745	.02592359	.02742741
14	.03901137	.04282780	.04662908	.05041526	.05418641	.05794258	.06168383	.06541023	.06912182
15	.08931248	.09779273	.10619401	.11451706	.12276261	.13093137	.13902406	.14704140	.15498408
16	.18286907	.19920599	.21521629	.23090649	.24628300	.26135208	.27611989	.29059245	.30477565
17	.33149022	.35787653	.38322136	.40756583	.43094941	.45341004	.47498413	.49570669	.51561132
18	.52598134	.56007871	.59172337	.62109174	.64834758	.67364283	.69711853	.71890557	.73912541
19	.72614795	.75941553	.78864176	.81431758	.83687430	.85669083	.87410004	.88939438	.90283076
20	.87968823	.90264934	.92122839	.93626169	.94842594	.95826868	.96623296	.97267729	.97789174
21	.96257774	.97305718	.98060204	.98603410	.98994500	.99276073	.99478796	.99624750	.99729832
22	.99231420	.99527657	.99709714	.99821600	.99890361	.99932620	.99958590	.99974551	.99984360
23	.99902344	.99951172	.99975586	.99987793	.99993896	.99996948	.99998474	.99999237	.99999619
24	.99992764	.99997211	.99998925	.99999586	.99999840	.99999938	.99999976	.99999991	.99999996
25	.99999703	.99999917	.99999977	.99999993	.99999998	.99999999	1.00000000	1.00000000	1.00000000
26	.99999994	.99999999	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
27	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
28	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
29	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
30	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
31	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
32	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000

Table 31: Sheet Z10. Probabilities of perfect ($i = 0$) and i-imperfect single columns ($k = 1$) in Bernoulli ($m \times n$)-matrices. The rows with all 1's are omitted

k m i	Width n of Bernoulli ($m \times n$)-matrix								
	10	11	12	13	14	15	16	17	18
1 47 33	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
34	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
35	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
1 48 0	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000
1	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000
2	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000
3	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000
4	.00000001	.00000001	.00000001	.00000001	.00000001	.00000001	.00000001	.00000001	.00000001
5	.00000007	.00000008	.00000008	.00000009	.00000010	.00000010	.00000011	.00000012	.00000012
6	.00000050	.00000055	.00000061	.00000066	.00000071	.00000076	.00000081	.00000086	.00000091
7	.00000312	.00000343	.00000374	.00000406	.00000437	.00000468	.00000499	.00000530	.00000562
8	.00001653	.00001818	.00001983	.00002148	.00002314	.00002479	.00002644	.00002809	.00002975
9	.00007611	.00008372	.00009133	.00009894	.00010655	.00011416	.00012177	.00012938	.00013699
10	.00030844	.00033928	.00037012	.00040095	.00043179	.00046262	.00049346	.00052429	.00055512
11	.00111067	.00122167	.00133266	.00144363	.00155459	.00166554	.00177648	.00188740	.00199832
12	.00358057	.00393792	.00429514	.00465223	.00500920	.00536604	.00572275	.00607933	.00643579
13	.01039162	.01142483	.01245695	.01348800	.01451797	.01554687	.01657469	.01760144	.01862711
14	.02723632	.02991881	.03259391	.03526163	.03792199	.04057502	.04322073	.04585914	.04849028
15	.06446615	.07067963	.07685184	.08298306	.08907355	.09512360	.10113346	.10710341	.11303371
16	.13723145	.14987314	.16232960	.17460354	.18669764	.19861453	.21035681	.22192704	.23332773
17	.26053354	.28251912	.30385104	.32454872	.34463103	.36411626	.38302215	.40136595	.41916435
18	.43624131	.46764348	.49729651	.52529782	.55173942	.57670818	.60028615	.62255079	.64357525
19	.63834899	.67332293	.70491467	.73345129	.75922823	.78251239	.80354481	.82254327	.83970446
20	.81694417	.84553079	.86965323	.89000862	.90718525	.92167952	.93391031	.94423110	.95294016
21	.93174751	.94781684	.96010282	.96949619	.97667799	.98216891	.98636706	.98957679	.99203083
22	.98250165	.98832393	.99220894	.99480128	.99653107	.99768530	.99845547	.99896939	.99931231
23	.99711092	.99838996	.99910275	.99949998	.99972135	.99984471	.99991346	.99995177	.99997312
24	.99971076	.99987195	.99994331	.99997490	.99998889	.99999508	.99999782	.99999904	.99999957
25	.99998337	.99999447	.99999816	.99999939	.99999980	.99999993	.99999998	.99999999	1.00000000
26	.99999948	.99999988	.99999997	.99999999	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
27	.99999999	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
28	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
29	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
30	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
31	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
32	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
33	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
34	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
35	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
36	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000

Table 32: Sheet A. Exact (in roman), approximate *(in italics)* and interpolated **(boldfaced)** probabilities of perfect ($i = 0$) and i-imperfect column pairs ($k = 2$) in Bernoulli ($m \times n$)-matrices. The rows with all 1's are omitted

k	m	i	Width n of Bernoulli ($m \times n$)-matrix								
			2	3	4	5	6	7	8	9	10
2	1	0	.75000000	.87500000	.93750000	.96875000	.98437500	.99218750	.99609375	.99804688	.99902344
2	2	0	.56250000	.76562500	.87890625	.93847656	.96899414	.98443604	.99220276	.99609756	.99804783
		1	.93750000	.98437500	.99609375	.99902344	.99975586	.99993896	.99998474	.99999619	.99999905
2	3	0	.42187500	.65820313	.80932617	.89724731	.94584274	.97187853	.98554820	.99262745	.99625873
		1	.84375000	.95703125	.98876953	.99713135	.99927521	.99981785	.99995434	.99998857	.99999714
		2	.98437500	.99804688	.99975586	.99996948	.99999619	.99999952	.99999994	.99999999	1.00000000
2	4	0	.31640625	.55395508	.72706604	.83994007	.90894467	.94937097	.97234511	.98510658	.99207073
		1	.73828125	.91528320	.97444153	.99255085	.99786454	.99939163	.99982666	.99995041	.99998572
		2	.94921875	.99291992	.99906921	.99988079	.99998492	.99999810	.99999976	.99999997	1.00000000
		3	.99609375	.99975586	.99998474	.99999905	.99999994	1.00000000	1.00000000	1.00000000	1.00000000
2	5	0	.23730469	.45706177	.63682079	.76744083	.85591626	.91303641	.94862327	.97018599	.98296031
		1	.63281250	.85913086	.94991684	.98295021	.99435489	.99816639	.99941260	.99981375	.99994140
		2	.89648438	.98211670	.99709511	.99953550	.99992519	.99998772	.99999794	.99999964	.99999994
		3	.98437500	.99890137	.99992752	.99999535	.99999971	.99999998	1.00000000	1.00000000	1.00000000
		4	.99902344	.99996948	.99999905	.99999997	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
2	6	0	.17797852	.37074661	.54485947	.68415703	.78781240	.86117292	.91116768	.94423140	.96556449
		1	.53393555	.79124832	.91349691	.96588820	.98702473	.99520175	.99826584	.99938509	.99978544
		2	.83056641	.96331024	.99255145	.99853246	.99971567	.99994552	.99998965	.99999804	.99999963
		3	.96240234	.99657440	.99970043	.99997344	.99999755	.99999976	.99999998	1.00000000	1.00000000
		4	.99536133	.99983597	.99999458	.99999983	.99999999	1.00000000	1.00000000	1.00000000	1.00000000
		5	.99975586	.99999619	.99999994	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
2	7	0	.13348389	.29650545	.45663340	.59582070	.70829581	.79469930	.85858020	.90438661	.93641152
		1	.44494629	.71559715	.86513076	.93932233	.97377519	.98902647	.99552985	.99821987	.99930471
		2	.75640869	.93506479	.98387891	.99617276	.99912165	.99980385	.99995717	.99999082	.99999806
		3	.92944336	.99155807	.99903189	.99989043	.99998768	.99999862	.99999985	.99999998	1.00000000
		4	.98712158	.99938536	.99997105	.99999857	.99999992	1.00000000	1.00000000	1.00000000	1.00000000
		5	.99865723	.99997616	.99999961	.99999999	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
		6	.99993896	.99999952	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
2	8	0	.10011292	.23439580	.37592543	.50792317	.62241199	.71685073	.79185026	**.8422**	**.8830**
		1	.36708069	.63633424	.80619312	.90205796	.95250528	**.9146**	**.9400**	**.9575**	**.9700**
		2	.67854309	.89700443	.96945792	.99141809	**.9806**	**.9898**	**.9947**	**.9972**	**.9986**
		3	.88618469	.98249370	.99745115	.99964252	**.9979**	**.9992**	**.9997**	**.9999**	**1.0000**
		4	.97270203	.99821919	.99988566	.99999264	**.9998**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
		5	.99577332	.99989480	.99999733	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
		6	.99961853	.99999660	.99999997	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
2	9	0	.07508469	.18355144	.30481599	.42482712	.53536575	.63227361	.71413493	**.7713**	**.8198**
		1	.30033875	.55717021	.73908883	.85402114	.92157455	**.8839**	**.9136**	**.9344**	**.9508**
		2	.60067749	.84973699	.94788072	.98294969	**.9679**	**.9816**	**.9895**	**.9937**	**.9963**
		3	.83427429	.96809000	.99425538	.99901746	**.9954**	**.9980**	**.9991**	**.9996**	**.9998**
		4	.95107269	.99574344	.99964228	.99997082	**.9995**	**.9999**	**1.0000**	**1.0000**	**1.0000**
		5	.99000549	.99964780	.99998744	**.9999**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
		6	.99865723	.99998263	.99999976	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
		7	.99989319	.99999952	1.00000000	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
2	10	0	.05631351	.14263227	.24400537	.34944906	.45161901	.54594319	.62993565	**.6922**	**.7459**
		1	.24402523	.48106438	.66678645	.79628643	.88020177	**.8458**	**.8804**	**.9048**	**.9244**
		2	.52559280	.79465447	.91820709	.96932543	**.9504**	**.9692**	**.9808**	**.9873**	**.9918**
		3	.77587509	.94731776	.98853032	.99764544	**.9912**	**.9957**	**.9979**	**.9989**	**.9994**
		4	.92187309	.99119191	.99905727	.99990368	**.9988**	**.9996**	**.9998**	**.9999**	**1.0000**
		5	.98027229	.99904287	.99995415	**.9996**	**.9999**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
		6	.99649429	.99993373	.99999870	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
		7	.99958420	.99999721	.99999998	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
2	11	0	.04223514	.11014066	.19323001	.28332781	.37442440	.46227659	.54408316	**.6104**	**.6668**
		1	.19709730	.41013887	.59236999	.73087584	.82867681	**.8034**	**.8424**	**.8705**	**.8937**
		2	.45520091	.73367123	.88013667	.94920239	**.9282**	**.9521**	**.9681**	**.9777**	**.9844**
		3	.71330452	.91956059	.97922915	.99496368	**.9845**	**.9916**	**.9954**	**.9973**	**.9984**
		4	.88537359	.98369060	.99782664	.99972614	**.9975**	**.9989**	**.9995**	**.9998**	**.9999**

Table 32: Sheet B. Exact (in roman), approximate *(in italics)* and interpolated **(boldfaced)** probabilities of perfect ($i = 0$) and i-imperfect column pairs ($k = 2$) in Bernoulli ($m \times n$)-matrices. The rows with all 1's are omitted

k	m	i	Width n of Bernoulli $(m \times n)$-matrix								
			2	3	4	5	6	7	8	9	10
2	11	5	.96567249	.99777547	.99986067	**.9991**	**.9997**	**.9999**	**1.0000**	**1.0000**	**1.0000**
		6	.99243879	.99979758	.99999453	**.9999**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
		7	.99881172	.99998802	.99999987	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
		8	.99987388	.99999956	1.00000000	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
2	12	0	.03167635	.08461506	.15164311	.22689331	.30573822	.38461535	.46088803	**.5303**	**.5872**
		1	.15838176	.34572525	.51867774	.66041064	.76833804	**.7337**	**.8009**	**.8326**	**.8591**
		2	.39067501	.66895089	.83406358	.92157039	**.9010**	**.9136**	**.9513**	**.9643**	**.9739**
		3	.64877862	.88469569	.96529537	.99021164	**.9748**	**.9790**	**.9913**	**.9945**	**.9965**
		4	.84235632	.97235313	.99550483	.99931305	**.9950**	**.9961**	**.9988**	**.9994**	**.9997**
		5	.94559777	.99541883	.99963330	**.9980**	**.9992**	**.9995**	**.9999**	**.9999**	**1.0000**
		6	.98574722	.99947645	.99998105	**.9997**	**.9999**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
		7	.99721849	.99995926	.99999938	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
		8	.99960834	.99999790	.99999999	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
2	13	0	.02375726	.06473252	.11811019	.17979540	.24638047	.31507086	.38363189	**.4547**	**.5104**
		1	.12670541	.28848225	.44806359	.58771947	.70134658	**.6870**	**.7320**	**.7912**	**.8206**
		2	.33260170	.60266324	.78101552	.88593953	**.8693**	**.8854**	**.9134**	**.9471**	**.9598**
		3	.58425272	.84309825	.94580481	.98246903	**.9619**	**.9684**	**.9788**	**.9900**	**.9933**
		4	.79396190	.95638749	.99150451	.99845071	**.9911**	**.9934**	**.9960**	**.9986**	**.9992**
		5	.91978741	.99143470	.99914191	**.9963**	**.9984**	**.9990**	**.9995**	**.9998**	**.9999**
		6	.97570986	.99880980	.99994361	**.9994**	**.9998**	**.9999**	**1.0000**	**1.0000**	**1.0000**
		7	.99435067	.99988365	.99999759	**.9999**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
		8	.99901088	.99999213	.99999993	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
		9	.99987388	.99999964	1.00000000	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
2	14	0	.01781795	.04935154	.09141364	.14120784	.19630835	.25462875	.31440448	**.3858**	**.4387**
		1	.10096837	.23854184	.38228244	.51548985	.63034052	**.6343**	**.6827**	**.7197**	**.7525**
		2	.28112762	.53679664	.72250648	.84244044	**.8331**	**.8458**	**.8824**	**.9056**	**.9241**
		3	.52133996	.79558056	.92009791	.97073790	**.9450**	**.9494**	**.9663**	**.9752**	**.9816**
		4	.74153460	.93519342	.98512281	.99681035	**.9853**	**.9867**	**.9924**	**.9948**	**.9964**
		5	.88833103	.98520537	.99817996	**.9934**	**.9968**	**.9973**	**.9987**	**.9992**	**.9995**
		6	.96172924	.99756069	.99985197	**.9987**	**.9995**	**.9996**	**.9998**	**.9999**	**.9999**
		7	.98969047	.99970996	.99999196	**.9998**	**.9999**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
		8	.99784582	.99997535	.99999971	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
		9	.99965813	.99999853	.99999999	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
2	15	0	.01336346	.03751882	.07037994	.11006967	.15490608	.20338866	.25419391	**.3243**	**.3734**
		1	.08018077	.19565561	.32247826	.44601569	.55806471	**.5871**	**.6299**	**.6669**	**.7042**
		2	.23608781	.47303494	.66034385	.79182228	**.7888**	**.8070**	**.8427**	**.8699**	**.8962**
		3	.46128688	.74328722	.88787747	.95405793	**.9239**	**.9288**	**.9467**	**.9590**	**.9708**
		4	.68648594	.90843468	.97559418	.99393186	**.9770**	**.9784**	**.9851**	**.9892**	**.9933**
		5	.85163192	.97608310	.99644869	**.9893**	**.9943**	**.9948**	**.9966**	**.9976**	**.9987**
		6	.94337969	.99540980	.99965017	**.9975**	**.9989**	**.9990**	**.9994**	**.9996**	**.9998**
		7	.98270016	.99935205	.99997651	**.9995**	**.9998**	**.9999**	**.9999**	**.9999**	**1.0000**
		8	.99580699	.99993300	.99999893	**.9999**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
		9	.99920505	.99999498	.99999997	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
		10	.99988466	.99999973	1.00000000	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
2	16	0	.01002260	.02845674	.05394884	.08525589	.12123568	.16083996	.20311397	**.2708**	**.3156**
		1	.06347644	.15932512	.26924328	.38105706	.48705389	**.5448**	**.5848**	**.6205**	**.6534**
		2	.19711105	.41269557	.59642944	.73535872	**.7375**	**.7694**	**.8071**	**.8370**	**.8619**
		3	.40498711	.68756956	.84925644	.93163407	**.8992**	**.9079**	**.9288**	**.9439**	**.9553**
		4	.63018618	.87607759	.96216309	.98922590	**.9662**	**.9696**	**.9783**	**.9839**	**.9879**
		5	.81034543	.96344949	.99355060	**.9837**	**.9906**	**.9919**	**.9947**	**.9963**	**.9973**
		6	.92044275	.99196010	.99924411	**.9957**	**.9979**	**.9983**	**.9990**	**.9993**	**.9995**
		7	.97287004	.99867797	.99993855	**.9991**	**.9996**	**.9997**	**.9999**	**.9999**	**.9999**
		8	.99253028	.99983767	.99999652	**.9998**	**.9999**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
		9	.99835553	.99998521	.99999986	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
		10	.99971476	.99999901	1.00000000	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
2	17	0	.00751695	.02154203	.04120302	.06568579	.09422735	.12611192	.16067127	**.2250**	**.2655**

Table 32: Sheet C. Exact (in roman), approximate *(in italics)* and interpolated **(boldfaced)** probabilities of perfect ($i = 0$) and i-imperfect column pairs ($k = 2$) in Bernoulli ($m \times n$)-matrices. The rows with all 1's are omitted

k	m	i	Width n of Bernoulli ($m \times n$)-matrix								
			2	3	4	5	6	7	8	9	10
2	17	1	.05011298	.12890875	.22271673	.32180143	.41941312	**.5016**	**.5400**	**.5738**	**.6053**
		2	.16370240	.35671752	.53258496	.67469038	**.6813**	**.7241**	**.7658**	**.7982**	**.8256**
		3	.35301810	.62985979	.80475201	.90295230	**.8699**	**.8780**	**.9047**	**.9229**	**.9372**
		4	.57388641	.83839408	.94416586	.98199952	**.9519**	**.9545**	**.9672**	**.9750**	**.9807**
		5	.76530561	.94677625	.98899648	**.9762**	**.9850**	**.9859**	**.9907**	**.9933**	**.9950**
		6	.89291842	.98675321	.99848825	**.9929**	**.9962**	**.9965**	**.9979**	**.9985**	**.9989**
		7	.95976322	.99750121	.99985380	**.9983**	**.9992**	**.9993**	**.9996**	**.9997**	**.9998**
		8	.98761522	.99964247	.99999000	**.9996**	**.9999**	**.9999**	**1.0000**	**1.0000**	**1.0000**
		9	.99689922	.99996131	.99999952	**.9999**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
		10	.99937495	.99999686	.99999998	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
		11	.99990011	.99999981	1.00000000	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
2	18	0	.00563771	.01628167	.03137294	.05038297	.07280873	.09817027	.12601086	*.15517082*	**.2227**
		1	.03946397	.10370308	.18269712	.26890065	.35670600	**.4637**	**.4976**	**.5332**	**.5594**
		2	.13530504	.30568542	.47042021	.61163945	**.6219**	**.6802**	**.7214**	**.7622**	**.7865**
		3	.30568917	.57155983	.75523207	.86786199	**.8368**	**.8467**	**.8756**	**.9031**	**.9157**
		4	.51866933	.79593319	.92110831	.97150602	**.9340**	**.9371**	**.9522**	**.9663**	**.9711**
		5	.71745081	.92567865	.98222784	**.9668**	**.9773**	**.9781**	**.9845**	**.9903**	**.9916**
		6	.86101522	.97929654	.99717387	**.9892**	**.9935**	**.9937**	**.9958**	**.9977**	**.9979**
		7	.94305202	.99557616	.99967961	**.9970**	**.9985**	**.9985**	**.9991**	**.9996**	**.9996**
		8	.98065222	.99927354	.99997393	**.9993**	**.9997**	**.9997**	**.9998**	**.9999**	**.9999**
		9	.99457822	.99990839	.99999847	**.9999**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
		10	.99875602	.99999117	.99999994	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
		11	.99976882	.99999936	1.00000000	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
2	19	0	.00422828	.01228971	.02382769	.03850151	.05598300	.07595496	.09811105	*.12186509*	**.1869**
		1	.03100741	.08300097	.14875028	.22255438	.29993705	**.3992**	**.4611**	**.4904**	**.5216**
		2	.11134477	.25987603	.41125018	.54802977	**.5613**	**.6419**	**.6801**	**.7164**	**.7534**
		3	.26309314	.51395369	.70182751	.82661116	**.8010**	**.8187**	**.8475**	**.8732**	**.8984**
		4	.46542429	.74946881	.89272737	.95701382	**.9130**	**.9216**	**.9372**	**.9508**	**.9640**
		5	.66775544	.89995507	.97265391	**.9554**	**.9674**	**.9710**	**.9780**	**.9839**	**.9894**
		6	.82512412	.96909837	.99502163	**.9841**	**.9897**	**.9911**	**.9936**	**.9956**	**.9975**
		7	.92254282	.99259975	.99934654	**.9952**	**.9973**	**.9978**	**.9985**	**.9990**	**.9995**
		8	.97125217	.99862259	.99993768	**.9988**	**.9994**	**.9996**	**.9997**	**.9998**	**.9999**
		9	.99109672	.99980060	.99999566	**.9997**	**.9999**	**.9999**	**1.0000**	**1.0000**	**1.0000**
		10	.99771157	.99997760	.99999978	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
		11	.99951562	.99999806	.99999999	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
		12	.99991652	.99999987	1.00000000	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
2	20	0	.00317121	.00926644	.01805900	.02933051	.04286870	.05846631	.07592094	*.09491296*	*.11511248*
		1	.02431262	.06612983	.12030271	.18261372	.24960193	**.1978**	**.4305**	**.4561**	**.4785**
		2	.09126043	.21931566	.35605866	.48553710	**.5000**	**.4448**	**.6426**	**.6778**	**.7050**
		3	.22515605	.45814698	.64582676	.77983140	**.7619**	**.6844**	**.8206**	**.8476**	**.8657**
		4	.41484150	.69993307	.85902787	.93788494	**.8876**	**.8465**	**.9221**	**.9372**	**.9462**
		5	.61717265	.86960862	.95969939	**.9415**	**.9541**	**.9362**	**.9709**	**.9779**	**.9815**
		6	.78578195	.95570672	.99168127	**.9777**	**.9840**	**.9778**	**.9909**	**.9935**	**.9946**
		7	.89818814	.98822004	.99874921	**.9926**	**.9953**	**.9937**	**.9977**	**.9984**	**.9987**
		8	.95907483	.99754075	.99986191	**.9979**	**.9989**	**.9985**	**.9995**	**.9997**	**.9997**
		9	.98613558	.99959635	.99998874	**.9995**	**.9998**	**.9997**	**.9999**	**1.0000**	**1.0000**
		10	.99605786	.99994794	.99999932	**.9999**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
		11	.99906461	.99999474	.99999997	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
		12	.99981630	.99999959	1.00000000	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
2	21	0	.00237841	.00698061	.01366293	.02228604	.03271354	.04481128	.05844711	*.07343745*	*.08958964*
		1	.01902727	.05247438	.09671566	.14868563	.20577705	**.1640**	**.1955**	**.3903**	**.4432**
		2	.07452348	.18383964	.30549989	.42558314	**.4410**	**.3925**	**.4440**	**.6376**	**.6623**
		3	.19168214	.40503290	.58856844	.72847851	**.7210**	**.6346**	**.6850**	**.8180**	**.8357**
		4	.36742014	.64834470	.82029064	.91365079	**.8587**	**.8110**	**.8461**	**.9202**	**.9291**
		5	.56658987	.83484944	.94285699	**.9245**	**.9376**	**.9155**	**.9352**	**.9696**	**.9733**
		6	.74362962	.93874747	.98674023	**.9697**	**.9762**	**.9680**	**.9771**	**.9903**	**.9915**

Table 32: Sheet D. Exact (in roman), approximate *(in italics)* and interpolated **(boldfaced)** probabilities of perfect ($i = 0$) and i-imperfect column pairs ($k = 2$) in Bernoulli ($m \times n$)-matrices. The rows with all 1's are omitted

k m i	Width n of Bernoulli $(m \times n)$-matrix								
	2	3	4	5	6	7	8	9	10
2 21 7	.87008659	.98205186	.99773743	**.9891**	**.9923**	**.9899**	**.9933**	**.9974**	**.9977**
8	.94385316	.99583495	.99971383	**.9966**	**.9979**	**.9974**	**.9984**	**.9994**	**.9995**
9	.97937040	.99923291	.99997300	**.9991**	**.9995**	**.9994**	**.9997**	**.9999**	**.9999**
10	.99357729	.99988781	.99999809	**.9998**	**.9999**	**.9999**	**1.0000**	**1.0000**	**1.0000**
11	.99831292	.99998699	.99999990	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
12	.99962837	.99999881	1.00000000	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
13	.99993194	.99999991	1.00000000	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
2 22 0	.00178381	.00525472	.01032191	.01689680	.02489223	.03422200	.04480064	*.05651919*	*.06926272*
1	.01486506	.04148788	.07733988	.12022600	.16822398	**.1349**	**.1637**	**.1874**	**.2126**
2	.06064943	.15314818	.25992757	.36927638	**.3840**	**.3425**	**.3954**	**.4333**	**.4718**
3	.16239248	.35528119	.53134500	.67374180	**.6753**	**.5822**	**.6400**	**.6759**	**.7099**
4	.32348564	.59574035	.77705266	.88407352	**.8265**	**.7700**	**.8146**	**.8387**	**.8603**
5	.51679743	.79607942	.92173784	**.9045**	**.9175**	**.8890**	**.9169**	**.9302**	**.9415**
6	.69936968	.91795717	.97974193	**.9600**	**.9658**	**.9539**	**.9683**	**.9744**	**.9792**
7	.83847235	.97369833	.99610944	**.9845**	**.9878**	**.9837**	**.9899**	**.9921**	**.9937**
8	.92541152	.99326797	.99944140	**.9948**	**.9963**	**.9952**	**.9973**	**.9980**	**.9984**
9	.97049109	.99862106	.99993964	**.9985**	**.9991**	**.9988**	**.9994**	**.9996**	**.9997**
10	.99002557	.99977365	.99999507	**.9996**	**.9998**	**.9998**	**.9999**	**.9999**	**.9999**
11	.99712901	.99997024	.99999970	**.9999**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
12	.99929951	.99999687	.99999999	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
13	.99985605	.99999974	1.00000000	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
2 23 0	.00133786	.00395310	.00778841	.01278765	.01889538	.02605651	.03421619	*.04330848*	*.05326387*
1	.01159474	.03269531	.06155282	.09661535	.13649104	**.1111**	**.1329**	**.1573**	**.1801**
2	.04920334	.12685498	.21944045	.31739516	**.3312**	**.2992**	**.3414**	**.3870**	**.4260**
3	.13695672	.30934523	.47532590	.61693797	**.6224**	**.5344**	**.5835**	**.6337**	**.6726**
4	.28321235	.54311487	.73006417	.84918376	**.7930**	**.7318**	**.7711**	**.8100**	**.8374**
5	.46846949	.75386253	.89611322	**.8815**	**.8952**	**.8637**	**.8891**	**.9137**	**.9296**
6	.65372662	.89320724	.97021284	**.9486**	**.9534**	**.9398**	**.9537**	**.9667**	**.9743**
7	.80369669	.96277628	.99360890	**.9789**	**.9820**	**.9773**	**.9836**	**.9892**	**.9922**
8	.90367673	.98956295	.99896668	**.9923**	**.9941**	**.9928**	**.9951**	**.9971**	**.9980**
9	.95922119	.99764050	.99987326	**.9976**	**.9984**	**.9981**	**.9988**	**.9994**	**.9996**
10	.98514195	.99956913	.99998815	**.9993**	**.9996**	**.9996**	**.9998**	**.9999**	**.9999**
11	.99535315	.99993641	.99999915	**.9998**	**.9999**	**.9999**	**1.0000**	**1.0000**	**1.0000**
12	.99875689	.99999242	.99999995	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
13	.99971691	.99999927	1.00000000	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
14	.99994549	.99999994	1.00000000	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
2 24 0	.00100339	.00297237	.00587081	.00966324	.01431449	.01978954	.02605339	*.03306568*	*.04078400*
1	.00903052	.02569134	.04878155	.07721580	.11000152	**.0717**	**.0862**	**.1010**	**.1124**
2	.03980119	.10452716	.18393534	.27040394	**.2803**	**.2096**	**.2411**	**.2716**	**.2921**
3	.11501837	.26748185	.42150438	.55940530	**.5615**	**.3920**	**.4311**	**.4660**	**.4862**
4	.24664844	.49137408	.68022989	.80928981	**.7555**	**.5465**	**.5782**	**.6045**	**.6171**
5	.42215520	.70888533	.86594232	**.8549**	**.8680**	**.6485**	**.6690**	**.6852**	**.6918**
6	.60741234	.86451727	.95769586	**.9350**	**.9367**	**.7116**	**.7230**	**.7318**	**.7349**
7	.76620417	.94894307	.98992704	**.9719**	**.9732**	**.7534**	**.7590**	**.7632**	**.7647**
8	.87868172	.98441260	.99817918	**.9891**	**.9902**	**.7892**	**.7918**	**.7938**	**.7947**
9	.94533508	.99613681	.99974848	**.9963**	**.9970**	**.8286**	**.8299**	**.8311**	**.8319**
10	.97866176	.99922111	.99997330	**.9989**	**.9992**	**.8728**	**.8737**	**.8745**	**.8754**
11	.99280035	.99987210	.99999781	**.9997**	**.9998**	**.9167**	**.9174**	**.9180**	**.9187**
12	.99790595	.99998290	.99999986	**.9999**	**1.0000**	**.9532**	**.9536**	**.9541**	**.9545**
13	.99947691	.99999814	.99999999	**1.0000**	**1.0000**	**.9781**	**.9783**	**.9785**	**.9788**
14	.99988835	.99999984	1.00000000	**1.0000**	**1.0000**	**.9917**	**.9918**	**.9919**	**.9920**
2 25 0	.00075254	.00223399	.00442162	.00729305	.01082604	.01499839	.01978789	*.02516966*	*.03111770*
1	.00702374	.02013514	.03851473	.06141055	.08812384	**.0589**	**.0717**	**.0832**	**.0927**
2	.03210852	.08571631	.15316025	.22849168	**.2377**	**.1819**	**.2126**	**.2374**	**.2560**
3	.09621407	.22977903	.37066794	.50241049	**.5072**	**.3574**	**.3991**	**.4296**	**.4501**
4	.21374092	.44130152	.62854276	.76495851	**.7173**	**.5169**	**.5535**	**.5775**	**.5912**

Table 32: Sheet E. Exact (in roman), approximate *(in italics)* and interpolated **(boldfaced)** probabilities of perfect ($i = 0$) and i-imperfect column pairs ($k = 2$) in Bernoulli ($m \times n$)-matrices. The rows with all 1's are omitted

k m i	Width n of Bernoulli $(m \times n)$-matrix								
	2	3	4	5	6	7	8	9	10
2 25 5	.37827851	.66191252	.83138328	**.8250**	**.8382**	**.6269**	**.6519**	**.6669**	**.6745**
6	.56109805	.83205680	.94178653	**.9190**	**.9171**	**.6956**	**.7103**	**.7186**	**.7224**
7	.72650621	.93192207	.98471122	**.9635**	**.9620**	**.7387**	**.7463**	**.7504**	**.7522**
8	.85056233	.97749271	.99693008	**.9849**	**.9847**	**.7716**	**.7751**	**.7770**	**.7779**
9	.92867174	.99392080	.99952575	**.9945**	**.9947**	**.8057**	**.8072**	**.8082**	**.8089**
10	.97033009	.99865567	.99994331	**.9983**	**.9984**	**.8456**	**.8464**	**.8471**	**.8478**
11	.98926570	.99975622	.99999473	**.9995**	**.9996**	**.8893**	**.8898**	**.8904**	**.8910**
12	.99662955	.99996373	.99999962	**.9999**	**.9999**	**.9303**	**.9307**	**.9312**	**.9316**
13	.99908417	.99999558	.99999998	**1.0000**	**1.0000**	**.9625**	**.9627**	**.9630**	**.9633**
14	.99978549	.99999956	1.00000000	**1.0000**	**1.0000**	**.9831**	**.9833**	**.9834**	**.9835**
2 26 0	.00056441	.00167845	.00332783	.00549846	.00817631	.01134735	.01499753	*.01911146*	*.02367314*
1	.00545594	.01574357	.03030678	.04862875	.07022297	**.0481**	**.0576**	**.0689**	**.0782**
2	.02583733	.06998126	.12676323	.19162236	**.1984**	**.1570**	**.1805**	**.2081**	**.2286**
3	.08018769	.19618713	.32339001	.44707628	**.4491**	**.3241**	**.3584**	**.3965**	**.4222**
4	.18435921	.39353904	.57601795	.71697185	**.6792**	**.4873**	**.5190**	**.5525**	**.5726**
5	.33714411	.61374207	.79278672	**.7920**	**.8071**	**.6055**	**.6277**	**.6508**	**.6634**
6	.51539317	.79613612	.92216808	**.9006**	**.8955**	**.6823**	**.6957**	**.7094**	**.7164**
7	.68515417	.91152441	.97757962	**.9537**	**.9488**	**.7313**	**.7386**	**.7457**	**.7493**
8	.81954830	.96847898	.99502840	**.9799**	**.9778**	**.7680**	**.7715**	**.7748**	**.7765**
9	.90914439	.99077066	.99914650	**.9922**	**.9916**	**.8040**	**.8056**	**.8071**	**.8079**
10	.95991550	.99777454	.99988591	**.9973**	**.9972**	**.8448**	**.8455**	**.8463**	**.8469**
11	.98453180	.99955726	.99998807	**.9992**	**.9992**	**.8888**	**.8893**	**.8898**	**.8903**
12	.99478859	.99992725	.99999902	**.9998**	**.9998**	**.9300**	**.9303**	**.9307**	**.9310**
13	.99847051	.99999013	.99999994	**1.0000**	**1.0000**	**.9623**	**.9625**	**.9627**	**.9629**
14	.99961016	.99999890	1.00000000	**1.0000**	**1.0000**	**.9830**	**.9831**	**.9832**	**.9834**
15	.99991406	.99999990	1.00000000	**1.0000**	**1.0000**	**.9938**	**.9939**	**.9939**	**.9939**
2 27 0	.00042331	.00126068	.00250315	.00414185	.00616794	.00857258	.01134691	*.01448139*	*.01796612*
1	.00423306	.01228381	.02377671	.03835900	.05569512	**.0188**	**.0227**	**.0561**	**.0640**
2	.02074198	.05690374	.10433391	.15958920	**.1399**	**.0756**	**.0888**	**.1789**	**.1979**
3	.06660010	.16655040	.28003855	.39433361	**.3112**	**.1890**	**.2129**	**.3593**	**.3856**
4	.15831633	.34857969	.52363347	.66626669	**.5143**	**.3257**	**.3506**	**.5215**	**.5439**
5	.29894789	.56516367	.75067379	**.7557**	**.6931**	**.4494**	**.4680**	**.6284**	**.6432**
6	.47083090	.75718784	.89864117	**.8794**	**.8265**	**.5542**	**.5658**	**.6929**	**.7016**
7	.64271392	.88766482	.96814180	**.9423**	**.9116**	**.6376**	**.6443**	**.7302**	**.7348**
8	.78594977	.95706598	.99224035	**.9737**	**.9598**	**.7016**	**.7050**	**.7544**	**.7564**
9	.88674536	.98643747	.99852798	**.9892**	**.9839**	**.7514**	**.7528**	**.7768**	**.7775**
10	.94722272	.99645252	.99978130	**.9960**	**.9943**	**.7949**	**.7955**	**.8051**	**.8054**
11	.97837772	.99923018	.99997443	**.9987**	**.9983**	**.8384**	**.8386**	**.8417**	**.8418**
12	.99222439	.99986121	.99999764	**.9996**	**.9995**	**.8828**	**.8828**	**.8836**	**.8837**
13	.99755003	.99997920	.99999983	**.9999**	**.9999**	**.9242**	**.9242**	**.9244**	**.9244**
14	.99932525	.99999741	.99999999	**1.0000**	**1.0000**	**.9575**	**.9575**	**.9575**	**.9575**
15	.99983809	.99999973	1.00000000	**1.0000**	**1.0000**	**.9798**	**.9799**	**.9799**	**.9799**
2 28 0	.00031748	.00094666	.00188192	.00311768	.00464839	.00646848	.00857237	*.01095413*	*.01360766*
1	.00328062	.00956607	.01860380	.03015452	.04398861	**.0148**	**.0178**	**.0210**	**.0241**
2	.01661475	.04609817	.08543697	.13206629	**.0861**	**.0621**	**.0733**	**.0844**	**.0954**
3	.05513557	.14063661	.24079653	.34489774	**.2078**	**.1637**	**.1860**	**.2066**	**.2255**
4	.13538727	.30677063	.47228184	.61386614	**.3600**	**.2947**	**.3199**	**.3412**	**.3594**
5	.26379000	.51692312	.70570186	**.7166**	**.4969**	**.4173**	**.4373**	**.4531**	**.4659**
6	.42786015	.71574162	.87114523	**.8552**	**.6023**	**.5240**	**.5369**	**.5468**	**.5548**
7	.59974317	.86037047	.95602313	**.9290**	**.6732**	**.6107**	**.6184**	**.6243**	**.6288**
8	.75014081	.94298636	.98829252	**.9663**	**.7201**	**.6785**	**.6826**	**.6856**	**.6879**
9	.86154647	.98065371	.99755856	**.9854**	**.7544**	**.7304**	**.7323**	**.7336**	**.7345**
10	.93210338	.99453646	.99959909	**.9943**	**.7858**	**.7731**	**.7738**	**.7743**	**.7747**
11	.97058897	.99871340	.99994790	**.9981**	**.8209**	**.8139**	**.8142**	**.8143**	**.8144**
12	.98876272	.99974694	.99999462	**.9994**	**.8611**	**.8564**	**.8565**	**.8566**	**.8566**
13	.99621862	.99995839	.99999956	**.9998**	**.9026**	**.8991**	**.8991**	**.8992**	**.8992**

Table 32: Sheet F. Exact (in roman), approximate *(in italics)* and interpolated **(boldfaced)** probabilities of perfect ($i = 0$) and i-imperfect column pairs ($k = 2$) in Bernoulli ($m \times n$)-matrices. The rows with all 1's are omitted

k m i	Width n of Bernoulli ($m \times n$)-matrix								
	2	3	4	5	6	7	8	9	10
2 28 14	.99888144	.99999428	.99999997	**1.0000**	**.9397**	**.9372**	**.9372**	**.9372**	**.9372**
15	.99970988	.99999934	1.00000000	**1.0000**	**.9677**	**.9662**	**.9662**	**.9662**	**.9662**
16	.99993424	.99999994	1.00000000	**1.0000**	**.9854**	**.9846**	**.9846**	**.9846**	**.9846**
2 29 0	.00023811	.00071072	.00141430	.00234535	.00350037	.00487588	.00646837	*.00827415*	*.01028944*
1	.00253983	.00743675	.01452154	.02363273	.03461430	**.0116**	**.0140**	**.0165**	**.0190**
2	.01328122	.03721695	.06963731	.10865417	**.0727**	**.0513**	**.0608**	**.0701**	**.0794**
3	.04550536	.11816320	.20568987	.29926551	**.1849**	**.1435**	**.1639**	**.1830**	**.2008**
4	.11532435	.26832378	.42273515	.56081081	**.3342**	**.2718**	**.2970**	**.3187**	**.3374**
5	.23168932	.46969446	.65862260	**.6752**	**.4748**	**.3980**	**.4192**	**.4362**	**.4501**
6	.38684261	.67239476	.83976921	**.8283**	**.5872**	**.5104**	**.5247**	**.5356**	**.5445**
7	.55677241	.82978232	.94089112	**.9138**	**.6639**	**.6018**	**.6106**	**.6174**	**.6227**
8	.71254140	.92602885	.98287957	**.9577**	**.7149**	**.6733**	**.6783**	**.6820**	**.6848**
9	.83369505	.97314461	.99609394	**.9808**	**.7516**	**.7278**	**.7302**	**.7320**	**.7332**
10	.91446415	.99184617	.99929463	**.9921**	**.7843**	**.7720**	**.7730**	**.7737**	**.7742**
11	.96096758	.99792587	.99989868	**.9971**	**.8199**	**.8135**	**.8139**	**.8141**	**.8143**
12	.98421929	.99955718	.99998838	**.9991**	**.8602**	**.8563**	**.8564**	**.8565**	**.8565**
13	.99435465	.99992055	.99999893	**.9997**	**.9019**	**.8991**	**.8991**	**.8991**	**.8991**
14	.99821574	.99998802	.99999992	**.9999**	**.9392**	**.9372**	**.9372**	**.9372**	**.9372**
15	.99950277	.99999848	1.00000000	**1.0000**	**.9674**	**.9662**	**.9662**	**.9662**	**.9662**
16	.99987815	.99999984	1.00000000	**1.0000**	**.9852**	**.9846**	**.9846**	**.9846**	**.9846**
2 30 0	.00017858	.00053349	.00106251	.00176345	.00263411	.00367231	.00487583	*.00624240*	*.00776966*
1	.00196440	.00577232	.01131078	.01847132	.02714852	**.0090**	**.0109**	**.0129**	**.0148**
2	.01059587	.02995235	.05651798	.08891716	**.0601**	**.0417**	**.0496**	**.0574**	**.0653**
3	.03744933	.09881929	.17461845	.25772881	**.1605**	**.1225**	**.1410**	**.1585**	**.1750**
4	.09786960	.23333141	.37562376	.50809707	**.3026**	**.2428**	**.2676**	**.2893**	**.3083**
5	.20259807	.42406037	.61023676	**.6317**	**.4440**	**.3661**	**.3885**	**.4066**	**.4215**
6	.34805429	.62778142	.80475086	**.7982**	**.5626**	**.4796**	**.4952**	**.5073**	**.5171**
7	.51428996	.79614917	.92248092	**.8962**	**.6455**	**.5737**	**.5837**	**.5913**	**.5973**
8	.67359915	.90605333	.97567636	**.9477**	**.7005**	**.6491**	**.6550**	**.6594**	**.6628**
9	.80340664	.96364148	.99395518	**.9752**	**.7381**	**.7068**	**.7099**	**.7121**	**.7137**
10	.89427188	.98817753	.99880512	**.9894**	**.7685**	**.7517**	**.7531**	**.7541**	**.7547**
11	.94934172	.99676496	.99981124	**.9959**	**.7997**	**.7913**	**.7919**	**.7922**	**.7925**
12	.97840636	.99925372	.99997606	**.9986**	**.8359**	**.8314**	**.8316**	**.8317**	**.8318**
13	.99182081	.99985465	.99999756	**.9996**	**.8763**	**.8734**	**.8734**	**.8735**	**.8735**
14	.99725047	.99997608	.99999980	**.9999**	**.9162**	**.9140**	**.9141**	**.9141**	**.9141**
15	.99918101	.99999667	.99999999	**1.0000**	**.9500**	**.9485**	**.9485**	**.9485**	**.9485**
16	.99978431	.99999961	1.00000000	**1.0000**	**.9742**	**.9734**	**.9734**	**.9734**	**.9734**
17	.99994992	.99999996	**1.0000**	**1.0000**	**.9888**	**.9884**	**.9884**	**.9884**	**.9884**
2 31 0	.00013394	.00040040	.00079800	.00132537	.00198112	.00276389	.00367229	*.00470489*	*.00586024*
1	.00151795	.00447401	.00879300	.01440232	.02123116	**.0070**	**.0085**	**.0100**	**.0116**
2	.00843800	.02403600	.04569176	.07241227	**.0502**	**.0338**	**.0403**	**.0468**	**.0533**
3	.03073596	.08228343	.14738714	.22039919	**.1406**	**.1041**	**.1206**	**.1363**	**.1513**
4	.08276453	.20178471	.33142782	.45662690	**.2762**	**.2158**	**.2399**	**.2613**	**.2803**
5	.17641596	.38050065	.56135031	**.5870**	**.4177**	**.3355**	**.3588**	**.3779**	**.3938**
6	.31169024	.58254283	.76646529	**.7651**	**.5417**	**.4496**	**.4665**	**.4797**	**.4903**
7	.47273104	.75981542	.90061748	**.8761**	**.6302**	**.5456**	**.5567**	**.5652**	**.5720**
8	.63377185	.88300189	.96635381	**.9361**	**.6890**	**.6244**	**.6312**	**.6364**	**.6404**
9	.77095477	.95189611	.99092924	**.9686**	**.7277**	**.6858**	**.6896**	**.6924**	**.6944**
10	.87155557	.98330798	.99804566	**.9859**	**.7563**	**.7327**	**.7346**	**.7358**	**.7367**
11	.93557426	.99510524	.99966197	**.9943**	**.7834**	**.7715**	**.7723**	**.7728**	**.7731**
12	.97114020	.99878493	.99995286	**.9979**	**.8147**	**.8089**	**.8092**	**.8093**	**.8095**
13	.98846720	.99974425	.99999468	**.9993**	**.8518**	**.8485**	**.8486**	**.8487**	**.8487**
14	.99589305	.99995431	.99999951	**.9998**	**.8917**	**.8895**	**.8895**	**.8895**	**.8895**
15	.99869837	.99999307	.99999996	**1.0000**	**.9292**	**.9276**	**.9276**	**.9276**	**.9276**
16	.99963348	.99999911	1.00000000	**1.0000**	**.9593**	**.9582**	**.9582**	**.9582**	**.9582**
17	.99990851	.99999990	**1.0000**	**1.0000**	**.9798**	**.9792**	**.9792**	**.9792**	**.9792**

Table 32: Sheet G. Exact (in roman), approximate *(in italics)* and interpolated **(boldfaced)** probabilities of perfect ($i = 0$) and i-imperfect column pairs ($k = 2$) in Bernoulli ($m \times n$)-matrices. The rows with all 1's are omitted

k m i	Width n of Bernoulli $(m \times n)$-matrix								
	2	3	4	5	6	7	8	9	10
2 32 0	.00010045	.00030048	.00059920	.00099577	.00148931	.00207897	.00276388	*.00354315*	*.00441588*
1	.00117194	.00346320	.00682385	.01120550	.01656075	**.0055**	**.0066**	**.0078**	**.0090**
2	.00670799	.01923679	.03680783	.05871010	**.0418**	**.0272**	**.0326**	**.0379**	**.0433**
3	.02516147	.06823714	.12373463	.18723878	**.1226**	**.0880**	**.1025**	**.1165**	**.1300**
4	.06975739	.17359331	.29047952	.40717215	**.2505**	**.1908**	**.2139**	**.2346**	**.2534**
5	.15300310	.33938802	.51273511	**.5416**	**.3909**	**.3063**	**.3301**	**.3500**	**.3668**
6	.27787167	.53730044	.72540424	**.7293**	**.5196**	**.4203**	**.4386**	**.4528**	**.4644**
7	.43247084	.72120395	.87523227	**.8534**	**.6140**	**.5178**	**.5300**	**.5393**	**.5469**
8	.59351165	.85690499	.95459759	**.9228**	**.6771**	**.5994**	**.6071**	**.6130**	**.6177**
9	.73665904	.93769504	.98677252	**.9608**	**.7181**	**.6645**	**.6690**	**.6724**	**.6749**
10	.84640537	.97700414	.99690589	**.9816**	**.7462**	**.7143**	**.7166**	**.7182**	**.7194**
11	.91956959	.99279864	.99941650	**.9922**	**.7702**	**.7535**	**.7545**	**.7552**	**.7557**
12	.96224871	.99808347	.99991090	**.9970**	**.7968**	**.7888**	**.7892**	**.7894**	**.7896**
13	.98413545	.99956582	.99998894	**.9990**	**.8295**	**.8255**	**.8256**	**.8257**	**.8258**
14	.99403659	.99991616	.99999888	**.9997**	**.8673**	**.8649**	**.8650**	**.8650**	**.8650**
15	.99799704	.99998619	.99999991	**.9999**	**.9062**	**.9045**	**.9045**	**.9045**	**.9045**
16	.99939971	.99999806	.99999999	**1.0000**	**.9408**	**.9396**	**.9396**	**.9396**	**.9396**
17	.99983976	.99999977	**1.0000**	**1.0000**	**.9672**	**.9664**	**.9664**	**.9664**	**.9664**
2 33 0	.00007534	.00022547	.00044984	.00074792	.00111916	.00156302	.00207897	*.00266644*	*.00332488*
1	.00090407	.00267757	.00528742	.00870147	.01288814	**.0042**	**.0051**	**.0061**	**.0070**
2	.00532398	.01535787	.02955461	.04740875	**.0331**	**.0220**	**.0264**	**.0308**	**.0352**
3	.02054810	.05637481	.10335859	.15809314	**.1013**	**.0748**	**.0874**	**.0997**	**.1117**
4	.05860841	.14860440	.25297349	.36035354	**.2164**	**.1706**	**.1924**	**.2122**	**.2303**
5	.13219167	.30099007	.46509679	**.4963**	**.3509**	**.2848**	**.3089**	**.3293**	**.3468**
6	.24665452	.49263338	.68214873	**.6913**	**.4826**	**.4030**	**.4224**	**.4378**	**.4503**
7	.39382105	.68079568	.84637285	**.8283**	**.5847**	**.5052**	**.5185**	**.5288**	**.5371**
8	.55325145	.82788248	.94012789	**.9079**	**.6563**	**.5907**	**.5994**	**.6060**	**.6114**
9	.70087219	.92087273	.98121769	**.9519**	**.7048**	**.6591**	**.6644**	**.6683**	**.6714**
10	.81896878	.96903125	.99524759	**.9766**	**.7386**	**.7113**	**.7142**	**.7162**	**.7178**
11	.90127853	.98967615	.99902655	**.9896**	**.7663**	**.7521**	**.7534**	**.7543**	**.7550**
12	.95157893	.99706440	.99983792	**.9958**	**.7950**	**.7882**	**.7888**	**.7891**	**.7894**
13	.97866376	.99928682	.99997798	**.9985**	**.8285**	**.8253**	**.8255**	**.8256**	**.8257**
14	.99156130	.99985176	.99999755	**.9995**	**.8668**	**.8649**	**.8649**	**.8650**	**.8650**
15	.99700693	.99997361	.99999978	**.9999**	**.9058**	**.9045**	**.9045**	**.9045**	**.9045**
16	.99904904	.99999597	.99999998	**1.0000**	**.9405**	**.9396**	**.9396**	**.9396**	**.9396**
17	.99972974	.99999947	**1.0000**	**1.0000**	**.9670**	**.9664**	**.9664**	**.9664**	**.9664**
18	.99993143	.99999994	**1.0000**	**1.0000**	**.9842**	**.9839**	**.9839**	**.9839**	**.9839**
2 34 0	.00005650	.00016917	.00033765	.00056162	.00084073	.00117464	.00156302	*.00200552*	*.00250179*
1	.00069689	.00206789	.00409115	.00674535	.01000952	**.0033**	**.0040**	**.0047**	**.0054**
2	.00421900	.01223316	.02365987	.03814215	**.0282**	**.0176**	**.0212**	**.0247**	**.0283**
3	.01674207	.04641037	.08593655	.13272332	**.0906**	**.0624**	**.0733**	**.0840**	**.0945**
4	.04909333	.12662036	.21898295	.31663323	**.2004**	**.1488**	**.1691**	**.1878**	**.2051**
5	.11379586	.26547587	.41905099	**.4515**	**.3336**	**.2568**	**.2809**	**.3016**	**.3195**
6	.21803881	.44906100	.63733772	**.6510**	**.4687**	**.3738**	**.3944**	**.4108**	**.4243**
7	.35702942	.63910758	.81420476	**.8000**	**.5751**	**.4768**	**.4913**	**.5024**	**.5115**
8	.51339385	.79613953	.92271877	**.8905**	**.6492**	**.5641**	**.5737**	**.5811**	**.5871**
9	.66396701	.90132251	.97398365	**.9414**	**.6976**	**.6353**	**.6414**	**.6460**	**.6496**
10	.78944464	.95916383	.99290385	**.9705**	**.7289**	**.6896**	**.6931**	**.6956**	**.6975**
11	.88070109	.98555131	.99842686	**.9863**	**.7509**	**.7295**	**.7313**	**.7325**	**.7333**
12	.93900383	.99562401	.99971545	**.9942**	**.7710**	**.7611**	**.7619**	**.7624**	**.7627**
13	.97189256	.99886377	.99995784	**.9978**	**.7951**	**.7912**	**.7915**	**.7917**	**.7918**
14	.98833692	.99974668	.99999487	**.9993**	**.8257**	**.8245**	**.8246**	**.8246**	**.8247**
15	.99564552	.99995145	.99999948	**.9998**	**.8621**	**.8618**	**.8618**	**.8618**	**.8618**
16	.99853851	.99999200	.99999996	**.9999**	**.9003**	**.9003**	**.9003**	**.9003**	**.9003**
17	.99955957	.99999886	**1.0000**	**1.0000**	**.9353**	**.9353**	**.9353**	**.9353**	**.9353**
18	.99988101	.99999986	**1.0000**	**1.0000**	**.9629**	**.9629**	**.9629**	**.9629**	**.9629**

Table 32: Sheet H. Exact (in roman), approximate *(in italics)* and interpolated **(boldfaced)** probabilities of perfect ($i = 0$) and i-imperfect column pairs ($k = 2$) in Bernoulli ($m \times n$)-matrices. The rows with all 1's are omitted

k m i	Width n of Bernoulli $(m \times n)$-matrix								
	2	3	4	5	6	7	8	9	10
2 35 0	.00004238	.00012692	.00025341	.00042164	.00063140	.00088247	.00117464	*.00150770*	*.00188143*
1	.00053679	.00159542	.00316148	.00522087	.00775970	**.0025**	**.0031**	**.0036**	**.0042**
2	.00333847	.00972363	.01888904	.03058404	**.0228**	**.0141**	**.0169**	**.0198**	**.0227**
3	.01361130	.03808137	.07114240	.11083462	**.0763**	**.0519**	**.0612**	**.0704**	**.0794**
4	.04100552	.10741442	.18847895	.27631890	**.1756**	**.1292**	**.1478**	**.1652**	**.1814**
5	.09762023	.23292605	.37510830	**.4080**	**.3024**	**.2306**	**.2542**	**.2749**	**.2931**
6	.19197807	.40703076	.59163580	**.6093**	**.4383**	**.3456**	**.3672**	**.3846**	**.3990**
7	.32228177	.59667074	.77900558	**.7692**	**.5503**	**.4492**	**.4648**	**.4770**	**.4868**
8	.47430274	.76195816	.90221521	**.8710**	**.6308**	**.5382**	**.5487**	**.5569**	**.5635**
9	.62632372	.87900457	.96478818	**.9296**	**.6847**	**.6126**	**.6195**	**.6247**	**.6289**
10	.75807523	.94719681	.98968025	**.9633**	**.7195**	**.6711**	**.6752**	**.6782**	**.6806**
11	.85788698	.98022533	.99753216	**.9824**	**.7426**	**.7144**	**.7166**	**.7181**	**.7192**
12	.92442815	.99363948	.99951674	**.9922**	**.7613**	**.7471**	**.7480**	**.7487**	**.7492**
13	.96367037	.99824031	.99992221	**.9969**	**.7816**	**.7755**	**.7759**	**.7762**	**.7763**
14	.98422583	.99958062	.99998967	**.9989**	**.8077**	**.8055**	**.8056**	**.8057**	**.8058**
15	.99381837	.99991379	.99999887	**.9996**	**.8404**	**.8397**	**.8398**	**.8398**	**.8398**
16	.99781527	.99998470	.99999990	**.9999**	**.8774**	**.8773**	**.8773**	**.8773**	**.8773**
17	.99930430	.99999765	**.9999**	**1.0000**	**.9143**	**.9143**	**.9143**	**.9143**	**.9143**
18	.99980065	.99999969	**1.0000**	**1.0000**	**.9463**	**.9463**	**.9463**	**.9463**	**.9463**
19	.99994868	.99999996	**1.0000**	**1.0000**	**.9703**	**.9703**	**.9703**	**.9703**	**.9703**
2 36 0	.00003178	.00009522	.00019017	.00031650	.00047408	.00066278	.00088247	*.00113301*	*.00141426*
1	.00041319	.00122976	.00244023	.00403530	.00600578	**.0020**	**.0024**	**.0029**	**.0033**
2	.00263805	.00771377	.01504237	.02444872	**.0188**	**.0113**	**.0135**	**.0159**	**.0182**
3	.01104310	.03115101	.05865866	.09210119	**.0657**	**.0433**	**.0512**	**.0590**	**.0668**
4	.03415696	.09074387	.16135087	.23957627	**.1569**	**.1130**	**.1299**	**.1458**	**.1609**
5	.08346655	.20334486	.33366748	**.3662**	**.2785**	**.2100**	**.2329**	**.2532**	**.2713**
6	.16838861	.36691126	.54570238	**.5666**	**.4149**	**.3263**	**.3487**	**.3670**	**.3822**
7	.28970584	.55400980	.74115211	**.7359**	**.5315**	**.4341**	**.4510**	**.4641**	**.4747**
8	.43629750	.72568503	.87854639	**.8493**	**.6172**	**.5272**	**.5387**	**.5476**	**.5548**
9	.58831847	.85395046	.95336259	**.9162**	**.6754**	**.6050**	**.6126**	**.6185**	**.6233**
10	.72513735	.93295653	.98535839	**.9551**	**.7136**	**.6663**	**.6710**	**.6746**	**.6774**
11	.83293404	.97349377	.99623481	**.9777**	**.7392**	**.7117**	**.7143**	**.7162**	**.7176**
12	.90779286	.99096976	.99920435	**.9898**	**.7595**	**.7457**	**.7470**	**.7479**	**.7485**
13	.95385982	.99734563	.99986134	**.9957**	**.7809**	**.7749**	**.7755**	**.7758**	**.7761**
14	.97908696	.99932580	.99998000	**.9984**	**.8074**	**.8052**	**.8055**	**.8056**	**.8057**
15	.99142024	.99985182	.99999761	**.9995**	**.8403**	**.8396**	**.8397**	**.8398**	**.8398**
16	.99681604	.99997179	.99999976	**.9998**	**.8774**	**.8772**	**.8773**	**.8773**	**.8773**
17	.99893204	.99999534	**.9999**	**1.0000**	**.9143**	**.9143**	**.9143**	**.9143**	**.9143**
18	.99967656	.99999933	**1.0000**	**1.0000**	**.9463**	**.9463**	**.9463**	**.9463**	**.9463**
19	.99991167	.99999992	**1.0000**	**1.0000**	**.9703**	**.9703**	**.9703**	**.9703**	**.9703**
2 37 0	.00002384	.00007143	.00014269	.00023754	.00035589	.00049767	.00066278	*.00085115*	*.00106269*
1	.00031784	.00094709	.00188152	.00311502	.00464151	**.0015**	**.0018**	**.0022**	**.0025**
2	.00208184	.00610822	.01195143	.01948970	**.0071**	**.0089**	**.0108**	**.0126**	**.0145**
3	.00894184	.02540862	.04818520	.07618587	**.0290**	**.0356**	**.0423**	**.0489**	**.0555**
4	.02837850	.07636082	.13742647	.20644742	**.0810**	**.0969**	**.1120**	**.1264**	**.1402**
5	.07113915	.17667326	.29501574	**.3265**	**.1618**	**.1861**	**.2080**	**.2277**	**.2456**
6	.14715809	.32898996	.50016441	**.5235**	**.2712**	**.2988**	**.3217**	**.3407**	**.3568**
7	.25937653	.51162487	.70110209	**.7001**	**.3829**	**.4064**	**.4244**	**.4385**	**.4500**
8	.39964958	.68771669	.85173410	**.8249**	**.4838**	**.5006**	**.5131**	**.5228**	**.5306**
9	.55031323	.82626385	.93946708	**.9009**	**.5696**	**.5809**	**.5894**	**.5959**	**.6012**
10	.69093263	.91631061	.97970187	**.9456**	**.6389**	**.6462**	**.6516**	**.6557**	**.6590**
11	.80598487	.96515436	.99440332	**.9720**	**.6912**	**.6956**	**.6987**	**.7011**	**.7028**
12	.88907815	.98745770	.99872761	**.9867**	**.7296**	**.7319**	**.7335**	**.7347**	**.7355**
13	.94234308	.99609325	.99976067	**.9942**	**.7597**	**.7608**	**.7615**	**.7620**	**.7624**
14	.97278018	.99894542	.99996263	**.9977**	**.7881**	**.7886**	**.7889**	**.7891**	**.7892**
15	.98833692	.99975295	.99999514	**.9992**	**.8194**	**.8196**	**.8197**	**.8198**	**.8198**

Table 32: Sheet I. Exact (in roman), approximate *(in italics)* and interpolated **(boldfaced)** probabilities of perfect ($i = 0$) and i-imperfect column pairs ($k = 2$) in Bernoulli $(m \times n)$-matrices. The rows with all 1's are omitted

k m i	Width n of Bernoulli $(m \times n)$-matrix								
	2	3	4	5	6	7	8	9	10
2 37 16	.99546709	.99994971	.99999947	**.9997**	**.8548**	**.8549**	**.8549**	**.8549**	**.8549**
17	.99840304	.99999110	**.9998**	**.9999**	**.8921**	**.8921**	**.8921**	**.8921**	**.8921**
18	.99949043	.99999863	**1.0000**	**1.0000**	**.9271**	**.9271**	**.9271**	**.9271**	**.9271**
19	.99985290	.99999982	**1.0000**	**1.0000**	**.9559**	**.9559**	**.9559**	**.9559**	**.9559**
2 38 0	.00001788	.00005358	.00010706	.00017826	.00026713	.00037362	.00049767	*.00063924*	*.00079826*
1	.00024434	.00072881	.00144933	.00240187	.00358245	**.0012**	**.0014**	**.0017**	**.0019**
2	.00164084	.00482865	.00947548	.01549698	**.0056**	**.0071**	**.0085**	**.0100**	**.0115**
3	.00722684	.02066904	.03944472	.06275533	**.0237**	**.0292**	**.0348**	**.0403**	**.0459**
4	.02351933	.06402059	.11649035	.17687214	**.0687**	**.0826**	**.0961**	**.1090**	**.1214**
5	.06044899	.15280194	.25933478	**.2891**	**.1416**	**.1642**	**.1849**	**.2037**	**.2209**
6	.12815336	.29347484	.45559405	**.4802**	**.2451**	**.2726**	**.2957**	**.3152**	**.3319**
7	.23132192	.46997660	.65937205	**.6620**	**.3548**	**.3795**	**.3986**	**.4137**	**.4260**
8	.36458132	.64848340	.82189560	**.7977**	**.4562**	**.4744**	**.4879**	**.4984**	**.5069**
9	.51264732	.79611772	.92290579	**.8834**	**.5442**	**.5566**	**.5658**	**.5730**	**.5789**
10	.65577778	.89717616	.97246460	**.9346**	**.6170**	**.6252**	**.6313**	**.6360**	**.6398**
11	.77722181	.95501569	.99188225	**.9653**	**.6737**	**.6787**	**.6824**	**.6852**	**.6873**
12	.86830483	.98293345	.99802007	**.9830**	**.7153**	**.7181**	**.7201**	**.7216**	**.7226**
13	.92902685	.99438052	.99959923	**.9923**	**.7465**	**.7479**	**.7489**	**.7495**	**.7500**
14	.96517090	.99839203	.99993245	**.9968**	**.7735**	**.7741**	**.7745**	**.7748**	**.7749**
15	.98444773	.99959959	.99999049	**.9988**	**.8017**	**.8019**	**.8021**	**.8022**	**.8022**
16	.99368455	.99991312	.99999888	**.9996**	**.8340**	**.8341**	**.8341**	**.8341**	**.8342**
17	.99766906	.99998356	**.9998**	**.9999**	**.8698**	**.8699**	**.8699**	**.8699**	**.8699**
18	.99921859	.99999728	**.9999**	**1.0000**	**.9061**	**.9061**	**.9061**	**.9061**	**.9061**
19	.99976228	.99999961	**1.0000**	**1.0000**	**.9386**	**.9386**	**.9386**	**.9386**	**.9386**
20	.99993445	.99999995	**1.0000**	**1.0000**	**.9642**	**.9642**	**.9642**	**.9642**	**.9642**
2 39 0	.00001341	.00004019	.00008032	.00013376	.00020048	.00028044	.00037362	*.00047997*	*.00059948*
1	.00018772	.00056042	.00111542	.00185007	.00276176	**.0009**	**.0011**	**.0013**	**.0015**
2	.00129171	.00381111	.00749777	.01229353	**.0045**	**.0056**	**.0068**	**.0079**	**.0091**
3	.00583034	.01677122	.03218587	.05149100	**.0195**	**.0241**	**.0287**	**.0333**	**.0379**
4	.01944621	.05348801	.09830022	.15071010	**.0587**	**.0709**	**.0827**	**.0941**	**.1052**
5	.05121657	.13158380	.22671091	**.2545**	**.1256**	**.1464**	**.1657**	**.1834**	**.1999**
6	.11122727	.26049913	.41249215	**.4377**	**.2258**	**.2527**	**.2758**	**.2956**	**.3127**
7	.20552978	.42947486	.61651343	**.6224**	**.3367**	**.3623**	**.3825**	**.3985**	**.4117**
8	.33126647	.60843250	.78924085	**.7682**	**.4416**	**.4612**	**.4758**	**.4871**	**.4963**
9	.47563082	.76374808	.90354025	**.8641**	**.5333**	**.5468**	**.5569**	**.5647**	**.5711**
10	.61999516	.87552580	.96340113	**.9224**	**.6092**	**.6183**	**.6251**	**.6304**	**.6347**
11	.74686080	.94290599	.98849369	**.9577**	**.6685**	**.6743**	**.6786**	**.6818**	**.6844**
12	.84553408	.97721923	.99699731	**.9785**	**.7122**	**.7156**	**.7180**	**.7198**	**.7211**
13	.91384634	.99208894	.99934774	**.9900**	**.7449**	**.7466**	**.7478**	**.7487**	**.7493**
14	.95613489	.99760619	.99988168	**.9957**	**.7727**	**.7735**	**.7740**	**.7744**	**.7747**
15	.97962853	.99936799	.99998202	**.9983**	**.8013**	**.8017**	**.8019**	**.8020**	**.8021**
16	.99137534	.99985421	.99999770	**.9994**	**.8338**	**.8340**	**.8340**	**.8341**	**.8341**
17	.99667293	.99997059	**.9996**	**.9998**	**.8698**	**.8698**	**.8699**	**.8699**	**.8699**
18	.99883120	.99999481	**.9999**	**.9999**	**.9061**	**.9061**	**.9061**	**.9061**	**.9061**
19	.99962636	.99999920	**1.0000**	**1.0000**	**.9386**	**.9386**	**.9386**	**.9386**	**.9386**
20	.99989141	.99999989	**1.0000**	**1.0000**	**.9642**	**.9642**	**.9642**	**.9642**	**.9642**
2 40 0	.00001006	.00003015	.00006026	.00010036	.00015044	.00021047	.00028044	*.00036032*	*.00045010*
1	.00014414	.00043064	.00085773	.00142369	.00212681	**.0007**	**.0009**	**.0010**	**.0012**
2	.00101572	.00300355	.00592213	.00973147	**.0035**	**.0044**	**.0054**	**.0063**	**.0072**
3	.00469568	.01357634	.02618435	.04209632	**.0158**	**.0196**	**.0234**	**.0272**	**.0310**
4	.01604224	.04454173	.08260050	.12776221	**.0493**	**.0598**	**.0701**	**.0801**	**.0899**
5	.04327398	.11284527	.19714793	**.2228**	**.1087**	**.1276**	**.1453**	**.1618**	**.1773**
6	.09622459	.23012837	.37127754	**.3966**	**.2019**	**.2280**	**.2507**	**.2705**	**.2879**
7	.18195415	.39047104	.57308854	**.5819**	**.3094**	**.3356**	**.3567**	**.3736**	**.3876**
8	.29983230	.56801242	.75406440	**.7362**	**.4139**	**.4348**	**.4505**	**.4626**	**.4725**
9	.43953973	.72944515	.88130023	**.8422**	**.5070**	**.5217**	**.5326**	**.5411**	**.5481**

Table 32: Sheet J. Exact (in roman), approximate *(in italics)* and interpolated **(boldfaced)** probabilities of perfect ($i = 0$) and i-imperfect column pairs ($k = 2$) in Bernoulli ($m \times n$)-matrices. The rows with all 1's are omitted

k m i	Width n of Bernoulli $(m \times n)$-matrix								
	2	3	4	5	6	7	8	9	10
2 40 10	.58390408	.85139101	.95227829	**.9082**	**.5860**	**.5960**	**.6035**	**.6093**	**.6141**
11	.71514439	.92868172	.98404063	**.9486**	**.6493**	**.6559**	**.6607**	**.6645**	**.6675**
12	.82086576	.97013500	.99555520	**.9731**	**.6970**	**.7009**	**.7038**	**.7059**	**.7076**
13	.89676828	.98908534	.99896652	**.9870**	**.7318**	**.7340**	**.7355**	**.7366**	**.7374**
14	.94556275	.99651532	.99979878	**.9942**	**.7595**	**.7605**	**.7612**	**.7617**	**.7621**
15	.97375512	.99902685	.99996709	**.9976**	**.7858**	**.7862**	**.7865**	**.7867**	**.7868**
16	.98843864	.99976197	.99999547	**.9991**	**.8150**	**.8152**	**.8153**	**.8154**	**.8154**
17	.99534853	.99994894	**.9995**	**.9997**	**.8485**	**.8485**	**.8486**	**.8486**	**.8486**
18	.99829163	.99999039	**.9998**	**.9999**	**.8844**	**.8844**	**.8844**	**.8844**	**.8844**
19	.99942757	.99999841	**.9999**	**1.0000**	**.9191**	**.9191**	**.9191**	**.9192**	**.9192**
20	.99982515	.99999977	**1.0000**	**1.0000**	**.9488**	**.9488**	**.9488**	**.9488**	**.9488**
2 41 0	.00000754	.00002261	.00004520	.00007530	.00011288	.00015794	.00021047	*.00027046*	*.00033788*
1	.00011062	.00033070	.00065908	.00109464	.00163624	**.0006**	**.0007**	**.0008**	**.0010**
2	.00079782	.00236384	.00466980	.00768829	**.0028**	**.0035**	**.0042**	**.0050**	**.0057**
3	.00377569	.01096574	.02124271	.03430101	**.0128**	**.0159**	**.0190**	**.0222**	**.0253**
4	.01320560	.03697710	.06913328	.10778996	**.0413**	**.0503**	**.0592**	**.0678**	**.0764**
5	.03646605	.09639623	.17058120	**.1298**	**.0936**	**.1107**	**.1268**	**.1420**	**.1563**
6	.08298694	.20236892	.33228198	**.2380**	**.1797**	**.2048**	**.2269**	**.2464**	**.2637**
7	.16052176	.35325383	.52964818	**.3629**	**.2832**	**.3099**	**.3316**	**.3494**	**.3642**
8	.27036276	.52765811	.71673292	**.4804**	**.3868**	**.4090**	**.4258**	**.4388**	**.4494**
9	.40461287	.69354228	.85619095	**.5750**	**.4808**	**.4967**	**.5086**	**.5177**	**.5252**
10	.54781299	.82486275	.93888750	**.6433**	**.5621**	**.5731**	**.5813**	**.5877**	**.5930**
11	.68233431	.91223523	.97831222	**.6896**	**.6292**	**.6365**	**.6420**	**.6462**	**.6496**
12	.79443542	.96150514	.99356916	**.7201**	**.6809**	**.6855**	**.6888**	**.6914**	**.6933**
13	.87779265	.98522422	.99840342	**.7412**	**.7188**	**.7214**	**.7233**	**.7246**	**.7256**
14	.93336413	.99503307	.99966719	**.7585**	**.7473**	**.7487**	**.7496**	**.7502**	**.7507**
15	.96670702	.99853604	.99994158	**.7770**	**.7722**	**.7728**	**.7732**	**.7735**	**.7736**
16	.98476776	.99962117	.99999134	**.8002**	**.7984**	**.7987**	**.7988**	**.7989**	**.7990**
17	.99362106	.99991383	**.9992**	**.8293**	**.8287**	**.8288**	**.8289**	**.8289**	**.8289**
18	.99755586	.99998275	**.9997**	**.8631**	**.8630**	**.8630**	**.8630**	**.8630**	**.8630**
19	.99914359	.99999696	**.9999**	**.8984**	**.8983**	**.8983**	**.8983**	**.8983**	**.8983**
20	.99972575	.99999953	**1.0000**	**.9310**	**.9310**	**.9310**	**.9310**	**.9310**	**.9310**
21	.99991981	.99999994	**1.0000**	**.9578**	**.9578**	**.9578**	**.9578**	**.9578**	**.9578**
2 42 0	.00000566	.00001696	.00003391	.00005649	.00008469	.00011851	.00015794	*.00020298*	*.00025360*
1	.00008485	.00025380	.00050608	.00084097	.00125772	**.0005**	**.0006**	**.0007**	**.0008**
2	.00062602	.00185797	.00367659	.00606315	**.0022**	**.0028**	**.0034**	**.0040**	**.0046**
3	.00303122	.00883871	.01718912	.02786303	**.0104**	**.0129**	**.0155**	**.0181**	**.0207**
4	.01084812	.03060764	.05764668	.09053209	**.0348**	**.0424**	**.0500**	**.0575**	**.0649**
5	.03065094	.08203841	.14689208	**.1113**	**.0816**	**.0968**	**.1113**	**.1251**	**.1383**
6	.07135672	.17717716	.29575002	**.2104**	**.1626**	**.1863**	**.2075**	**.2265**	**.2436**
7	.14113806	.31804814	.48671204	**.3305**	**.2647**	**.2915**	**.3137**	**.3321**	**.3476**
8	.24290251	.48777850	.67766930	**.4492**	**.3705**	**.3938**	**.4117**	**.4257**	**.4370**
9	.37105035	.65640369	.82829631	**.5491**	**.4679**	**.4851**	**.4979**	**.5077**	**.5157**
10	.51201296	.79608936	.92305674	**.6241**	**.5524**	**.5644**	**.5733**	**.5803**	**.5860**
11	.64870398	.89350108	.97109074	**.6764**	**.6222**	**.6303**	**.6364**	**.6412**	**.6450**
12	.76641014	.95116548	.99089439	**.7118**	**.6762**	**.6815**	**.6853**	**.6883**	**.6906**
13	.85695334	.98035098	.99759178	**.7364**	**.7160**	**.7191**	**.7213**	**.7230**	**.7242**
14	.91947126	.99305927	.99946383	**.7560**	**.7458**	**.7475**	**.7486**	**.7494**	**.7500**
15	.95837130	.99784536	.99989922	**.7758**	**.7714**	**.7722**	**.7728**	**.7731**	**.7734**
16	.98025257	.99941144	.99998396	**.7997**	**.7981**	**.7984**	**.7986**	**.7988**	**.7989**
17	.99140773	.99985836	**.9989**	**.8291**	**.8286**	**.8287**	**.8288**	**.8289**	**.8289**
18	.99657216	.99996994	**.9996**	**.8630**	**.8629**	**.8630**	**.8630**	**.8630**	**.8630**
19	.99874665	.99999437	**.9999**	**.8983**	**.8983**	**.8983**	**.8983**	**.8983**	**.8983**
20	.99958021	.99999907	**1.0000**	**.9310**	**.9310**	**.9310**	**.9310**	**.9310**	**.9310**
21	.99987129	.99999986	**1.0000**	**.9578**	**.9578**	**.9578**	**.9578**	**.9578**	**.9578**
2 43 0	.00000424	.00001272	.00002544	.00004238	.00006354	.00008892	.00011851	*.00015232*	*.00019032*

Table 32: Sheet K. Exact (in roman), approximate *(in italics)* and interpolated **(boldfaced)** probabilities of perfect ($i = 0$) and i-imperfect column pairs ($k = 2$) in Bernoulli ($m \times n$)-matrices. The rows with all 1's are omitted

k	m	i	Width n of Bernoulli ($m \times n$)-matrix								
			2	3	4	5	6	7	8	9	10
2	43	1	.00006505	.00019466	.00038834	.00064561	.00096598	**.0004**	**.0005**	**.0006**	**.0007**
		2	.00049073	.00145858	.00289046	.00477361	**.0017**	**.0022**	**.0026**	**.0031**	**.0035**
		3	.00242992	.00711034	.01387556	.02256880	**.0084**	**.0104**	**.0125**	**.0146**	**.0168**
		4	.00889390	.02526557	.04790098	.07571835	**.0288**	**.0354**	**.0418**	**.0482**	**.0545**
		5	.02570023	.06957223	.12592158	**.0959**	**.0696**	**.0831**	**.0960**	**.1084**	**.1203**
		6	.06118027	.15446896	.26184291	**.1872**	**.1432**	**.1654**	**.1855**	**.2038**	**.2204**
		7	.12369272	.28501670	.44475276	**.3026**	**.2400**	**.2666**	**.2890**	**.3079**	**.3239**
		8	.21746140	.44874634	.63733477	**.4217**	**.3437**	**.3680**	**.3869**	**.4019**	**.4140**
		9	.33901339	.61841162	.79777788	**.5256**	**.4414**	**.4599**	**.4736**	**.4842**	**.4928**
		10	.47677231	.76527193	.90466142	**.6059**	**.5276**	**.5406**	**.5503**	**.5579**	**.5641**
		11	.61453123	.87246065	.96216018	**.6632**	**.6006**	**.6095**	**.6162**	**.6215**	**.6258**
		12	.73698360	.93897041	.98736759	**.7021**	**.6586**	**.6646**	**.6690**	**.6724**	**.6751**
		13	.83431754	.97430616	.99644876	**.7286**	**.7021**	**.7057**	**.7083**	**.7103**	**.7118**
		14	.90384178	.99048061	.99915744	**.7481**	**.7339**	**.7359**	**.7373**	**.7383**	**.7391**
		15	.94864629	.99689354	.99983083	**.7659**	**.7592**	**.7602**	**.7609**	**.7614**	**.7617**
		16	.97478226	.99910606	.99997118	**.7863**	**.7836**	**.7840**	**.7843**	**.7845**	**.7847**
		17	.98861894	.99977287	**.9985**	**.8118**	**.8109**	**.8111**	**.8112**	**.8113**	**.8113**
		18	.99528105	.99994899	**.9994**	**.8428**	**.8426**	**.8426**	**.8427**	**.8427**	**.8427**
		19	.99820303	.99998986	**.9998**	**.8772**	**.8771**	**.8772**	**.8772**	**.8772**	**.8772**
		20	.99937182	.99999822	**.9999**	**.9114**	**.9114**	**.9114**	**.9114**	**.9114**	**.9114**
		21	.99979852	.99999972	**1.0000**	**.9417**	**.9417**	**.9417**	**.9417**	**.9417**	**.9417**
		22	.99994076	.99999996	**1.0000**	**.9654**	**.9654**	**.9654**	**.9654**	**.9654**	**.9654**
2	44	0	.00000318	.00000954	.00001908	.00003179	.00004767	.00006671	.00008892	*.00011429*	*.00014281*
		1	.00004985	.00014923	.00029782	.00049530	.00074137	**.0003**	**.0004**	**.0005**	**.0006**
		2	.00038431	.00114373	.00226938	.00375258	**.0014**	**.0017**	**.0021**	**.0024**	**.0028**
		3	.00194512	.00570940	.01117561	.01823219	**.0068**	**.0084**	**.0101**	**.0118**	**.0136**
		4	.00727790	.02080163	.03967267	.06308020	**.0239**	**.0294**	**.0348**	**.0402**	**.0456**
		5	.02149865	.05880215	.10748296	**.0824**	**.0592**	**.0710**	**.0824**	**.0935**	**.1041**
		6	.05231026	.13412873	.23064557	**.1660**	**.1256**	**.1462**	**.1651**	**.1824**	**.1984**
		7	.10806461	.25426372	.40418414	**.2762**	**.2168**	**.2429**	**.2652**	**.2843**	**.3008**
		8	.19401923	.41089066	.59621016	**.3947**	**.3179**	**.3431**	**.3629**	**.3787**	**.3917**
		9	.30862539	.57995370	.76486980	**.5019**	**.4153**	**.4351**	**.4498**	**.4612**	**.4704**
		10	.44233258	.73265756	.88363327	**.5871**	**.5028**	**.5169**	**.5273**	**.5354**	**.5421**
		11	.58009150	.84914473	.95131568	**.6492**	**.5783**	**.5881**	**.5954**	**.6012**	**.6060**
		12	.70637051	.92479970	.98281003	**.6921**	**.6401**	**.6467**	**.6516**	**.6555**	**.6586**
		13	.80998405	.96693043	.99487422	**.7209**	**.6874**	**.6915**	**.6946**	**.6969**	**.6988**
		14	.88646072	.98717207	.99870679	**.7411**	**.7219**	**.7244**	**.7261**	**.7273**	**.7283**
		15	.93744516	.99560754	.99972325	**.7577**	**.7480**	**.7493**	**.7502**	**.7508**	**.7512**
		16	.96824827	.99867092	.99994963	**.7752**	**.7710**	**.7716**	**.7720**	**.7722**	**.7724**
		17	.98515977	.99964420	**.9980**	**.7970**	**.7954**	**.7957**	**.7958**	**.7959**	**.7960**
		18	.99361553	.99991563	**.9992**	**.8244**	**.8239**	**.8240**	**.8241**	**.8241**	**.8241**
		19	.99747254	.99998226	**.9997**	**.8566**	**.8565**	**.8565**	**.8566**	**.8566**	**.8566**
		20	.99907962	.99999669	**.9999**	**.8909**	**.8909**	**.8909**	**.8909**	**.8909**	**.8909**
		21	.99969185	.99999945	**1.0000**	**.9236**	**.9235**	**.9236**	**.9236**	**.9236**	**.9236**
		22	.99990520	.99999992	**1.0000**	**.9512**	**.9512**	**.9512**	**.9512**	**.9512**	**.9512**
2	45	0	.00000239	.00000716	.00001431	.00002385	.00003576	.00005005	.00006671	*.00008575*	*.00010716*
		1	.00003818	.00011434	.00022827	.00037975	.00056860	**.0002**	**.0003**	**.0003**	**.0004**
		2	.00030070	.00089588	.00177953	.00294577	**.0011**	**.0014**	**.0016**	**.0019**	**.0022**
		3	.00155492	.00457653	.00898216	.01469276	**.0054**	**.0068**	**.0081**	**.0095**	**.0109**
		4	.00594471	.01708429	.03275696	.05235889	**.0197**	**.0243**	**.0289**	**.0335**	**.0380**
		5	.01794346	.04954073	.09137261	**.0702**	**.0501**	**.0604**	**.0704**	**.0802**	**.0897**
		6	.04460736	.11601794	.20217559	**.1456**	**.1098**	**.1286**	**.1461**	**.1624**	**.1775**
		7	.09412602	.22584021	.36535351	**.2494**	**.1951**	**.2204**	**.2424**	**.2615**	**.2783**
		8	.17253058	.37449184	.55477775	**.3658**	**.2931**	**.3188**	**.3395**	**.3561**	**.3699**
		9	.27997385	.54141112	.72987004	**.4753**	**.3899**	**.4109**	**.4267**	**.4389**	**.4487**
		10	.40890578	.69853094	.85996665	**.5654**	**.4781**	**.4933**	**.5045**	**.5133**	**.5204**

Table 32: Sheet L. Exact (in roman), approximate *(in italics)* and interpolated **(boldfaced)** probabilities of perfect ($i = 0$) and i-imperfect column pairs ($k = 2$) in Bernoulli ($m \times n$)-matrices. The rows with all 1's are omitted

k m i	Width n of Bernoulli ($m \times n$)-matrix								
	2	3	4	5	6	7	8	9	10
2 45 11	.54565177	.82363404	.93837360	**.6328**	**.5556**	**.5662**	**.5742**	**.5805**	**.5857**
12	.67480076	.90856460	.97703220	**.6802**	**.6206**	**.6279**	**.6334**	**.6377**	**.6412**
13	.78408067	.95807016	.99275029	**.7124**	**.6717**	**.6765**	**.6800**	**.6827**	**.6849**
14	.86734155	.98299918	.99805896	**.7342**	**.7096**	**.7125**	**.7145**	**.7160**	**.7172**
15	.92469905	.99390228	.99955817	**.7505**	**.7374**	**.7389**	**.7400**	**.7408**	**.7413**
16	.96054749	.99806343	.99991428	**.7661**	**.7599**	**.7606**	**.7612**	**.7615**	**.7618**
17	.98093190	.99945479	**.9973**	**.7846**	**.7820**	**.7824**	**.7826**	**.7827**	**.7828**
18	.99150159	.99986377	**.9989**	**.8083**	**.8073**	**.8075**	**.8076**	**.8076**	**.8077**
19	.99650828	.99996976	**.9996**	**.8374**	**.8372**	**.8372**	**.8372**	**.8372**	**.8373**
20	.99867785	.99999403	**.9999**	**.8704**	**.8703**	**.8703**	**.8703**	**.8704**	**.8704**
21	.99953879	.99999895	**1.0000**	**.9040**	**.9040**	**.9040**	**.9040**	**.9040**	**.9040**
22	.99985186	.99999984	**1.0000**	**.9346**	**.9346**	**.9346**	**.9346**	**.9346**	**.9346**
2 46 0	.00000179	.00000537	.00001073	.00001789	.00002682	.00003754	.00005005	*.00006433*	*.00008040*
1	.00002923	.00008757	.00017486	.00029099	.00043581	**.0002**	**.0002**	**.0003**	**.0003**
2	.00023507	.00070102	.00139379	.00230939	**.0009**	**.0011**	**.0013**	**.0016**	**.0018**
3	.00124136	.00366244	.00720509	.01181350	**.0043**	**.0054**	**.0065**	**.0077**	**.0088**
4	.00484726	.01399864	.02696889	.04331100	**.0163**	**.0202**	**.0240**	**.0278**	**.0317**
5	.01494377	.04161149	.07737926	**.0596**	**.0428**	**.0517**	**.0604**	**.0689**	**.0772**
6	.03794138	.09998260	.17639354	**.1272**	**.0970**	**.1141**	**.1302**	**.1452**	**.1594**
7	.08174636	.19975027	.32853809	**.2242**	**.1783**	**.2026**	**.2240**	**.2429**	**.2596**
8	.15292944	.33977907	.51350448	**.3376**	**.2757**	**.3017**	**.3229**	**.3403**	**.3548**
9	.25311303	.50314806	.69312893	**.4486**	**.3746**	**.3968**	**.4136**	**.4267**	**.4372**
10	.37667280	.66320481	.83372177	**.5430**	**.4659**	**.4823**	**.4944**	**.5038**	**.5114**
11	.51146527	.79605763	.92318119	**.6156**	**.5462**	**.5578**	**.5664**	**.5732**	**.5788**
12	.64251351	.89021276	.96983975	**.6680**	**.6137**	**.6217**	**.6278**	**.6326**	**.6365**
13	.75676069	.94758325	.98994200	**.7042**	**.6670**	**.6723**	**.6764**	**.6795**	**.6819**
14	.84652633	.97782109	.99714775	**.7291**	**.7066**	**.7099**	**.7124**	**.7142**	**.7156**
15	.91035968	.99168094	.99931080	**.7477**	**.7357**	**.7375**	**.7389**	**.7398**	**.7405**
16	.95158538	.99723160	.99985775	**.7647**	**.7590**	**.7599**	**.7606**	**.7611**	**.7614**
17	.97583579	.99918177	**.9964**	**.7839**	**.7816**	**.7820**	**.7823**	**.7825**	**.7827**
18	.98885916	.99978497	**.9985**	**.8080**	**.8072**	**.8074**	**.8075**	**.8076**	**.8076**
19	.99525661	.99994970	**.9994**	**.8373**	**.8371**	**.8372**	**.8372**	**.8372**	**.8372**
20	.99813546	.99998952	**.9998**	**.8704**	**.8703**	**.8703**	**.8703**	**.8703**	**.8704**
21	.99932356	.99999805	**.9999**	**.9040**	**.9040**	**.9040**	**.9040**	**.9040**	**.9040**
22	.99977359	.99999968	**1.0000**	**.9346**	**.9346**	**.9346**	**.9346**	**.9346**	**.9346**
23	.99993013	.99999995	**1.0000**	**.9595**	**.9595**	**.9595**	**.9595**	**.9595**	**.9595**
2 47 0	.00000134	.00000403	.00000805	.00001342	.00002012	.00002816	.00003754	*.00004826*	*.00006032*
1	.00002237	.00006703	.00013389	.00022285	.00033384	**.0002**	**.0002**	**.0002**	**.0003**
2	.00018361	.00054801	.00109046	.00180827	**.0007**	**.0009**	**.0011**	**.0013**	**.0015**
3	.00098979	.00292638	.00576900	.00947839	**.0035**	**.0044**	**.0052**	**.0061**	**.0071**
4	.00394579	.01144506	.02214353	.03571203	**.0134**	**.0166**	**.0197**	**.0229**	**.0261**
5	.01241965	.03485070	.06529145	**.0504**	**.0359**	**.0436**	**.0511**	**.0585**	**.0658**
6	.03219198	.08585992	.15321352	**.1107**	**.0839**	**.0993**	**.1139**	**.1277**	**.1408**
7	.07079511	.17595796	.29394494	**.2007**	**.1588**	**.1819**	**.2025**	**.2209**	**.2375**
8	.13513367	.30693008	.47282756	**.3101**	**.2520**	**.2781**	**.2997**	**.3177**	**.3328**
9	.22806713	.46550283	.65503547	**.4215**	**.3494**	**.3726**	**.3904**	**.4044**	**.4157**
10	.34578286	.62700984	.80502467	**.5194**	**.4409**	**.4585**	**.4715**	**.4815**	**.4897**
11	.47776715	.76658933	.90562558	**.5969**	**.5227**	**.5352**	**.5445**	**.5518**	**.5578**
12	.60975145	.86973189	.96104066	**.6539**	**.5929**	**.6017**	**.6083**	**.6135**	**.6179**
13	.72819889	.93534499	.98629893	**.6939**	**.6499**	**.6559**	**.6604**	**.6639**	**.6667**
14	.82408492	.97149435	.99589243	**.7214**	**.6932**	**.6970**	**.6998**	**.7019**	**.7036**
15	.89440134	.98883581	.99894837	**.7408**	**.7247**	**.7270**	**.7286**	**.7297**	**.7306**
16	.94127896	.99611333	.99976955	**.7568**	**.7487**	**.7499**	**.7507**	**.7513**	**.7517**
17	.96977319	.99879604	**.9954**	**.7735**	**.7699**	**.7705**	**.7709**	**.7711**	**.7713**
18	.98560332	.99966779	**.9980**	**.7941**	**.7927**	**.7929**	**.7931**	**.7932**	**.7933**
19	.99365725	.99991826	**.9992**	**.8199**	**.8195**	**.8196**	**.8196**	**.8197**	**.8197**

Table 32: Sheet M. Exact (in roman), approximate *(in italics)* and interpolated **(boldfaced)** probabilities of perfect ($i = 0$) and i-imperfect column pairs ($k = 2$) in Bernoulli ($m \times n$)-matrices. The rows with all 1's are omitted

k m i	Width n of Bernoulli $(m \times n)$-matrix								
	2	3	4	5	6	7	8	9	10
2 47 20	.99741575	.99998205	**.9997**	**.8506**	**.8505**	**.8505**	**.8505**	**.8506**	**.8506**
21	.99902653	.99999648	**.9999**	**.8838**	**.8838**	**.8838**	**.8838**	**.8838**	**.8838**
22	.99966108	.99999938	**1.0000**	**.9162**	**.9162**	**.9162**	**.9162**	**.9162**	**.9162**
23	.99989099	.99999990	**1.0000**	**.9446**	**.9446**	**.9446**	**.9446**	**.9446**	**.9446**
2 48 0	.00000101	.00000302	.00000604	.00001006	.00001509	.00002113	.00002816	*.00003620*	*.00004525*
1	.00001712	.00005129	.00010246	.00017058	.00025559	**.0001**	**.0002**	**.0002**	**.0002**
2	.00014330	.00042801	.00085228	.00141429	**.0006**	**.0007**	**.0009**	**.0011**	**.0012**
3	.00078825	.00233482	.00461121	.00758989	**.0028**	**.0035**	**.0042**	**.0049**	**.0057**
4	.00320679	.00933779	.01813535	.02935831	**.0109**	**.0136**	**.0162**	**.0189**	**.0215**
5	.01030118	.02910848	.05490320	**.0426**	**.0301**	**.0367**	**.0431**	**.0495**	**.0558**
6	.02724890	.07348380	.13251364	**.0962**	**.0724**	**.0861**	**.0992**	**.1118**	**.1238**
7	.06114433	.15439438	.26171377	**.1793**	**.1409**	**.1626**	**.1823**	**.2000**	**.2162**
8	.11904903	.27607267	.43314285	**.2843**	**.2296**	**.2554**	**.2772**	**.2956**	**.3112**
9	.20483377	.42878084	.61600278	**.3952**	**.3250**	**.3491**	**.3678**	**.3827**	**.3947**
10	.31635393	.59028447	.77406407	**.4960**	**.4163**	**.4352**	**.4491**	**.4598**	**.4685**
11	.44477108	.73544252	.88564115	**.5778**	**.4991**	**.5126**	**.5226**	**.5304**	**.5368**
12	.57675538	.84715178	.95045310	**.6394**	**.5715**	**.5810**	**.5882**	**.5939**	**.5986**
13	.69858703	.92125356	.98165809	**.6833**	**.6318**	**.6384**	**.6434**	**.6474**	**.6506**
14	.80011342	.96387719	.99419696	**.7135**	**.6788**	**.6832**	**.6864**	**.6889**	**.6908**
15	.87682224	.98524985	.99842865	**.7343**	**.7135**	**.7161**	**.7180**	**.7194**	**.7205**
16	.92955955	.99463607	.99963508	**.7500**	**.7389**	**.7403**	**.7413**	**.7421**	**.7426**
17	.96264963	.99826129	**.9941**	**.7649**	**.7596**	**.7603**	**.7608**	**.7612**	**.7614**
18	.98164579	.99949709	**.9973**	**.7824**	**.7802**	**.7805**	**.7807**	**.7809**	**.7810**
19	.99164377	.99987006	**.9989**	**.8047**	**.8039**	**.8040**	**.8041**	**.8042**	**.8042**
20	.99647612	.99996998	**.9996**	**.8323**	**.8321**	**.8321**	**.8322**	**.8322**	**.8322**
21	.99862384	.99999379	**.9998**	**.8639**	**.8639**	**.8639**	**.8639**	**.8639**	**.8639**
22	.99950245	.99999885	**.9999**	**.8968**	**.8968**	**.8968**	**.8968**	**.8968**	**.8968**
23	.99983352	.99999981	**1.0000**	**.9276**	**.9276**	**.9276**	**.9276**	**.9276**	**.9276**
24	.99994847	.99999997	**1.0000**	**.9534**	**.9534**	**.9534**	**.9534**	**.9534**	**.9534**

Table 32: Sheet N. Exact (in roman), approximate *(in italics)* and interpolated **(boldfaced)** probabilities of perfect ($i = 0$) and i-imperfect column pairs ($k = 2$) in Bernoulli ($m \times n$)-matrices. The rows with all 1's are omitted

k m i	Width n of Bernoulli ($m \times n$)-matrix								
	11	12	13	14	15	16	17	18	19
2 1 0	.99951172	.99975586	.99987793	.99993896	.99996948	.99998474	.99999237	.99999619	.99999809
2 2 0	.99902368	.99951178	.99975587	.99987793	.99993897	.99996948	.99998474	.99999237	.99999619
1	.99999976	.99999994	.99999999	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
2 3 0	.99810874	.99904664	.99952042	.99975912	.99987915	.99993942	.99996965	.99998481	.99999239
1	.99999928	.99999982	.99999996	.99999999	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
2	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
2 4 0	.99581803	.99781157	.99886225	.99941173	.99969725	.99984481	.99992072	.99995961	.99997948
1	.99999585	.99999879	.99999964	.99999989	.99999997	.99999999	1.00000000	1.00000000	1.00000000
2	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
3	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
2 5 0	.99038813	.99463956	.99704029	.99838020	.99912043	.99952571	.99974585	.99986458	.99992821
1	.99998167	.99999429	.99999823	.99999945	.99999983	.99999995	.99999998	1.00000000	1.00000000
2	.99999999	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
3	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
4	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
2 6 0	.97904511	.98741265	.99252615	.99560849	.99744394	.99852504	.99915557	.99952004	.99972902
1	.99992615	.99997487	.99999154	.99999717	.99999906	.99999969	.99999990	.99999997	.99999999
2	.99999993	.99999999	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
3	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
4	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
5	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
2 7 0	.95832546	.97304482	.98277197	.98910788	.99318181	.99577088	.99739892	.99841275	.99903849
1	.99973294	.99989890	.99996222	.99998604	.99999489	.99999815	.99999933	.99999976	.99999992
2	.99999959	.99999992	.99999998	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
3	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
4	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
5	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
6	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
2 8 0	**.9131**	**.9350**	**.9509**	**.9622**	**.9703**	**.9757**	**.9799**	**.9832**	**.9944**
1	**.9787**	**.9846**	**.9887**	**.9914**	**.9933**	**.9944**	**.9953**	**.9960**	**.9994**
2	**.9993**	**.9996**	**.9998**	**.9999**	**.9999**	**.9999**	**1.0000**	**1.0000**	**1.0000**
3	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
4	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
5	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
6	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
2 9 0	**.8570**	**.8854**	**.9068**	**.9228**	**.9348**	**.9426**	**.9497**	**.9558**	**.9880**
1	**.9625**	**.9708**	**.9767**	**.9808**	**.9838**	**.9854**	**.9870**	**.9884**	**.9986**
2	**.9978**	**.9986**	**.9991**	**.9994**	**.9996**	**.9996**	**.9997**	**.9998**	**1.0000**
3	**.9999**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
4	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
5	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
6	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
7	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
2 10 0	**.7887**	**.8226**	**.8489**	**.8695**	**.8855**	**.8957**	**.9060**	**.9154**	**.9781**
1	**.9391**	**.9501**	**.9585**	**.9646**	**.9691**	**.9714**	**.9741**	**.9764**	**.9974**
2	**.9945**	**.9962**	**.9973**	**.9980**	**.9984**	**.9986**	**.9989**	**.9990**	**1.0000**
3	**.9997**	**.9998**	**.9999**	**.9999**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
4	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
5	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
6	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
7	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
2 11 0	**.7132**	**.7512**	**.7817**	**.8063**	**.8259**	**.8381**	**.8517**	**.8646**	**.9646**
1	**.9115**	**.9250**	**.9353**	**.9431**	**.9490**	**.9522**	**.9559**	**.9595**	**.9954**
2	**.9888**	**.9917**	**.9936**	**.9949**	**.9958**	**.9962**	**.9967**	**.9972**	**1.0000**
3	**.9991**	**.9994**	**.9996**	**.9997**	**.9998**	**.9998**	**.9998**	**.9999**	**1.0000**
4	**.9999**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**

Table 32: Sheet O. Exact (in roman), approximate *(in italics)* and interpolated **(boldfaced)** probabilities of perfect ($i = 0$) and i-imperfect column pairs ($k = 2$) in Bernoulli ($m \times n$)-matrices. The rows with all 1's are omitted

k m i	Width n of Bernoulli ($m \times n$)-matrix								
	11	12	13	14	15	16	17	18	19
2 11 5	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
6	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
7	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
8	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
2 12 0	**.6355**	**.6760**	**.7094**	**.7371**	**.7598**	**.7737**	**.7903**	**.8065**	**.9483**
1	**.8797**	**.8959**	**.9085**	**.9182**	**.9258**	**.9299**	**.9350**	**.9399**	**.9922**
2	**.9803**	**.9848**	**.9878**	**.9900**	**.9915**	**.9922**	**.9932**	**.9941**	**.9999**
3	**.9977**	**.9985**	**.9989**	**.9992**	**.9994**	**.9994**	**.9995**	**.9996**	**1.0000**
4	**.9998**	**.9999**	**.9999**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
5	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
6	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
7	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
8	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
2 13 0	**.5588**	**.6005**	**.6356**	**.6655**	**.6906**	**.7058**	**.7249**	**.7440**	**.9298**
1	**.8441**	**.8629**	**.8778**	**.8897**	**.8992**	**.9045**	**.9110**	**.9175**	**.9873**
2	**.9687**	**.9750**	**.9795**	**.9828**	**.9852**	**.9863**	**.9878**	**.9893**	**.9998**
3	**.9954**	**.9967**	**.9976**	**.9981**	**.9985**	**.9987**	**.9989**	**.9991**	**1.0000**
4	**.9995**	**.9997**	**.9998**	**.9999**	**.9999**	**.9999**	**.9999**	**.9999**	**1.0000**
5	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
6	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
7	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
8	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
9	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
2 14 0	**.4857**	**.5273**	**.5632**	**.5943**	**.6209**	**.6372**	**.6581**	**.6797**	**.9094**
1	**.8054**	**.8266**	**.8438**	**.8578**	**.8691**	**.8757**	**.8839**	**.8920**	**.9794**
2	**.9536**	**.9621**	**.9683**	**.9729**	**.9764**	**.9780**	**.9803**	**.9826**	**.9995**
3	**.9915**	**.9937**	**.9952**	**.9962**	**.9969**	**.9972**	**.9976**	**.9980**	**1.0000**
4	**.9988**	**.9992**	**.9995**	**.9996**	**.9997**	**.9997**	**.9998**	**.9998**	**1.0000**
5	**.9999**	**.9999**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
6	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
7	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
8	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
9	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
2 15 0	**.4183**	**.4587**	**.4943**	**.5257**	**.5532**	**.5700**	**.5923**	**.6156**	**.8870**
1	**.7329**	**.7574**	**.7781**	**.8230**	**.8362**	**.8441**	**.8539**	**.8638**	**.9686**
2	**.9131**	**.9262**	**.9362**	**.9603**	**.9650**	**.9673**	**.9705**	**.9738**	**.9990**
3	**.9771**	**.9816**	**.9848**	**.9931**	**.9943**	**.9948**	**.9955**	**.9962**	**1.0000**
4	**.9950**	**.9961**	**.9969**	**.9991**	**.9993**	**.9994**	**.9995**	**.9996**	**1.0000**
5	**.9991**	**.9993**	**.9995**	**.9999**	**.9999**	**.9999**	**1.0000**	**1.0000**	**1.0000**
6	**.9999**	**.9999**	**.9999**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
7	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
8	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
9	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
10	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
2 16 0	**.3574**	**.3959**	**.4304**	**.4614**	**.4890**	**.5062**	**.5291**	**.5536**	**.8629**
1	**.6861**	**.7115**	**.7333**	**.7521**	**.7683**	**.7782**	**.8192**	**.8310**	**.9543**
2	**.8860**	**.9016**	**.9138**	**.9236**	**.9313**	**.9351**	**.9578**	**.9623**	**.9982**
3	**.9666**	**.9727**	**.9771**	**.9805**	**.9830**	**.9842**	**.9922**	**.9934**	**.9999**
4	**.9920**	**.9937**	**.9950**	**.9958**	**.9965**	**.9967**	**.9989**	**.9991**	**1.0000**
5	**.9985**	**.9988**	**.9991**	**.9993**	**.9994**	**.9994**	**.9999**	**.9999**	**1.0000**
6	**.9998**	**.9998**	**.9999**	**.9999**	**.9999**	**.9999**	**1.0000**	**1.0000**	**1.0000**
7	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
8	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
9	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
10	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
2 17 0	**.3039**	**.3398**	**.3726**	**.4026**	**.4296**	**.4469**	**.4700**	**.4949**	**.8369**

Table 32: Sheet P. Exact (in roman), approximate *(in italics)* and interpolated **(boldfaced)** probabilities of perfect ($i = 0$) and i-imperfect column pairs ($k = 2$) in Bernoulli ($m \times n$)-matrices. The rows with all 1's are omitted

k	m	i	Width n of Bernoulli ($m \times n$)-matrix								
			11	12	13	14	15	16	17	18	19
2	17	1	.6371	.6574	.6855	.7053	.7226	.7337	.7477	.7624	.9356
		2	.8523	.8642	.8858	.8977	.9074	.9124	.9195	.9269	.9967
		3	.9511	.9546	.9656	.9703	.9739	.9756	.9782	.9808	.9999
		4	.9864	.9866	.9913	.9927	.9938	.9942	.9950	.9957	1.0000
		5	.9969	.9965	.9981	.9985	.9987	.9988	.9990	.9991	1.0000
		6	.9994	.9992	.9997	.9997	.9998	.9998	.9998	.9998	1.0000
		7	.9999	.9999	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
		8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
		9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
		10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
		11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	18	0	.2576	.2906	.3213	.3498	.3758	.3930	.4157	.4405	.8090
		1	.5867	.6116	.6343	.6570	.6771	.6874	.7024	.7182	.9129
		2	.8113	.8321	.8498	.8671	.8813	.8859	.8947	.9039	.9941
		3	.9290	.9395	.9480	.9566	.9633	.9644	.9680	.9717	.9998
		4	.9767	.9809	.9842	.9878	.9904	.9905	.9917	.9928	1.0000
		5	.9935	.9948	.9958	.9971	.9979	.9978	.9981	.9984	1.0000
		6	.9984	.9988	.9990	.9994	.9996	.9996	.9996	.9997	1.0000
		7	.9997	.9998	.9998	.9999	.9999	.9999	.9999	.9999	1.0000
		8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
		9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
		10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
		11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	19	0	.2182	.2484	.2768	.3034	.3280	.3449	.3669	.3910	.7793
		1	.5432	.5668	.5884	.6081	.6260	.6390	.6563	.6750	.8867
		2	.7738	.7962	.8153	.8314	.8451	.8539	.8668	.8804	.9899
		3	.9088	.9212	.9310	.9389	.9453	.9492	.9557	.9623	.9996
		4	.9678	.9733	.9774	.9806	.9830	.9846	.9873	.9900	1.0000
		5	.9904	.9923	.9937	.9947	.9954	.9960	.9969	.9977	1.0000
		6	.9976	.9981	.9985	.9987	.9989	.9991	.9993	.9996	1.0000
		7	.9995	.9996	.9997	.9997	.9998	.9998	.9999	.9999	1.0000
		8	.9999	.9999	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
		9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
		10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
		11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
		12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	20	0	.1853	.2126	.2386	.2632	.2863	.3026	.3237	.3469	.7477
		1	.5017	.5235	.5439	.5628	.5813	.5949	.6123	.6312	.8574
		2	.7322	.7561	.7767	.7946	.8116	.8227	.8375	.8532	.9834
		3	.8835	.8980	.9098	.9195	.9288	.9343	.9422	.9504	.9993
		4	.9551	.9621	.9676	.9719	.9761	.9786	.9822	.9857	1.0000
		5	.9852	.9879	.9900	.9915	.9931	.9941	.9954	.9966	1.0000
		6	.9959	.9967	.9973	.9978	.9983	.9986	.9990	.9993	1.0000
		7	.9990	.9992	.9994	.9995	.9996	.9997	.9998	.9999	1.0000
		8	.9998	.9999	.9999	.9999	.9999	1.0000	1.0000	1.0000	1.0000
		9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
		10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
		11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
		12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	21	0	*.10656303*	.1827	.2063	.2290	.2503	.2663	.2860	.3079	.7143
		1	.4640	.4838	.5026	.5209	.5397	.5530	.5700	.5872	.8260
		2	.6897	.7143	.7360	.7563	.7765	.7893	.8058	.8214	.9740
		3	.8555	.8721	.8858	.8982	.9107	.9175	.9269	.9353	.9986
		4	.9399	.9486	.9555	.9617	.9681	.9714	.9759	.9796	.9999
		5	.9783	.9821	.9849	.9875	.9903	.9916	.9934	.9946	1.0000
		6	.9933	.9946	.9956	.9965	.9975	.9979	.9985	.9988	1.0000

Table 32: Sheet Q. Exact (in roman), *approximate (in italics)* and interpolated **(boldfaced)** probabilities of perfect ($i = 0$) and i-imperfect column pairs ($k = 2$) in Bernoulli ($m \times n$)-matrices. The rows with all 1's are omitted

k	m	i	Width n of Bernoulli ($m \times n$)-matrix								
			11	12	13	14	15	16	17	18	19
2	21	7	.9982	.9986	.9989	.9991	.9994	.9996	.9997	.9998	1.0000
		8	.9996	.9997	.9998	.9998	.9999	.9999	1.0000	1.0000	1.0000
		9	.9999	.9999	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
		10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
		11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
		12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
		13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	22	0	*.08284786*	*.09699770*	.1794	.2001	.2199	.2351	.2535	.2740	.6794
		1	.4213	.4480	.4650	.4835	.5011	.5139	.5286	.5442	.7935
		2	.6471	.6717	.6941	.7182	.7396	.7539	.7695	.7853	.9612
		3	.8250	.8433	.8590	.8762	.8905	.8987	.9077	.9167	.9975
		4	.9222	.9325	.9410	.9508	.9585	.9626	.9670	.9711	.9999
		5	.9695	.9744	.9783	.9831	.9866	.9883	.9901	.9916	1.0000
		6	.9897	.9917	.9931	.9950	.9963	.9969	.9975	.9979	1.0000
		7	.9970	.9976	.9981	.9987	.9991	.9993	.9995	.9996	1.0000
		8	.9993	.9994	.9995	.9997	.9998	.9999	.9999	.9999	1.0000
		9	.9998	.9999	.9999	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
		10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
		11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
		12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
		13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	23	0	*.06398035*	*.07531109*	.1573	.1762	.1943	.2089	.2259	.2448	.6431
		1	.1990	.3780	.4318	.4462	.4600	.4718	.4868	.5040	.7616
		2	.4542	.6306	.6527	.6728	.6911	.7064	.7256	.7466	.9452
		3	.6965	.8139	.8309	.8456	.8581	.8681	.8813	.8955	.9954
		4	.8514	.9156	.9253	.9334	.9402	.9457	.9531	.9609	.9998
		5	.9364	.9660	.9709	.9747	.9779	.9806	.9841	.9876	1.0000
		6	.9769	.9883	.9902	.9918	.9929	.9940	.9954	.9967	1.0000
		7	.9929	.9965	.9972	.9977	.9980	.9984	.9989	.9992	1.0000
		8	.9982	.9991	.9993	.9994	.9995	.9996	.9998	.9999	1.0000
		9	.9996	.9998	.9999	.9999	.9999	.9999	1.0000	1.0000	1.0000
		10	.9999	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
		11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
		12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
		13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
		14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	24	0	*.04915022*	*.05808432*	*.06746975*	.1372	.1514	.1636	.1769	.1911	.6057
		1	.1258	.1391	.1521	.1648	.1789	.1900	.2039	.2184	.7211
		2	.3164	.3392	.3606	.3805	.4023	.4187	.4385	.4576	.9215
		3	.5100	.5309	.5493	.5654	.5831	.5951	.6098	.6233	.9910
		4	.6327	.6460	.6572	.6667	.6776	.6844	.6931	.7006	.9993
		5	.7008	.7081	.7142	.7192	.7252	.7288	.7332	.7368	.9999
		6	.7395	.7431	.7460	.7483	.7512	.7529	.7550	.7566	1.0000
		7	.7668	.7685	.7700	.7712	.7727	.7736	.7747	.7758	1.0000
		8	.7959	.7970	.7981	.7991	.8001	.8008	.8018	.8030	1.0000
		9	.8329	.8339	.8349	.8359	.8370	.8377	.8388	.8401	1.0000
		10	.8763	.8772	.8782	.8792	.8802	.8809	.8820	.8833	1.0000
		11	.9194	.9202	.9210	.9219	.9227	.9233	.9242	.9252	1.0000
		12	.9550	.9556	.9561	.9567	.9572	.9576	.9582	.9589	1.0000
		13	.9790	.9793	.9796	.9799	.9802	.9804	.9807	.9811	1.0000
		14	.9921	.9922	.9923	.9924	.9925	.9926	.9927	.9929	1.0000
2	25	0	*.03759827*	*.04456681*	*.05196082*	*.05969066*	.1387	.1504	.1629	.1761	.5676
		1	.1040	.1153	.1265	.1390	.1518	.1621	.1732	.1854	.6887
		2	.2784	.2998	.3200	.3424	.3643	.3813	.3982	.4160	.8979
		3	.4744	.4961	.5155	.5366	.5562	.5700	.5831	.5966	.9855
		4	.6083	.6229	.6354	.6494	.6621	.6702	.6777	.6855	.9989

Table 32: Sheet R. Exact (in roman), approximate *(in italics)* and interpolated **(boldfaced)** probabilities of perfect ($i = 0$) and i-imperfect column pairs ($k = 2$) in Bernoulli ($m \times n$)-matrices. The rows with all 1's are omitted

k	m	i	Width n of Bernoulli $(m \times n)$-matrix								
			11	12	13	14	15	16	17	18	19
2	25	5	**.6848**	**.6933**	**.7004**	**.7086**	**.7158**	**.7202**	**.7241**	**.7279**	**.9999**
		6	**.7279**	**.7322**	**.7358**	**.7400**	**.7435**	**.7455**	**.7473**	**.7490**	**1.0000**
		7	**.7547**	**.7568**	**.7584**	**.7604**	**.7620**	**.7629**	**.7638**	**.7647**	**1.0000**
		8	**.7791**	**.7802**	**.7811**	**.7822**	**.7831**	**.7837**	**.7844**	**.7853**	**1.0000**
		9	**.8097**	**.8105**	**.8113**	**.8122**	**.8130**	**.8136**	**.8144**	**.8154**	**1.0000**
		10	**.8485**	**.8493**	**.8501**	**.8510**	**.8518**	**.8525**	**.8533**	**.8545**	**1.0000**
		11	**.8917**	**.8924**	**.8932**	**.8940**	**.8948**	**.8954**	**.8962**	**.8973**	**1.0000**
		12	**.9322**	**.9327**	**.9333**	**.9339**	**.9346**	**.9350**	**.9357**	**.9365**	**1.0000**
		13	**.9636**	**.9640**	**.9643**	**.9647**	**.9651**	**.9654**	**.9658**	**.9664**	**1.0000**
		14	**.9837**	**.9839**	**.9841**	**.9843**	**.9845**	**.9846**	**.9848**	**.9851**	**1.0000**
2	26	0	*.02866270*	*.03405490*	*.03981561*	*.04589725*	*.05223295*	**.1394**	**.1512**	**.1635**	**.5292**
		1	**.0859**	**.0954**	**.1049**	**.1143**	**.1236**	**.1324**	**.1434**	**.1566**	**.6723**
		2	**.2441**	**.2637**	**.2824**	**.3003**	**.3174**	**.3335**	**.3530**	**.3752**	**.8788**
		3	**.4397**	**.4617**	**.4816**	**.4996**	**.5158**	**.5306**	**.5484**	**.5682**	**.9798**
		4	**.5843**	**.6000**	**.6135**	**.6254**	**.6357**	**.6451**	**.6568**	**.6697**	**.9985**
		5	**.6698**	**.6794**	**.6875**	**.6944**	**.7003**	**.7058**	**.7126**	**.7198**	**.9999**
		6	**.7197**	**.7249**	**.7292**	**.7327**	**.7357**	**.7385**	**.7419**	**.7454**	**1.0000**
		7	**.7508**	**.7533**	**.7553**	**.7570**	**.7584**	**.7597**	**.7613**	**.7629**	**1.0000**
		8	**.7772**	**.7784**	**.7794**	**.7803**	**.7811**	**.7818**	**.7827**	**.7837**	**1.0000**
		9	**.8085**	**.8092**	**.8099**	**.8106**	**.8113**	**.8119**	**.8126**	**.8136**	**1.0000**
		10	**.8475**	**.8481**	**.8488**	**.8494**	**.8501**	**.8507**	**.8515**	**.8525**	**1.0000**
		11	**.8908**	**.8913**	**.8919**	**.8926**	**.8932**	**.8937**	**.8944**	**.8954**	**1.0000**
		12	**.9314**	**.9319**	**.9323**	**.9328**	**.9333**	**.9337**	**.9343**	**.9350**	**1.0000**
		13	**.9632**	**.9634**	**.9637**	**.9640**	**.9644**	**.9646**	**.9650**	**.9654**	**1.0000**
		14	**.9835**	**.9836**	**.9838**	**.9839**	**.9841**	**.9842**	**.9844**	**.9846**	**1.0000**
		15	**.9940**	**.9940**	**.9941**	**.9942**	**.9942**	**.9943**	**.9943**	**.9944**	**1.0000**
2	27	0	*.02178932*	*.02593646*	*.03038859*	*.03512006*	*.04009562*	**.1303**	**.1414**	**.1530**	**.3710**
		1	**.0703**	**.0782**	**.0861**	**.0941**	**.1030**	**.1113**	**.1211**	**.1313**	**.3633**
		2	**.2114**	**.2291**	**.2462**	**.2626**	**.2810**	**.2977**	**.3166**	**.3348**	**.5815**
		3	**.4026**	**.4246**	**.4447**	**.4631**	**.4831**	**.5004**	**.5194**	**.5367**	**.7107**
		4	**.5560**	**.5729**	**.5876**	**.6005**	**.6148**	**.6265**	**.6396**	**.6509**	**.7440**
		5	**.6501**	**.6608**	**.6699**	**.6778**	**.6868**	**.6939**	**.7018**	**.7082**	**.7492**
		6	**.7051**	**.7112**	**.7163**	**.7205**	**.7255**	**.7293**	**.7332**	**.7363**	**.7516**
		7	**.7364**	**.7394**	**.7418**	**.7437**	**.7460**	**.7477**	**.7494**	**.7506**	**.7556**
		8	**.7570**	**.7582**	**.7592**	**.7599**	**.7608**	**.7614**	**.7621**	**.7625**	**.7644**
		9	**.7777**	**.7782**	**.7785**	**.7787**	**.7790**	**.7792**	**.7794**	**.7796**	**.7810**
		10	**.8054**	**.8056**	**.8057**	**.8057**	**.8058**	**.8059**	**.8060**	**.8060**	**.8078**
		11	**.8419**	**.8419**	**.8419**	**.8420**	**.8420**	**.8421**	**.8421**	**.8421**	**.8441**
		12	**.8837**	**.8837**	**.8837**	**.8838**	**.8838**	**.8838**	**.8838**	**.8839**	**.8859**
		13	**.9244**	**.9244**	**.9244**	**.9244**	**.9245**	**.9245**	**.9245**	**.9245**	**.9262**
		14	**.9575**	**.9575**	**.9576**	**.9576**	**.9576**	**.9576**	**.9576**	**.9576**	**.9587**
		15	**.9799**	**.9799**	**.9799**	**.9799**	**.9799**	**.9799**	**.9799**	**.9799**	**.9805**
2	28	0	*.01652588*	*.01970033*	*.02312036*	*.02677192*	*.03063619*	**.1227**	**.1332**	**.1441**	**.3448**
		1	**.0274**	**.0306**	**.0339**	**.0372**	**.0406**	**.0942**	**.1011**	**.1096**	**.3329**
		2	**.1061**	**.1166**	**.1269**	**.1370**	**.1468**	**.2663**	**.2800**	**.2961**	**.5496**
		3	**.2430**	**.2592**	**.2741**	**.2878**	**.3005**	**.4719**	**.4864**	**.5032**	**.6953**
		4	**.3749**	**.3884**	**.4001**	**.4103**	**.4193**	**.6094**	**.6190**	**.6304**	**.7409**
		5	**.4764**	**.4852**	**.4928**	**.4992**	**.5048**	**.6837**	**.6894**	**.6962**	**.7483**
		6	**.5612**	**.5665**	**.5710**	**.5747**	**.5779**	**.7233**	**.7263**	**.7297**	**.7505**
		7	**.6324**	**.6352**	**.6375**	**.6393**	**.6408**	**.7433**	**.7446**	**.7460**	**.7533**
		8	**.6895**	**.6908**	**.6917**	**.6925**	**.6931**	**.7556**	**.7561**	**.7566**	**.7591**
		9	**.7351**	**.7356**	**.7360**	**.7362**	**.7364**	**.7692**	**.7693**	**.7695**	**.7708**
		10	**.7749**	**.7750**	**.7751**	**.7752**	**.7753**	**.7899**	**.7899**	**.7900**	**.7914**
		11	**.8145**	**.8146**	**.8146**	**.8146**	**.8147**	**.8200**	**.8201**	**.8201**	**.8219**
		12	**.8566**	**.8566**	**.8567**	**.8567**	**.8567**	**.8583**	**.8583**	**.8584**	**.8603**
		13	**.8992**	**.8992**	**.8992**	**.8992**	**.8993**	**.8996**	**.8997**	**.8997**	**.9014**

Table 32: Sheet S. Exact (in roman), approximate *(in italics)* and interpolated **(boldfaced)** probabilities of perfect ($i = 0$) and i-imperfect column pairs ($k = 2$) in Bernoulli ($m \times n$)-matrices. The rows with all 1's are omitted

k m i	Width n of Bernoulli ($m \times n$)-matrix								
	11	12	13	14	15	16	17	18	19
2 28 14	.9372	.9372	.9372	.9373	.9373	.9373	.9374	.9374	.9387
15	.9662	.9662	.9662	.9662	.9662	.9662	.9663	.9663	.9671
16	.9846	.9846	.9846	.9846	.9847	.9847	.9847	.9847	.9851
2 29 0	*.01250996*	*.01493076*	*.01754573*	*.02034711*	*.02332471*	.1098	.1189	.1366	.3193
1	.0215	.0241	.0267	.0294	.0321	.0347	.0375	.0917	.3045
2	.0886	.0977	.1066	.1154	.1240	.1322	.1407	.2619	.5175
3	.2175	.2330	.2475	.2611	.2737	.2849	.2964	.4705	.6770
4	.3536	.3678	.3802	.3912	.4009	.4090	.4174	.6105	.7366
5	.4616	.4713	.4796	.4867	.4929	.4980	.5033	.6845	.7476
6	.5517	.5577	.5628	.5672	.5709	.5738	.5769	.7236	.7504
7	.6269	.6303	.6331	.6355	.6374	.6388	.6403	.7434	.7532
8	.6869	.6885	.6898	.6909	.6917	.6923	.6929	.7556	.7590
9	.7341	.7348	.7353	.7356	.7359	.7362	.7364	.7692	.7707
10	.7745	.7747	.7749	.7750	.7751	.7752	.7753	.7899	.7912
11	.8144	.8145	.8145	.8146	.8146	.8146	.8147	.8200	.8216
12	.8566	.8566	.8566	.8566	.8567	.8567	.8567	.8583	.8601
13	.8992	.8992	.8992	.8992	.8992	.8992	.8993	.8996	.9012
14	.9372	.9372	.9372	.9372	.9373	.9373	.9373	.9374	.9386
15	.9662	.9662	.9662	.9662	.9662	.9662	.9662	.9662	.9670
16	.9846	.9846	.9846	.9846	.9846	.9846	.9847	.9847	.9850
2 30 0	*.00945501*	*.01129552*	*.01328767*	*.01542710*	*.01770822*	.1060	.1148	.1235	.2947
1	.0169	.0189	.0210	.0231	.0252	.0273	.0295	.0320	.2768
2	.0730	.0807	.0884	.0959	.1034	.1105	.1181	.1264	.4842
3	.1905	.2052	.2191	.2321	.2444	.2557	.2671	.2795	.6547
4	.3250	.3398	.3528	.3645	.3749	.3838	.3928	.4028	.7302
5	.4340	.4446	.4536	.4614	.4683	.4740	.4799	.4867	.7462
6	.5251	.5318	.5375	.5424	.5467	.5501	.5538	.5578	.7496
7	.6023	.6063	.6096	.6125	.6148	.6167	.6186	.6206	.7517
8	.6654	.6675	.6692	.6705	.6716	.6724	.6733	.6741	.7555
9	.7149	.7158	.7165	.7170	.7175	.7178	.7181	.7184	.7636
10	.7552	.7555	.7558	.7560	.7561	.7562	.7563	.7564	.7787
11	.7926	.7927	.7928	.7929	.7929	.7930	.7930	.7930	.8030
12	.8319	.8319	.8319	.8320	.8320	.8320	.8320	.8321	.8365
13	.8735	.8735	.8736	.8736	.8736	.8736	.8736	.8736	.8760
14	.9141	.9141	.9141	.9141	.9141	.9141	.9142	.9142	.9157
15	.9485	.9485	.9485	.9485	.9486	.9486	.9486	.9486	.9496
16	.9734	.9734	.9734	.9734	.9734	.9734	.9734	.9734	.9740
17	.9884	.9884	.9884	.9884	.9884	.9884	.9884	.9884	.9887
2 31 0	*.00713676*	*.00853269*	*.01004598*	*.01167414*	*.01341409*	.1026	.1113	.1197	.2712
1	.0132	.0148	.0164	.0180	.0197	.0214	.0232	.0252	.2497
2	.0598	.0663	.0727	.0792	.0855	.0917	.0982	.1054	.4500
3	.1657	.1793	.1923	.2046	.2163	.2273	.2384	.2506	.6286
4	.2972	.3123	.3259	.3380	.3490	.3586	.3682	.3789	.7209
5	.4072	.4186	.4284	.4369	.4444	.4508	.4573	.4648	.7439
6	.4991	.5065	.5128	.5183	.5230	.5271	.5312	.5360	.7487
7	.5776	.5822	.5861	.5895	.5923	.5946	.5969	.5995	.7506
8	.6436	.6462	.6482	.6500	.6514	.6525	.6536	.6547	.7532
9	.6960	.6972	.6981	.6989	.6994	.6999	.7003	.7008	.7587
10	.7374	.7378	.7382	.7385	.7387	.7389	.7390	.7391	.7695
11	.7733	.7735	.7736	.7737	.7738	.7738	.7739	.7739	.7882
12	.8095	.8096	.8096	.8097	.8097	.8097	.8097	.8097	.8161
13	.8487	.8488	.8488	.8488	.8488	.8488	.8488	.8489	.8519
14	.8896	.8896	.8896	.8896	.8896	.8896	.8896	.8896	.8915
15	.9276	.9276	.9276	.9276	.9276	.9276	.9276	.9276	.9289
16	.9582	.9582	.9582	.9582	.9583	.9583	.9583	.9583	.9591
17	.9792	.9792	.9792	.9792	.9792	.9792	.9792	.9793	.9797

Table 32: Sheet T. Exact (in roman), approximate *(in italics)* and interpolated **(boldfaced)** probabilities of perfect ($i = 0$) and i-imperfect column pairs ($k = 2$) in Bernoulli ($m \times n$)-matrices. The rows with all 1's are omitted

k m i	Width n of Bernoulli $(m \times n)$-matrix								
	11	12	13	14	15	16	17	18	19
2 32 0	*.00538110*	*.00643774*	*.00758461*	*.00882028*	*.01014302*	**.0995**	**.1080**	**.1163**	**.1843**
1	**.0102**	**.0115**	**.0128**	**.0140**	**.0154**	**.0167**	**.0181**	**.0196**	**.0828**
2	**.0487**	**.0541**	**.0595**	**.0649**	**.0702**	**.0755**	**.0810**	**.0872**	**.2233**
3	**.1429**	**.1554**	**.1673**	**.1788**	**.1898**	**.2002**	**.2108**	**.2224**	**.3792**
4	**.2702**	**.2855**	**.2993**	**.3118**	**.3232**	**.3333**	**.3435**	**.3547**	**.4803**
5	**.3810**	**.3932**	**.4038**	**.4130**	**.4212**	**.4282**	**.4353**	**.4434**	**.5224**
6	**.4740**	**.4820**	**.4889**	**.4949**	**.5002**	**.5047**	**.5093**	**.5147**	**.5562**
7	**.5532**	**.5584**	**.5628**	**.5666**	**.5699**	**.5727**	**.5755**	**.5786**	**.5973**
8	**.6214**	**.6245**	**.6270**	**.6291**	**.6309**	**.6323**	**.6337**	**.6353**	**.6426**
9	**.6769**	**.6784**	**.6797**	**.6806**	**.6814**	**.6821**	**.6827**	**.6833**	**.6859**
10	**.7203**	**.7210**	**.7215**	**.7219**	**.7222**	**.7224**	**.7226**	**.7228**	**.7240**
11	**.7560**	**.7563**	**.7564**	**.7566**	**.7567**	**.7568**	**.7568**	**.7569**	**.7578**
12	**.7897**	**.7898**	**.7899**	**.7899**	**.7900**	**.7900**	**.7900**	**.7900**	**.7911**
13	**.8258**	**.8258**	**.8259**	**.8259**	**.8259**	**.8259**	**.8259**	**.8259**	**.8272**
14	**.8650**	**.8651**	**.8651**	**.8651**	**.8651**	**.8651**	**.8651**	**.8651**	**.8665**
15	**.9045**	**.9045**	**.9045**	**.9046**	**.9046**	**.9046**	**.9046**	**.9046**	**.9058**
16	**.9396**	**.9396**	**.9396**	**.9396**	**.9396**	**.9396**	**.9396**	**.9396**	**.9406**
17	**.9665**	**.9665**	**.9665**	**.9665**	**.9665**	**.9665**	**.9665**	**.9665**	**.9671**
2 33 0	*.00405369*	*.00485223*	*.00571979*	*.00665553*	*.00765848*	**.0968**	**.1051**	**.1132**	**.1729**
1	**.0080**	**.0089**	**.0099**	**.0109**	**.0119**	**.0130**	**.0141**	**.0153**	**.0701**
2	**.0396**	**.0441**	**.0486**	**.0530**	**.0575**	**.0619**	**.0665**	**.0717**	**.1992**
3	**.1232**	**.1344**	**.1452**	**.1557**	**.1658**	**.1754**	**.1853**	**.1961**	**.3539**
4	**.2469**	**.2620**	**.2759**	**.2885**	**.3002**	**.3107**	**.3212**	**.3327**	**.4682**
5	**.3618**	**.3748**	**.3861**	**.3960**	**.4048**	**.4124**	**.4201**	**.4288**	**.5195**
6	**.4607**	**.4694**	**.4769**	**.4834**	**.4891**	**.4941**	**.4991**	**.5051**	**.5559**
7	**.5440**	**.5498**	**.5547**	**.5590**	**.5628**	**.5659**	**.5692**	**.5729**	**.5972**
8	**.6157**	**.6193**	**.6223**	**.6248**	**.6270**	**.6287**	**.6305**	**.6324**	**.6425**
9	**.6738**	**.6757**	**.6773**	**.6785**	**.6796**	**.6804**	**.6812**	**.6821**	**.6859**
10	**.7189**	**.7198**	**.7205**	**.7210**	**.7214**	**.7218**	**.7221**	**.7224**	**.7239**
11	**.7554**	**.7558**	**.7561**	**.7563**	**.7564**	**.7565**	**.7566**	**.7568**	**.7576**
12	**.7895**	**.7897**	**.7897**	**.7898**	**.7899**	**.7899**	**.7899**	**.7900**	**.7909**
13	**.8257**	**.8258**	**.8258**	**.8258**	**.8259**	**.8259**	**.8259**	**.8259**	**.8269**
14	**.8650**	**.8650**	**.8650**	**.8651**	**.8651**	**.8651**	**.8651**	**.8651**	**.8662**
15	**.9045**	**.9045**	**.9045**	**.9045**	**.9045**	**.9046**	**.9046**	**.9046**	**.9056**
16	**.9396**	**.9396**	**.9396**	**.9396**	**.9396**	**.9396**	**.9396**	**.9396**	**.9404**
17	**.9664**	**.9664**	**.9665**	**.9665**	**.9665**	**.9665**	**.9665**	**.9665**	**.9670**
18	**.9839**	**.9839**	**.9839**	**.9839**	**.9839**	**.9839**	**.9839**	**.9840**	**.9842**
2 34 0	*.00305146*	*.00365415*	*.00430942*	*.00501679*	*.00577571*	*.00658549*	**.1025**	**.1104**	**.1624**
1	**.0062**	**.0069**	**.0077**	**.0085**	**.0093**	**.0101**	**.0109**	**.0119**	**.0589**
2	**.0319**	**.0356**	**.0393**	**.0429**	**.0466**	**.0502**	**.0541**	**.0584**	**.1762**
3	**.1047**	**.1146**	**.1242**	**.1336**	**.1428**	**.1515**	**.1605**	**.1704**	**.3267**
4	**.2211**	**.2358**	**.2495**	**.2621**	**.2738**	**.2845**	**.2951**	**.3068**	**.4503**
5	**.3351**	**.3488**	**.3608**	**.3713**	**.3808**	**.3890**	**.3972**	**.4065**	**.5088**
6	**.4354**	**.4449**	**.4530**	**.4600**	**.4662**	**.4716**	**.4770**	**.4835**	**.5442**
7	**.5190**	**.5253**	**.5308**	**.5355**	**.5397**	**.5433**	**.5469**	**.5512**	**.5821**
8	**.5920**	**.5961**	**.5996**	**.6026**	**.6051**	**.6072**	**.6093**	**.6117**	**.6253**
9	**.6525**	**.6548**	**.6567**	**.6583**	**.6596**	**.6607**	**.6617**	**.6628**	**.6681**
10	**.6990**	**.7001**	**.7010**	**.7017**	**.7023**	**.7028**	**.7032**	**.7036**	**.7054**
11	**.7340**	**.7344**	**.7348**	**.7351**	**.7353**	**.7355**	**.7356**	**.7358**	**.7363**
12	**.7629**	**.7631**	**.7633**	**.7634**	**.7634**	**.7635**	**.7635**	**.7636**	**.7637**
13	**.7919**	**.7919**	**.7920**	**.7920**	**.7920**	**.7920**	**.7920**	**.7921**	**.7921**
14	**.8247**	**.8247**	**.8247**	**.8247**	**.8247**	**.8247**	**.8247**	**.8247**	**.8247**
15	**.8618**	**.8618**	**.8618**	**.8618**	**.8618**	**.8618**	**.8618**	**.8618**	**.8618**
16	**.9003**	**.9003**	**.9003**	**.9003**	**.9003**	**.9003**	**.9003**	**.9003**	**.9003**
17	**.9353**	**.9353**	**.9353**	**.9353**	**.9353**	**.9353**	**.9353**	**.9353**	**.9353**
18	**.9629**	**.9629**	**.9629**	**.9629**	**.9629**	**.9629**	**.9629**	**.9629**	**.9629**

Table 32: Sheet U. Exact (in roman), approximate *(in italics)* and interpolated **(boldfaced)** probabilities of perfect ($i = 0$) and i-imperfect column pairs ($k = 2$) in Bernoulli ($m \times n$)-matrices. The rows with all 1's are omitted

k m i	Width n of Bernoulli ($m \times n$)-matrix								
	11	12	13	14	15	16	17	18	19
2 35 0	*.00229560*	*.00274998*	*.00324430*	*.00377828*	*.00435160*	*.00496388*	**.1000**	**.1078**	**.1527**
1	**.0048**	**.0054**	**.0060**	**.0066**	**.0072**	**.0078**	**.0085**	**.0093**	**.0491**
2	**.0257**	**.0286**	**.0316**	**.0346**	**.0376**	**.0406**	**.0437**	**.0473**	**.1547**
3	**.0883**	**.0970**	**.1055**	**.1138**	**.1220**	**.1298**	**.1378**	**.1467**	**.2992**
4	**.1966**	**.2108**	**.2240**	**.2364**	**.2479**	**.2586**	**.2692**	**.2808**	**.4304**
5	**.3090**	**.3232**	**.3357**	**.3469**	**.3569**	**.3658**	**.3744**	**.3842**	**.4975**
6	**.4110**	**.4212**	**.4299**	**.4375**	**.4442**	**.4500**	**.4558**	**.4627**	**.5338**
7	**.4949**	**.5018**	**.5077**	**.5129**	**.5174**	**.5214**	**.5253**	**.5301**	**.5685**
8	**.5690**	**.5736**	**.5775**	**.5809**	**.5838**	**.5863**	**.5887**	**.5916**	**.6096**
9	**.6323**	**.6351**	**.6374**	**.6393**	**.6409**	**.6422**	**.6435**	**.6449**	**.6523**
10	**.6824**	**.6838**	**.6850**	**.6859**	**.6867**	**.6873**	**.6879**	**.6885**	**.6911**
11	**.7201**	**.7207**	**.7212**	**.7216**	**.7219**	**.7222**	**.7224**	**.7226**	**.7235**
12	**.7495**	**.7498**	**.7500**	**.7501**	**.7502**	**.7503**	**.7504**	**.7505**	**.7507**
13	**.7765**	**.7765**	**.7766**	**.7767**	**.7767**	**.7767**	**.7767**	**.7768**	**.7768**
14	**.8058**	**.8058**	**.8058**	**.8059**	**.8059**	**.8059**	**.8059**	**.8059**	**.8059**
15	**.8398**	**.8398**	**.8398**	**.8398**	**.8398**	**.8398**	**.8398**	**.8398**	**.8399**
16	**.8773**	**.8773**	**.8773**	**.8773**	**.8773**	**.8773**	**.8773**	**.8773**	**.8773**
17	**.9143**	**.9143**	**.9143**	**.9143**	**.9143**	**.9143**	**.9143**	**.9143**	**.9143**
18	**.9463**	**.9463**	**.9463**	**.9463**	**.9463**	**.9463**	**.9463**	**.9463**	**.9463**
19	**.9703**	**.9703**	**.9703**	**.9703**	**.9703**	**.9703**	**.9703**	**.9703**	**.9703**
2 36 0	*.00172609*	*.00206834*	*.00244087*	*.00284351*	*.00327605*	*.00373829*	**.0977**	**.1054**	**.1439**
1	**.0038**	**.0042**	**.0047**	**.0052**	**.0057**	**.0062**	**.0067**	**.0073**	**.0407**
2	**.0206**	**.0230**	**.0254**	**.0278**	**.0303**	**.0327**	**.0353**	**.0382**	**.1348**
3	**.0744**	**.0819**	**.0893**	**.0966**	**.1037**	**.1107**	**.1177**	**.1256**	**.2722**
4	**.1750**	**.1884**	**.2010**	**.2129**	**.2242**	**.2346**	**.2449**	**.2563**	**.4097**
5	**.2875**	**.3019**	**.3149**	**.3266**	**.3371**	**.3464**	**.3555**	**.3656**	**.4890**
6	**.3950**	**.4060**	**.4153**	**.4235**	**.4307**	**.4369**	**.4431**	**.4504**	**.5321**
7	**.4835**	**.4909**	**.4973**	**.5028**	**.5078**	**.5121**	**.5163**	**.5216**	**.5682**
8	**.5609**	**.5659**	**.5703**	**.5741**	**.5774**	**.5802**	**.5830**	**.5863**	**.6095**
9	**.6271**	**.6304**	**.6330**	**.6353**	**.6373**	**.6389**	**.6404**	**.6422**	**.6523**
10	**.6796**	**.6813**	**.6828**	**.6839**	**.6849**	**.6857**	**.6864**	**.6873**	**.6911**
11	**.7187**	**.7196**	**.7202**	**.7208**	**.7212**	**.7215**	**.7218**	**.7222**	**.7235**
12	**.7490**	**.7493**	**.7496**	**.7498**	**.7500**	**.7501**	**.7502**	**.7503**	**.7507**
13	**.7762**	**.7764**	**.7765**	**.7765**	**.7766**	**.7766**	**.7767**	**.7767**	**.7768**
14	**.8057**	**.8058**	**.8058**	**.8058**	**.8058**	**.8059**	**.8059**	**.8059**	**.8059**
15	**.8398**	**.8398**	**.8398**	**.8398**	**.8398**	**.8398**	**.8398**	**.8398**	**.8399**
16	**.8773**	**.8773**	**.8773**	**.8773**	**.8773**	**.8773**	**.8773**	**.8773**	**.8773**
17	**.9143**	**.9143**	**.9143**	**.9143**	**.9143**	**.9143**	**.9143**	**.9143**	**.9143**
18	**.9463**	**.9463**	**.9463**	**.9463**	**.9463**	**.9463**	**.9463**	**.9463**	**.9463**
19	**.9703**	**.9703**	**.9703**	**.9703**	**.9703**	**.9703**	**.9703**	**.9703**	**.9703**
2 37 0	*.00129731*	*.00155492*	*.00183543*	*.00213874*	*.00246473*	*.00281328*	**.0956**	**.1032**	**.1359**
1	**.0028**	**.0032**	**.0036**	**.0039**	**.0043**	**.0047**	**.0050**	**.0055**	**.0332**
2	**.0164**	**.0184**	**.0203**	**.0223**	**.0243**	**.0263**	**.0283**	**.0306**	**.1165**
3	**.0620**	**.0684**	**.0748**	**.0810**	**.0872**	**.0933**	**.0994**	**.1062**	**.2451**
4	**.1533**	**.1657**	**.1775**	**.1888**	**.1995**	**.2096**	**.2194**	**.2303**	**.3851**
5	**.2616**	**.2762**	**.2894**	**.3013**	**.3123**	**.3220**	**.3314**	**.3418**	**.4739**
6	**.3703**	**.3820**	**.3921**	**.4008**	**.4086**	**.4153**	**.4217**	**.4294**	**.5217**
7	**.4594**	**.4674**	**.4742**	**.4802**	**.4855**	**.4901**	**.4947**	**.5002**	**.5558**
8	**.5372**	**.5427**	**.5475**	**.5517**	**.5554**	**.5586**	**.5616**	**.5654**	**.5945**
9	**.6056**	**.6093**	**.6124**	**.6150**	**.6173**	**.6192**	**.6210**	**.6232**	**.6366**
10	**.6616**	**.6637**	**.6655**	**.6669**	**.6682**	**.6692**	**.6701**	**.6711**	**.6766**
11	**.7042**	**.7053**	**.7061**	**.7068**	**.7074**	**.7078**	**.7082**	**.7087**	**.7107**
12	**.7361**	**.7366**	**.7370**	**.7373**	**.7375**	**.7377**	**.7378**	**.7380**	**.7386**
13	**.7626**	**.7628**	**.7630**	**.7631**	**.7632**	**.7632**	**.7633**	**.7633**	**.7635**
14	**.7893**	**.7894**	**.7894**	**.7894**	**.7895**	**.7895**	**.7895**	**.7895**	**.7896**
15	**.8199**	**.8199**	**.8199**	**.8199**	**.8199**	**.8199**	**.8199**	**.8199**	**.8199**

Table 32: Sheet V. Exact (in roman), approximate *(in italics)* and interpolated **(boldfaced)** probabilities of perfect ($i = 0$) and i-imperfect column pairs ($k = 2$) in Bernoulli ($m \times n$)-matrices. The rows with all 1's are omitted

k m i	Width n of Bernoulli $(m \times n)$-matrix								
	11	12	13	14	15	16	17	18	19
2 37 16	**.8550**	**.8550**	**.8550**	**.8550**	**.8550**	**.8550**	**.8550**	**.8550**	**.8550**
17	**.8921**	**.8921**	**.8921**	**.8921**	**.8921**	**.8921**	**.8921**	**.8921**	**.8921**
18	**.9271**	**.9271**	**.9271**	**.9271**	**.9271**	**.9271**	**.9271**	**.9271**	**.9271**
19	**.9559**	**.9559**	**.9559**	**.9559**	**.9559**	**.9559**	**.9559**	**.9559**	**.9559**
2 38 0	*.00097470*	*.00116848*	*.00137956*	*.00160787*	*.00185335*	*.00211592*	**.0936**	**.1011**	**.1289**
1	**.0022**	**.0025**	**.0028**	**.0030**	**.0033**	**.0036**	**.0039**	**.0043**	**.0269**
2	**.0130**	**.0146**	**.0162**	**.0177**	**.0193**	**.0209**	**.0225**	**.0244**	**.0996**
3	**.0514**	**.0568**	**.0622**	**.0676**	**.0729**	**.0781**	**.0833**	**.0892**	**.2189**
4	**.1333**	**.1447**	**.1556**	**.1661**	**.1761**	**.1856**	**.1950**	**.2051**	**.3589**
5	**.2367**	**.2511**	**.2643**	**.2764**	**.2875**	**.2976**	**.3072**	**.3178**	**.4569**
6	**.3462**	**.3585**	**.3692**	**.3786**	**.3869**	**.3942**	**.4010**	**.4089**	**.5114**
7	**.4361**	**.4447**	**.4520**	**.4585**	**.4641**	**.4691**	**.4738**	**.4796**	**.5445**
8	**.5139**	**.5199**	**.5251**	**.5297**	**.5337**	**.5372**	**.5405**	**.5446**	**.5806**
9	**.5838**	**.5879**	**.5914**	**.5944**	**.5970**	**.5992**	**.6013**	**.6038**	**.6213**
10	**.6428**	**.6453**	**.6474**	**.6492**	**.6507**	**.6519**	**.6530**	**.6543**	**.6619**
11	**.6890**	**.6903**	**.6914**	**.6923**	**.6930**	**.6936**	**.6941**	**.6947**	**.6976**
12	**.7234**	**.7241**	**.7246**	**.7249**	**.7253**	**.7255**	**.7257**	**.7260**	**.7269**
13	**.7503**	**.7506**	**.7508**	**.7509**	**.7511**	**.7512**	**.7512**	**.7513**	**.7516**
14	**.7751**	**.7752**	**.7752**	**.7753**	**.7753**	**.7754**	**.7754**	**.7754**	**.7755**
15	**.8023**	**.8023**	**.8023**	**.8023**	**.8024**	**.8024**	**.8024**	**.8024**	**.8024**
16	**.8342**	**.8342**	**.8342**	**.8342**	**.8342**	**.8342**	**.8342**	**.8342**	**.8342**
17	**.8699**	**.8699**	**.8699**	**.8699**	**.8699**	**.8699**	**.8699**	**.8699**	**.8699**
18	**.9061**	**.9061**	**.9061**	**.9061**	**.9061**	**.9061**	**.9061**	**.9061**	**.9061**
19	**.9386**	**.9386**	**.9386**	**.9386**	**.9386**	**.9386**	**.9386**	**.9386**	**.9386**
20	**.9642**	**.9642**	**.9642**	**.9642**	**.9642**	**.9642**	**.9642**	**.9642**	**.9642**
2 39 0	*.00073210*	*.00087780*	*.00103654*	*.00120829*	*.00139301*	*.00159065*	**.0916**	**.0991**	**.1226**
1	**.0017**	**.0019**	**.0022**	**.0024**	**.0026**	**.0028**	**.0031**	**.0033**	**.0216**
2	**.0104**	**.0116**	**.0128**	**.0141**	**.0154**	**.0167**	**.0180**	**.0194**	**.0845**
3	**.0425**	**.0471**	**.0517**	**.0562**	**.0607**	**.0651**	**.0696**	**.0745**	**.1940**
4	**.1159**	**.1262**	**.1361**	**.1457**	**.1550**	**.1638**	**.1725**	**.1819**	**.3323**
5	**.2151**	**.2291**	**.2421**	**.2542**	**.2654**	**.2756**	**.2853**	**.2958**	**.4400**
6	**.3276**	**.3405**	**.3519**	**.3619**	**.3707**	**.3785**	**.3857**	**.3939**	**.5059**
7	**.4226**	**.4318**	**.4397**	**.4465**	**.4526**	**.4578**	**.4628**	**.4688**	**.5432**
8	**.5038**	**.5103**	**.5159**	**.5208**	**.5251**	**.5289**	**.5325**	**.5369**	**.5802**
9	**.5765**	**.5810**	**.5849**	**.5883**	**.5913**	**.5938**	**.5962**	**.5990**	**.6212**
10	**.6382**	**.6411**	**.6435**	**.6456**	**.6473**	**.6488**	**.6502**	**.6517**	**.6618**
11	**.6864**	**.6880**	**.6893**	**.6904**	**.6913**	**.6921**	**.6927**	**.6935**	**.6976**
12	**.7221**	**.7229**	**.7236**	**.7241**	**.7245**	**.7248**	**.7251**	**.7255**	**.7269**
13	**.7498**	**.7501**	**.7504**	**.7506**	**.7508**	**.7509**	**.7510**	**.7511**	**.7516**
14	**.7749**	**.7750**	**.7751**	**.7752**	**.7752**	**.7753**	**.7753**	**.7754**	**.7755**
15	**.8022**	**.8022**	**.8023**	**.8023**	**.8023**	**.8023**	**.8023**	**.8024**	**.8024**
16	**.8341**	**.8342**	**.8342**	**.8342**	**.8342**	**.8342**	**.8342**	**.8342**	**.8342**
17	**.8699**	**.8699**	**.8699**	**.8699**	**.8699**	**.8699**	**.8699**	**.8699**	**.8699**
18	**.9061**	**.9061**	**.9061**	**.9061**	**.9061**	**.9061**	**.9061**	**.9061**	**.9061**
19	**.9386**	**.9386**	**.9386**	**.9386**	**.9386**	**.9386**	**.9386**	**.9386**	**.9386**
20	**.9642**	**.9642**	**.9642**	**.9642**	**.9642**	**.9642**	**.9642**	**.9642**	**.9642**
2 40 0	*.00054974*	*.00065924*	*.00077857*	*.00090771*	*.00104662*	*.00119530*	*.00135370*	**.0971**	**.1171**
1	**.0014**	**.0015**	**.0017**	**.0019**	**.0021**	**.0023**	**.0024**	**.0026**	**.0172**
2	**.0082**	**.0092**	**.0102**	**.0112**	**.0122**	**.0132**	**.0143**	**.0154**	**.0709**
3	**.0349**	**.0387**	**.0425**	**.0463**	**.0501**	**.0539**	**.0576**	**.0617**	**.1702**
4	**.0993**	**.1085**	**.1175**	**.1262**	**.1346**	**.1427**	**.1506**	**.1591**	**.3040**
5	**.1918**	**.2052**	**.2178**	**.2296**	**.2407**	**.2509**	**.2606**	**.2709**	**.4179**
6	**.3031**	**.3165**	**.3284**	**.3390**	**.3484**	**.3567**	**.3644**	**.3727**	**.4931**
7	**.3993**	**.4091**	**.4176**	**.4250**	**.4314**	**.4370**	**.4423**	**.4483**	**.5323**
8	**.4806**	**.4875**	**.4935**	**.4987**	**.5034**	**.5074**	**.5113**	**.5158**	**.5672**
9	**.5539**	**.5588**	**.5631**	**.5668**	**.5701**	**.5730**	**.5756**	**.5787**	**.6064**

Table 32: Sheet W. Exact (in roman), approximate *(in italics)* and interpolated **(boldfaced)** probabilities of perfect ($i = 0$) and i-imperfect column pairs ($k = 2$) in Bernoulli ($m \times n$)-matrices. The rows with all 1's are omitted

k	m	i	Width n of Bernoulli ($m \times n$)-matrix								
			11	12	13	14	15	16	17	18	19
2	40	10	**.6180**	**.6213**	**.6241**	**.6265**	**.6286**	**.6303**	**.6319**	**.6337**	**.6470**
		11	**.6698**	**.6718**	**.6734**	**.6747**	**.6758**	**.6768**	**.6776**	**.6785**	**.6842**
		12	**.7088**	**.7099**	**.7107**	**.7113**	**.7119**	**.7123**	**.7127**	**.7131**	**.7153**
		13	**.7380**	**.7385**	**.7388**	**.7391**	**.7393**	**.7395**	**.7397**	**.7398**	**.7406**
		14	**.7623**	**.7625**	**.7626**	**.7628**	**.7628**	**.7629**	**.7630**	**.7630**	**.7633**
		15	**.7869**	**.7870**	**.7871**	**.7871**	**.7871**	**.7872**	**.7872**	**.7872**	**.7873**
		16	**.8154**	**.8155**	**.8155**	**.8155**	**.8155**	**.8155**	**.8155**	**.8155**	**.8155**
		17	**.8486**	**.8486**	**.8486**	**.8486**	**.8486**	**.8486**	**.8486**	**.8486**	**.8486**
		18	**.8844**	**.8844**	**.8844**	**.8844**	**.8844**	**.8844**	**.8844**	**.8844**	**.8844**
		19	**.9192**	**.9192**	**.9192**	**.9192**	**.9192**	**.9192**	**.9192**	**.9192**	**.9192**
		20	**.9488**	**.9488**	**.9488**	**.9488**	**.9488**	**.9488**	**.9488**	**.9488**	**.9488**
2	41	0	*.00041273*	*.00049499*	*.00058466*	*.00068171*	*.00078613*	*.00089791*	*.00101703*	**.0953**	**.1123**
		1	**.0011**	**.0013**	**.0014**	**.0015**	**.0017**	**.0018**	**.0020**	**.0021**	**.0136**
		2	**.0065**	**.0073**	**.0081**	**.0089**	**.0097**	**.0105**	**.0114**	**.0123**	**.0589**
		3	**.0285**	**.0317**	**.0349**	**.0380**	**.0412**	**.0443**	**.0475**	**.0509**	**.1478**
		4	**.0847**	**.0928**	**.1007**	**.1085**	**.1160**	**.1233**	**.1305**	**.1381**	**.2755**
		5	**.1699**	**.1826**	**.1947**	**.2060**	**.2167**	**.2267**	**.2363**	**.2462**	**.3937**
		6	**.2792**	**.2929**	**.3052**	**.3162**	**.3261**	**.3349**	**.3431**	**.3516**	**.4789**
		7	**.3766**	**.3871**	**.3962**	**.4041**	**.4110**	**.4170**	**.4226**	**.4287**	**.5218**
		8	**.4581**	**.4655**	**.4718**	**.4774**	**.4824**	**.4867**	**.4908**	**.4954**	**.5551**
		9	**.5315**	**.5368**	**.5414**	**.5455**	**.5491**	**.5522**	**.5551**	**.5584**	**.5922**
		10	**.5974**	**.6011**	**.6042**	**.6069**	**.6093**	**.6113**	**.6132**	**.6152**	**.6322**
		11	**.6524**	**.6547**	**.6566**	**.6582**	**.6596**	**.6607**	**.6617**	**.6628**	**.6704**
		12	**.6949**	**.6961**	**.6971**	**.6980**	**.6987**	**.6992**	**.6997**	**.7002**	**.7033**
		13	**.7264**	**.7270**	**.7275**	**.7278**	**.7282**	**.7284**	**.7286**	**.7288**	**.7299**
		14	**.7510**	**.7513**	**.7515**	**.7516**	**.7517**	**.7518**	**.7519**	**.7520**	**.7524**
		15	**.7738**	**.7739**	**.7740**	**.7740**	**.7741**	**.7741**	**.7741**	**.7741**	**.7743**
		16	**.7990**	**.7991**	**.7991**	**.7991**	**.7991**	**.7991**	**.7991**	**.7992**	**.7992**
		17	**.8290**	**.8290**	**.8290**	**.8290**	**.8290**	**.8290**	**.8290**	**.8290**	**.8290**
		18	**.8630**	**.8630**	**.8630**	**.8630**	**.8630**	**.8630**	**.8630**	**.8630**	**.8630**
		19	**.8983**	**.8983**	**.8984**	**.8984**	**.8984**	**.8984**	**.8984**	**.8984**	**.8984**
		20	**.9310**	**.9310**	**.9310**	**.9310**	**.9310**	**.9310**	**.9310**	**.9310**	**.9310**
		21	**.9578**	**.9578**	**.9578**	**.9578**	**.9578**	**.9578**	**.9578**	**.9578**	**.9578**
2	42	0	*.00030981*	*.00037159*	*.00043895*	*.00051186*	*.00059033*	*.00067434*	*.00076387*	**.0935**	**.1081**
		1	**.0009**	**.0010**	**.0012**	**.0013**	**.0014**	**.0015**	**.0016**	**.0018**	**.0107**
		2	**.0052**	**.0058**	**.0065**	**.0071**	**.0078**	**.0084**	**.0091**	**.0098**	**.0485**
		3	**.0233**	**.0259**	**.0285**	**.0311**	**.0338**	**.0364**	**.0390**	**.0418**	**.1272**
		4	**.0721**	**.0792**	**.0862**	**.0930**	**.0997**	**.1062**	**.1126**	**.1193**	**.2476**
		5	**.1508**	**.1627**	**.1740**	**.1847**	**.1950**	**.2046**	**.2138**	**.2233**	**.3690**
		6	**.2590**	**.2729**	**.2854**	**.2968**	**.3071**	**.3163**	**.3249**	**.3335**	**.4661**
		7	**.3607**	**.3720**	**.3816**	**.3901**	**.3975**	**.4040**	**.4099**	**.4161**	**.5177**
		8	**.4463**	**.4542**	**.4610**	**.4669**	**.4722**	**.4767**	**.4811**	**.4857**	**.5538**
		9	**.5224**	**.5282**	**.5331**	**.5375**	**.5414**	**.5448**	**.5480**	**.5514**	**.5918**
		10	**.5908**	**.5949**	**.5984**	**.6015**	**.6041**	**.6064**	**.6085**	**.6107**	**.6321**
		11	**.6482**	**.6508**	**.6530**	**.6549**	**.6565**	**.6579**	**.6591**	**.6603**	**.6704**
		12	**.6925**	**.6940**	**.6952**	**.6962**	**.6971**	**.6978**	**.6984**	**.6990**	**.7033**
		13	**.7252**	**.7259**	**.7265**	**.7270**	**.7274**	**.7277**	**.7280**	**.7283**	**.7299**
		14	**.7504**	**.7508**	**.7511**	**.7513**	**.7514**	**.7516**	**.7517**	**.7518**	**.7524**
		15	**.7736**	**.7737**	**.7738**	**.7739**	**.7739**	**.7740**	**.7740**	**.7741**	**.7743**
		16	**.7990**	**.7990**	**.7990**	**.7991**	**.7991**	**.7991**	**.7991**	**.7991**	**.7992**
		17	**.8289**	**.8290**	**.8290**	**.8290**	**.8290**	**.8290**	**.8290**	**.8290**	**.8290**
		18	**.8630**	**.8630**	**.8630**	**.8630**	**.8630**	**.8630**	**.8630**	**.8630**	**.8630**
		19	**.8983**	**.8983**	**.8983**	**.8983**	**.8984**	**.8984**	**.8984**	**.8984**	**.8984**
		20	**.9310**	**.9310**	**.9310**	**.9310**	**.9310**	**.9310**	**.9310**	**.9310**	**.9310**
		21	**.9578**	**.9578**	**.9578**	**.9578**	**.9578**	**.9578**	**.9578**	**.9578**	**.9578**
2	43	0	*.00023252*	*.00027892*	*.00032950*	*.00038426*	*.00044320*	*.00050631*	*.00057359*	**.0918**	**.1044**

Table 32: Sheet X. Exact (in roman), approximate *(in italics)* and interpolated **(boldfaced)** probabilities of perfect ($i = 0$) and i-imperfect column pairs ($k = 2$) in Bernoulli ($m \times n$)-matrices. The rows with all 1's are omitted

k m i	Width n of Bernoulli $(m \times n)$-matrix								
	11	12	13	14	15	16	17	18	19
2 43 1	**.0008**	**.0009**	**.0010**	**.0011**	**.0012**	**.0013**	**.0014**	**.0015**	**.0084**
2	**.0040**	**.0045**	**.0050**	**.0055**	**.0060**	**.0065**	**.0070**	**.0076**	**.0392**
3	**.0189**	**.0210**	**.0232**	**.0254**	**.0275**	**.0297**	**.0318**	**.0341**	**.1083**
4	**.0608**	**.0669**	**.0729**	**.0789**	**.0848**	**.0905**	**.0961**	**.1019**	**.2200**
5	**.1318**	**.1427**	**.1532**	**.1632**	**.1728**	**.1819**	**.1907**	**.1996**	**.3412**
6	**.2356**	**.2494**	**.2620**	**.2735**	**.2840**	**.2936**	**.3025**	**.3112**	**.4472**
7	**.3377**	**.3496**	**.3599**	**.3689**	**.3769**	**.3838**	**.3902**	**.3965**	**.5056**
8	**.4240**	**.4324**	**.4397**	**.4460**	**.4515**	**.4564**	**.4609**	**.4656**	**.5420**
9	**.4999**	**.5060**	**.5113**	**.5160**	**.5202**	**.5238**	**.5272**	**.5307**	**.5782**
10	**.5693**	**.5737**	**.5776**	**.5809**	**.5839**	**.5865**	**.5888**	**.5912**	**.6175**
11	**.6294**	**.6324**	**.6349**	**.6371**	**.6389**	**.6405**	**.6420**	**.6434**	**.6564**
12	**.6773**	**.6790**	**.6805**	**.6817**	**.6828**	**.6836**	**.6844**	**.6852**	**.6909**
13	**.7130**	**.7139**	**.7147**	**.7153**	**.7158**	**.7162**	**.7166**	**.7170**	**.7193**
14	**.7396**	**.7401**	**.7404**	**.7407**	**.7410**	**.7411**	**.7413**	**.7414**	**.7423**
15	**.7620**	**.7622**	**.7623**	**.7624**	**.7625**	**.7626**	**.7627**	**.7627**	**.7630**
16	**.7848**	**.7849**	**.7849**	**.7849**	**.7850**	**.7850**	**.7850**	**.7850**	**.7851**
17	**.8114**	**.8114**	**.8114**	**.8114**	**.8114**	**.8115**	**.8115**	**.8115**	**.8115**
18	**.8427**	**.8427**	**.8427**	**.8427**	**.8427**	**.8427**	**.8427**	**.8427**	**.8427**
19	**.8772**	**.8772**	**.8772**	**.8772**	**.8772**	**.8772**	**.8772**	**.8772**	**.8772**
20	**.9114**	**.9114**	**.9114**	**.9114**	**.9114**	**.9114**	**.9114**	**.9114**	**.9114**
21	**.9417**	**.9417**	**.9417**	**.9417**	**.9417**	**.9417**	**.9417**	**.9417**	**.9417**
22	**.9654**	**.9654**	**.9654**	**.9654**	**.9654**	**.9654**	**.9654**	**.9654**	**.9654**
2 44 0	*.00017449*	*.00020932*	*.00024730*	*.00028842*	*.00033269*	*.00038009*	*.00043062*	**.0901**	**.1012**
1	**.0007**	**.0008**	**.0009**	**.0009**	**.0010**	**.0011**	**.0012**	**.0013**	**.0065**
2	**.0032**	**.0036**	**.0040**	**.0044**	**.0048**	**.0052**	**.0056**	**.0060**	**.0315**
3	**.0153**	**.0171**	**.0188**	**.0206**	**.0224**	**.0242**	**.0260**	**.0278**	**.0913**
4	**.0509**	**.0562**	**.0614**	**.0666**	**.0716**	**.0766**	**.0816**	**.0866**	**.1934**
5	**.1144**	**.1244**	**.1339**	**.1432**	**.1521**	**.1606**	**.1688**	**.1770**	**.3124**
6	**.2131**	**.2266**	**.2391**	**.2506**	**.2612**	**.2709**	**.2801**	**.2889**	**.4259**
7	**.3151**	**.3276**	**.3385**	**.3481**	**.3566**	**.3640**	**.3708**	**.3774**	**.4927**
8	**.4024**	**.4114**	**.4191**	**.4258**	**.4317**	**.4368**	**.4416**	**.4463**	**.5308**
9	**.4780**	**.4845**	**.4901**	**.4951**	**.4995**	**.5034**	**.5070**	**.5106**	**.5654**
10	**.5477**	**.5525**	**.5567**	**.5604**	**.5636**	**.5664**	**.5691**	**.5716**	**.6033**
11	**.6099**	**.6133**	**.6161**	**.6186**	**.6207**	**.6225**	**.6242**	**.6258**	**.6422**
12	**.6611**	**.6632**	**.6649**	**.6664**	**.6676**	**.6687**	**.6696**	**.6705**	**.6781**
13	**.7002**	**.7013**	**.7023**	**.7031**	**.7037**	**.7042**	**.7047**	**.7051**	**.7084**
14	**.7290**	**.7296**	**.7300**	**.7304**	**.7307**	**.7309**	**.7311**	**.7313**	**.7326**
15	**.7516**	**.7518**	**.7520**	**.7522**	**.7523**	**.7524**	**.7525**	**.7525**	**.7530**
16	**.7726**	**.7727**	**.7728**	**.7728**	**.7729**	**.7729**	**.7729**	**.7730**	**.7731**
17	**.7961**	**.7961**	**.7961**	**.7961**	**.7962**	**.7962**	**.7962**	**.7962**	**.7962**
18	**.8242**	**.8242**	**.8242**	**.8242**	**.8242**	**.8242**	**.8242**	**.8242**	**.8242**
19	**.8566**	**.8566**	**.8566**	**.8566**	**.8566**	**.8566**	**.8566**	**.8566**	**.8566**
20	**.8909**	**.8909**	**.8909**	**.8909**	**.8909**	**.8909**	**.8909**	**.8909**	**.8909**
21	**.9236**	**.9236**	**.9236**	**.9236**	**.9236**	**.9236**	**.9236**	**.9236**	**.9236**
22	**.9512**	**.9512**	**.9512**	**.9512**	**.9512**	**.9512**	**.9512**	**.9512**	**.9512**
2 45 0	*.00013093*	*.00015708*	*.00018559*	*.00021646*	*.00024969*	*.00028528*	*.00032323*	**.0886**	**.0984**
1	**.0004**	**.0005**	**.0005**	**.0006**	**.0007**	**.0007**	**.0008**	**.0008**	**.0047**
2	**.0025**	**.0028**	**.0032**	**.0035**	**.0038**	**.0041**	**.0044**	**.0048**	**.0250**
3	**.0123**	**.0137**	**.0152**	**.0166**	**.0181**	**.0195**	**.0210**	**.0225**	**.0759**
4	**.0425**	**.0470**	**.0515**	**.0559**	**.0602**	**.0645**	**.0688**	**.0731**	**.1682**
5	**.0989**	**.1078**	**.1164**	**.1248**	**.1329**	**.1407**	**.1484**	**.1559**	**.2832**
6	**.1916**	**.2047**	**.2168**	**.2282**	**.2387**	**.2485**	**.2577**	**.2665**	**.4024**
7	**.2929**	**.3058**	**.3172**	**.3274**	**.3364**	**.3443**	**.3516**	**.3584**	**.4783**
8	**.3813**	**.3909**	**.3992**	**.4064**	**.4127**	**.4181**	**.4232**	**.4280**	**.5196**
9	**.4568**	**.4637**	**.4697**	**.4749**	**.4796**	**.4836**	**.4875**	**.4912**	**.5533**
10	**.5264**	**.5315**	**.5360**	**.5400**	**.5435**	**.5465**	**.5494**	**.5522**	**.5896**

Table 32: Sheet Y. Exact (in roman), approximate *(in italics)* and interpolated **(boldfaced)** probabilities of perfect ($i = 0$) and i-imperfect column pairs ($k = 2$) in Bernoulli ($m \times n$)-matrices. The rows with all 1's are omitted

k m i	Width n of Bernoulli ($m \times n$)-matrix								
	11	12	13	14	15	16	17	18	19
2 45 11	.5900	.5937	.5969	.5996	.6020	.6041	.6060	.6078	.6280
12	.6441	.6465	.6485	.6502	.6517	.6529	.6541	.6551	.6650
13	.6866	.6880	.6891	.6901	.6909	.6915	.6921	.6926	.6970
14	.7181	.7188	.7194	.7199	.7203	.7206	.7208	.7211	.7228
15	.7418	.7421	.7424	.7426	.7428	.7429	.7430	.7431	.7437
16	.7619	.7621	.7622	.7623	.7623	.7624	.7624	.7625	.7627
17	.7829	.7830	.7830	.7830	.7830	.7831	.7831	.7831	.7832
18	.8077	.8077	.8077	.8077	.8077	.8077	.8077	.8078	.8078
19	.8373	.8373	.8373	.8373	.8373	.8373	.8373	.8373	.8373
20	.8704	.8704	.8704	.8704	.8704	.8704	.8704	.8704	.8704
21	.9040	.9040	.9040	.9040	.9040	.9040	.9040	.9040	.9040
22	.9346	.9346	.9346	.9346	.9346	.9346	.9346	.9346	.9346
2 46 0	*.00009824*	*.00011786*	*.00013926*	*.00016243*	*.00018738*	*.00021410*	*.00024259*	*.00027285*	.0960
1	.0004	.0004	.0005	.0005	.0006	.0006	.0007	.0007	.0036
2	.0020	.0023	.0025	.0028	.0031	.0033	.0036	.0039	.0197
3	.0099	.0111	.0122	.0134	.0146	.0158	.0170	.0182	.0625
4	.0355	.0393	.0430	.0468	.0505	.0542	.0578	.0615	.1449
5	.0854	.0933	.1010	.1085	.1159	.1229	.1299	.1367	.2545
6	.1727	.1851	.1969	.2078	.2182	.2278	.2370	.2457	.3781
7	.2745	.2877	.2995	.3100	.3195	.3279	.3357	.3429	.4656
8	.3669	.3772	.3861	.3937	.4005	.4063	.4117	.4167	.5145
9	.4459	.4532	.4595	.4650	.4700	.4742	.4783	.4821	.5512
10	.5178	.5232	.5280	.5322	.5360	.5393	.5424	.5453	.5888
11	.5835	.5875	.5910	.5941	.5968	.5991	.6012	.6032	.6278
12	.6397	.6424	.6447	.6467	.6484	.6499	.6512	.6524	.6649
13	.6839	.6856	.6869	.6881	.6890	.6898	.6906	.6912	.6970
14	.7167	.7176	.7183	.7189	.7194	.7198	.7201	.7204	.7228
15	.7411	.7415	.7419	.7421	.7423	.7425	.7427	.7428	.7437
16	.7616	.7618	.7620	.7621	.7622	.7623	.7623	.7624	.7627
17	.7828	.7829	.7829	.7830	.7830	.7830	.7830	.7831	.7832
18	.8076	.8077	.8077	.8077	.8077	.8077	.8077	.8077	.8078
19	.8373	.8373	.8373	.8373	.8373	.8373	.8373	.8373	.8373
20	.8704	.8704	.8704	.8704	.8704	.8704	.8704	.8704	.8704
21	.9040	.9040	.9040	.9040	.9040	.9040	.9040	.9040	.9040
22	.9346	.9346	.9346	.9346	.9346	.9346	.9346	.9346	.9346
23	.9595	.9595	.9595	.9595	.9595	.9595	.9595	.9595	.9595
2 47 0	*.00007371*	*.00008843*	*.00010449*	*.00012188*	*.00014060*	*.00016066*	*.00018205*	*.00020476*	.0938
1	.0003	.0004	.0004	.0004	.0005	.0005	.0006	.0006	.0027
2	.0017	.0019	.0021	.0023	.0025	.0027	.0029	.0032	.0154
3	.0080	.0089	.0099	.0108	.0118	.0127	.0137	.0147	.0508
4	.0293	.0325	.0357	.0388	.0420	.0451	.0482	.0513	.1232
5	.0729	.0799	.0867	.0934	.0999	.1062	.1125	.1187	.2257
6	.1532	.1649	.1760	.1865	.1965	.2058	.2148	.2234	.3503
7	.2524	.2657	.2778	.2886	.2985	.3073	.3155	.3231	.4465
8	.3456	.3566	.3660	.3742	.3814	.3877	.3934	.3987	.5015
9	.4250	.4328	.4394	.4453	.4504	.4549	.4592	.4632	.5388
10	.4964	.5022	.5073	.5117	.5157	.5192	.5225	.5257	.5753
11	.5629	.5672	.5710	.5743	.5773	.5798	.5822	.5844	.6136
12	.6214	.6245	.6271	.6293	.6313	.6329	.6345	.6359	.6513
13	.6690	.6709	.6725	.6739	.6750	.6760	.6768	.6776	.6851
14	.7049	.7060	.7069	.7076	.7082	.7087	.7092	.7096	.7128
15	.7313	.7319	.7323	.7326	.7329	.7332	.7334	.7336	.7348
16	.7520	.7523	.7525	.7526	.7527	.7528	.7529	.7530	.7535
17	.7714	.7715	.7716	.7717	.7717	.7718	.7718	.7718	.7720
18	.7933	.7934	.7934	.7934	.7934	.7934	.7935	.7935	.7935
19	.8197	.8197	.8197	.8198	.8198	.8198	.8198	.8198	.8198

Table 32: Sheet Z. Exact (in roman), approximate *(in italics)* and interpolated **(boldfaced)** probabilities of perfect ($i = 0$) and i-imperfect column pairs ($k = 2$) in Bernoulli ($m \times n$)-matrices. The rows with all 1's are omitted

k m i	Width n of Bernoulli $(m \times n)$-matrix								
	11	12	13	14	15	16	17	18	19
2 47 20	**.8506**	**.8506**	**.8506**	**.8506**	**.8506**	**.8506**	**.8506**	**.8506**	**.8506**
21	**.8838**	**.8838**	**.8838**	**.8838**	**.8838**	**.8838**	**.8838**	**.8838**	**.8838**
22	**.9162**	**.9162**	**.9162**	**.9162**	**.9162**	**.9162**	**.9162**	**.9162**	**.9162**
23	**.9446**	**.9446**	**.9446**	**.9446**	**.9446**	**.9446**	**.9446**	**.9446**	**.9446**
2 48 0	*.00005530*	*.00006634*	*.00007839*	*.00009144*	*.00010550*	*.00012055*	*.00013660*	*.00015365*	**.0918**
1	**.0003**	**.0003**	**.0004**	**.0004**	**.0004**	**.0005**	**.0005**	**.0006**	**.0021**
2	**.0014**	**.0016**	**.0017**	**.0019**	**.0021**	**.0023**	**.0024**	**.0026**	**.0119**
3	**.0064**	**.0072**	**.0080**	**.0087**	**.0095**	**.0103**	**.0111**	**.0119**	**.0409**
4	**.0242**	**.0269**	**.0295**	**.0322**	**.0348**	**.0374**	**.0400**	**.0427**	**.1036**
5	**.0619**	**.0680**	**.0740**	**.0799**	**.0856**	**.0913**	**.0968**	**.1023**	**.1980**
6	**.1352**	**.1461**	**.1564**	**.1663**	**.1758**	**.1847**	**.1933**	**.2016**	**.3212**
7	**.2308**	**.2442**	**.2563**	**.2674**	**.2775**	**.2866**	**.2951**	**.3031**	**.4252**
8	**.3247**	**.3362**	**.3462**	**.3550**	**.3627**	**.3694**	**.3756**	**.3812**	**.4874**
9	**.4047**	**.4130**	**.4201**	**.4263**	**.4317**	**.4365**	**.4409**	**.4451**	**.5266**
10	**.4757**	**.4818**	**.4871**	**.4918**	**.4960**	**.4997**	**.5032**	**.5065**	**.5622**
11	**.5422**	**.5469**	**.5510**	**.5546**	**.5578**	**.5605**	**.5632**	**.5656**	**.5998**
12	**.6026**	**.6059**	**.6088**	**.6113**	**.6135**	**.6154**	**.6172**	**.6188**	**.6376**
13	**.6532**	**.6554**	**.6572**	**.6588**	**.6601**	**.6613**	**.6623**	**.6632**	**.6727**
14	**.6924**	**.6937**	**.6947**	**.6956**	**.6964**	**.6970**	**.6975**	**.6980**	**.7023**
15	**.7213**	**.7220**	**.7225**	**.7230**	**.7234**	**.7237**	**.7239**	**.7242**	**.7260**
16	**.7430**	**.7433**	**.7436**	**.7438**	**.7440**	**.7441**	**.7442**	**.7443**	**.7450**
17	**.7616**	**.7617**	**.7618**	**.7619**	**.7620**	**.7620**	**.7621**	**.7621**	**.7624**
18	**.7810**	**.7811**	**.7811**	**.7812**	**.7812**	**.7812**	**.7812**	**.7813**	**.7813**
19	**.8042**	**.8043**	**.8043**	**.8043**	**.8043**	**.8043**	**.8043**	**.8043**	**.8043**
20	**.8322**	**.8322**	**.8322**	**.8322**	**.8322**	**.8322**	**.8322**	**.8322**	**.8322**
21	**.8639**	**.8639**	**.8639**	**.8639**	**.8639**	**.8639**	**.8639**	**.8639**	**.8639**
22	**.8968**	**.8968**	**.8968**	**.8968**	**.8968**	**.8968**	**.8968**	**.8968**	**.8968**
23	**.9276**	**.9276**	**.9276**	**.9276**	**.9276**	**.9276**	**.9276**	**.9276**	**.9276**
24	**.9534**	**.9534**	**.9534**	**.9534**	**.9534**	**.9534**	**.9534**	**.9534**	**.9534**

Table 33: Sheet A. Exact (in roman), approximate *(in italics)* and interpolated **(boldfaced)** probabilities of perfect ($i = 0$) and i-imperfect column triplets ($k = 3$) in Bernoulli ($m \times n$)-matrices. The rows with all 1's are omitted

k	m	i	Width n of Bernoulli ($m \times n$)-matrix								
			3	4	5	6	7	8	9	10	11
3	1	0	.50000000	.68750000	.81250000	.89062500	.93750000	.96484375	.98046875	.98925781	.99414063
3	2	0	.25000000	.44921875	.61132813	.72973633	.81225586	.86875916	.90737152	.93393040	.95239639
		1	.75000000	.90234375	.96484375	.98803711	.99609375	.99876404	.99961853	.99988461	.99996567
3	3	0	.12500000	.27514648	.42376709	.55218887	.65610695	.73748273	.80019028	.84815205	.88471509
		1	.50000000	.74609375	.88024902	.94555664	.97567177	.98919678	.99520132	.99785921	.99903835
		2	.87500000	.96948242	.99340820	.99869156	.99975586	.99995655	.99999255	.99999876	.99999980
3	4	0	.06250000	.15995789	.27330971	.38646346	.49073754	.58249769	.66097559	.72683604	.78136257
		1	.31250000	.56596375	.74513435	.85620672	.92062546	.95666145	.97643659	.98719067	.99302234
		2	.68750000	.89280701	.96704102	.99048918	.99735689	.99928118	.99980669	.99994823	.99998613
		3	.93750000	.99046326	.99876404	.99985689	.99998474	.99999847	.99999985	.99999999	1.00000000
3	5	0	.03125000	.08944607	.16627449	.25237670	.34059513	.42613960	.50611353	.57896691	.64405187
		1	.18750000	.39891815	.58403063	.72412922	.82214329	.88755600	.92991492	.95679268	.97360034
		2	.50000000	.76921082	.90341794	.96181602	.98540344	.99452482	.99796478	.99924523	.99971947
		3	.81250000	.95708466	.99152851	.99847241	.99973840	.99995606	.99999253	.99999867	.99999975
		4	.96875000	.99701977	.99976826	.99998435	.99999905	.99999995	1.00000000	1.00000000	1.00000000
3	6	0	.01562500	.04860848	.09663510	.15557551	.22129020	.29024047	.35964297	.42742799	.49213106
		1	.10937500	.26506132	.42682210	.57051109	.68746276	.77756953	.84445699	.89279104	.92700715
		2	.34375000	.61914063	.79807311	.89882601	.95117467	.97704082	.98940236	.99517487	.99782588
		3	.65625000	.88675117	.96696451	.99105526	.99768827	.99941821	.99985497	.99996358	.99999062
		4	.89062500	.98342246	.99792156	.99976830	.99997558	.99999736	.99999967	.99999995	.99999999
		5	.98437500	.99906868	.99995655	.99999829	.99999994	1.00000000	1.00000000	1.00000000	1.00000000
3	7	0	.00781250	.02586606	.05421108	.09154705	.13604801	.18578767	.23895803	.29396719	.34947075
		1	.06250000	.16808704	.29376464	.42062329	.53723984	.63820408	.72205837	.78960331	.84273826
		2	.22656250	.46849843	.66322248	.79787613	.88326472	.93448100	.96403628	.98061241	.98970549
		3	.50000000	.77972507	.91295352	.96788341	.98868921	.99614568	.99871750	.99958047	.99986429
		4	.77343750	.94755490	.98950133	.99808440	.99967079	.99994523	.99999089	.99999841	.99999969
		5	.93750000	.99376014	.99950709	.99996632	.99999782	.99999984	.99999998	1.00000000	1.00000000
		6	.99218750	.99970896	.99999185	.99999981	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
3	8	0	.00390625	.01355145	.02959583	.05192179	.07996495	.11292569	**.1673**	**.2096**	**.2525**
		1	.03515625	.10272465	.19230925	.29245405	.39403851	**.5023**	**.5508**	**.5964**	**.6380**
		2	.14453125	.33612457	.51938407	.66882364	**.6798**	**.7485**	**.8000**	**.8409**	**.8728**
		3	.36328125	.64922988	.82507384	**.8307**	**.8821**	**.9098**	**.9369**	**.9551**	**.9674**
		4	.63671875	.88122084	.96569079	**.9454**	**.9644**	**.9760**	**.9848**	**.9901**	**.9934**
		5	.85546875	.97675404	.99684412	**.9851**	**.9917**	**.9952**	**.9973**	**.9984**	**.9991**
		6	.96484375	.99769716	.99988611	**.9971**	**.9986**	**.9993**	**.9997**	**.9998**	**.9999**
		7	.99609375	.99990905	.99999847	**.9997**	**.9999**	**.9999**	**1.0000**	**1.0000**	**1.0000**
3	9	0	.00195313	.00701787	.01582380	.02860887	.04534006	.06580189	**.1030**	**.1334**	**.1647**
		1	.01953125	.06095828	.12079652	.19337858	.27304478	**.3675**	**.4600**	**.4982**	**.5349**
		2	.08984375	.23055155	.38529038	.52946132	**.5521**	**.6538**	**.7017**	**.7497**	**.7900**
		3	.25390625	.51328401	.70970503	**.7356**	**.8127**	**.8483**	**.8822**	**.9119**	**.9332**
		4	.50000000	.78520431	.91760455	**.9055**	**.9326**	**.9506**	**.9652**	**.9762**	**.9832**
		5	.74609375	.93939156	.98739264	**.9685**	**.9803**	**.9874**	**.9920**	**.9949**	**.9967**
		6	.91015625	.99004467	.99909197	**.9916**	**.9955**	**.9975**	**.9986**	**.9992**	**.9995**
		7	.98046875	.99916326	.99997422	**.9984**	**.9993**	**.9997**	**.9998**	**.9999**	**1.0000**
		8	.99804688	.99997158	.99999971	**.9998**	**.9999**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
3	10	0	.00097656	.00360293	.00832623	.01541214	.02498582	.03706493	**.0600**	**.0810**	**.1026**
		1	.01074219	.03532874	.07334450	.12257006	.18014215	**.1574**	**.1926**	**.3734**	**.4565**
		2	.05468750	.15228751	.27247535	.39726873	**.4225**	**.4121**	**.4683**	**.6610**	**.7039**
		3	.17187500	.38721516	.58058229	**.6205**	**.7241**	**.6902**	**.7376**	**.8550**	**.8848**
		4	.37695313	.66869562	.84132869	**.8467**	**.8803**	**.8807**	**.9034**	**.9523**	**.9647**
		5	.62304688	.87631803	.96414011	**.9414**	**.9586**	**.9658**	**.9731**	**.9870**	**.9910**
		6	.82812500	.97042244	.99562178	**.9810**	**.9884**	**.9925**	**.9945**	**.9973**	**.9982**
		7	.94531250	.99585327	.99974783	**.9952**	**.9975**	**.9986**	**.9992**	**.9996**	**.9998**
		8	.98925781	.99969980	.99999426	**.9992**	**.9996**	**.9998**	**.9999**	**1.0000**	**1.0000**
		9	.99902344	.99999112	**.9998**	**.9999**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
3	11	0	.00048828	.00183772	.00432787	.00815887	.01346445	.02032335	**.0332**	**.0470**	**.0613**

Table 33: Sheet B. Exact (in roman), approximate *(in italics)* and interpolated **(boldfaced)** probabilities of perfect ($i = 0$) and i-imperfect column triplets ($k = 3$) in Bernoulli ($m \times n$)-matrices. The rows with all 1's are omitted

k	m	i	Width n of Bernoulli $(m \times n)$-matrix								
			3	4	5	6	7	8	9	10	11
3	11	1	.00585938	.02008717	.04330965	.07499382	.11399238	**.1022**	**.1271**	**.1576**	**.2563**
		2	.03271484	.09746228	.18487259	.28402070	**.3072**	**.3033**	**.3531**	**.4113**	**.6159**
		3	.11328125	.28025851	.45274932	**.4994**	**.6198**	**.5673**	**.6224**	**.6829**	**.8211**
		4	.27441406	.54480577	.74032733	**.7627**	**.8099**	**.7933**	**.8294**	**.8679**	**.9357**
		5	.50000000	.78842099	.92023210	**.9006**	**.9213**	**.9220**	**.9379**	**.9552**	**.9799**
		6	.72558594	.93233270	.98537861	**.9629**	**.9739**	**.9774**	**.9825**	**.9883**	**.9949**
		7	.88671875	.98608666	.99854882	**.9884**	**.9931**	**.9951**	**.9964**	**.9978**	**.9990**
		8	.96728516	.99831191	.99993199	**.9972**	**.9986**	**.9992**	**.9995**	**.9997**	**.9999**
		9	.99414063	.99989342	**.9989**	**.9996**	**.9998**	**.9999**	**1.0000**	**1.0000**	**1.0000**
		10	.99951172	.99999722	**.9999**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
3	12	0	.00024414	.00093276	.00222866	.00426117	.00713022	.01090990	**.0172**	**.0260**	**.0352**
		1	.00317383	.01124445	.02499600	.04455901	.06964662	**.0644**	**.0819**	**.1033**	**.1259**
		2	.01928711	.06073989	.12105036	.19458622	**.2108**	**.2166**	**.2586**	**.3075**	**.3551**
		3	.07299805	.19564226	.33786760	**.3838**	**.4894**	**.4511**	**.5087**	**.5703**	**.6250**
		4	.19384766	.42585254	.62403203	**.6600**	**.7265**	**.6942**	**.7431**	**.7911**	**.8299**
		5	.38720703	.68246301	.85226852	**.8446**	**.8664**	**.8606**	**.8886**	**.9148**	**.9348**
		6	.61279297	.87207152	.96263449	**.9341**	**.9465**	**.9485**	**.9606**	**.9716**	**.9798**
		7	.80615234	.96453842	.99435595	**.9763**	**.9832**	**.9852**	**.9892**	**.9928**	**.9953**
		8	.92700195	.99365382	.99953750	**.9929**	**.9959**	**.9968**	**.9978**	**.9987**	**.9992**
		9	.98071289	.99932592	**.9966**	**.9984**	**.9992**	**.9995**	**.9997**	**.9998**	**.9999**
		10	.99682617	.99996250	**.9994**	**.9998**	**.9999**	**.9999**	**1.0000**	**1.0000**	**1.0000**
		11	.99975586	.99999913	**.9999**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
3	13	0	.00012207	.00047169	.00113951	.00220247	.00372507	.00576038	**.0082**	**.0138**	**.0196**
		1	.00170898	.00621440	.01415764	.02584095	.04132432	**.0412**	**.0518**	**.0631**	**.0760**
		2	.01123047	.03701507	.07688676	.12846237	**.1188**	**.1571**	**.1861**	**.2144**	**.2460**
		3	.04614258	.13235109	.24236121	**.2835**	**.2909**	**.3618**	**.4091**	**.4511**	**.4955**
		4	.13342285	.32052877	.50420745	**.5504**	**.5283**	**.6100**	**.6599**	**.6988**	**.7368**
		5	.29052734	.56839550	.76214695	**.7763**	**.7436**	**.8051**	**.8403**	**.8638**	**.8855**
		6	.50000000	.79049869	.92189710	**.8957**	**.8870**	**.9194**	**.9381**	**.9492**	**.9586**
		7	.70947266	.92620640	.98352191	**.9576**	**.9611**	**.9729**	**.9808**	**.9850**	**.9882**
		8	.86657715	.98208536	.99792065	**.9851**	**.9897**	**.9928**	**.9954**	**.9966**	**.9975**
		9	.95385742	.99718035	**.9919**	**.9957**	**.9976**	**.9985**	**.9991**	**.9994**	**.9996**
		10	.98876953	.99973523	**.9980**	**.9991**	**.9995**	**.9998**	**.9999**	**.9999**	**1.0000**
		11	.99829102	.99998690	**.9997**	**.9999**	**.9999**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
		12	.99987793	.99999973	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
3	14	0	.00006104	.00023787	.00057947	.00112935	.00192587	.00300258	**.0033**	**.0068**	**.0105**
		1	.00091553	.00339833	.00789559	.01468840	.02393019	**.0255**	**.0318**	**.0392**	**.0478**
		2	.00646973	.02213274	.04758492	.08213886	**.0807**	**.1086**	**.1279**	**.1493**	**.1741**
		3	.02868652	.08712874	.16785937	**.1994**	**.2205**	**.2739**	**.3092**	**.3471**	**.3877**
		4	.08978271	.23319945	.39138723	**.4336**	**.4389**	**.5068**	**.5498**	**.5944**	**.6366**
		5	.21197510	.45622635	.65654232	**.6797**	**.6638**	**.7220**	**.7553**	**.7902**	**.8186**
		6	.39526367	.69281545	.86020815	**.8413**	**.8314**	**.8677**	**.8863**	**.9073**	**.9217**
		7	.60473633	.86840856	.96126109	**.9279**	**.9290**	**.9474**	**.9558**	**.9661**	**.9720**
		8	.78802490	.95913461	.99310215	**.9717**	**.9751**	**.9830**	**.9862**	**.9902**	**.9921**
		9	.91021729	.99123143	**.9837**	**.9905**	**.9929**	**.9957**	**.9967**	**.9978**	**.9983**
		10	.97131348	.99877525	**.9949**	**.9974**	**.9984**	**.9991**	**.9994**	**.9996**	**.9997**
		11	.99353027	.99989747	**.9989**	**.9995**	**.9997**	**.9999**	**.9999**	**1.0000**	**1.0000**
		12	.99908447	.99999546	**.9998**	**.9999**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
		13	.99993896	.99999992	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
3	15	0	.00003052	.00011970	.00029347	.00057557	.00098771	.00154961	*.00226603*	**.0031**	**.0055**
		1	.00048828	.00184209	.00434735	.00821204	.01358123	**.0154**	**.0198**	**.0240**	**.0299**
		2	.00369263	.01302177	.02880576	.05109700	**.0524**	**.0734**	**.0885**	**.1021**	**.1219**
		3	.01757813	.05601775	.11272416	**.1294**	**.1588**	**.2018**	**.2327**	**.2598**	**.2971**
		4	.05923462	.16460366	.29276740	**.3059**	**.3475**	**.4076**	**.4527**	**.4895**	**.5363**
		5	.15087891	.35378410	.54467207	**.5356**	**.5701**	**.6282**	**.6723**	**.7041**	**.7428**
		6	.30361938	.58683934	.77857153	**.7434**	**.7592**	**.8002**	**.8313**	**.8507**	**.8746**

Table 33: Sheet C. Exact (in roman), approximate *(in italics)* and interpolated **(boldfaced)** probabilities of perfect ($i = 0$) and i-imperfect column triplets ($k = 3$) in Bernoulli ($m \times n$)-matrices. The rows with all 1's are omitted

k m i	Width n of Bernoulli ($m \times n$)-matrix								
	3	4	5	6	7	8	9	10	11
3 15 7	.50000000	.79194221	.92304470	**.8796**	**.8840**	**.9077**	**.9254**	**.9351**	**.9473**
8	.69638062	.92085441	.98182955	**.9495**	**.9521**	**.9644**	**.9728**	**.9768**	**.9819**
9	.84912109	.97816128	**.9708**	**.9811**	**.9835**	**.9889**	**.9921**	**.9934**	**.9951**
10	.94076538	.99582462	**.9894**	**.9940**	**.9954**	**.9973**	**.9982**	**.9986**	**.9990**
11	.98242188	.99947840	**.9969**	**.9984**	**.9990**	**.9995**	**.9997**	**.9998**	**.9998**
12	.99630737	.99996079	**.9993**	**.9997**	**.9998**	**.9999**	**1.0000**	**1.0000**	**1.0000**
13	.99951172	.99999843	**.9999**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
3 16 0	.00001526	.00006014	.00014816	.00029198	.00050346	.00079367	*.00117084*	*.00164321*	*.00221247*
1	.00025940	.00099120	.00236846	.00452888	.00758063	**.0101**	**.0124**	**.0151**	**.0185**
2	.00209045	.00755614	.01711168	.03104809	**.0346**	**.0531**	**.0622**	**.0722**	**.0837**
3	.01063538	.03528250	.07367744	**.0822**	**.1178**	**.1561**	**.1774**	**.1995**	**.2232**
4	.03840637	.11310318	.21174396	**.2103**	**.2826**	**.3364**	**.3728**	**.4081**	**.4432**
5	.10505676	.26579346	.43567944	**.4038**	**.5007**	**.5541**	**.5959**	**.6334**	**.6679**
6	.22724915	.48082716	.68184003	**.6163**	**.7057**	**.7439**	**.7772**	**.8050**	**.8290**
7	.40180969	.70094718	.86628061	**.7920**	**.8507**	**.8724**	**.8931**	**.9096**	**.9234**
8	.59819031	.86523346	.96002987	**.9053**	**.9337**	**.9454**	**.9563**	**.9646**	**.9713**
9	.77275085	.95419160	**.9518**	**.9634**	**.9744**	**.9805**	**.9852**	**.9887**	**.9913**
10	.89494324	.98868915	**.9804**	**.9874**	**.9916**	**.9942**	**.9959**	**.9971**	**.9979**
11	.96159363	.99805930	**.9932**	**.9962**	**.9977**	**.9986**	**.9991**	**.9994**	**.9996**
12	.98936462	.99978169	**.9981**	**.9991**	**.9995**	**.9997**	**.9998**	**.9999**	**.9999**
13	.99790955	.99998517	**.9996**	**.9998**	**.9999**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
14	.99974060	.99999946	**.9999**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
3 17 0	.00000763	.00003018	.00007462	.00014759	.00025542	.00040413	*.00059908*	*.00084551*	*.00114741*
1	.00013733	.00053005	.00127906	.00246959	.00417356	**.0071**	**.0080**	**.0092**	**.0113**
2	.00117493	.00433283	.01000242	.01849070	**.0213**	**.0394**	**.0435**	**.0484**	**.0567**
3	.00636292	.02182772	.04703001	**.0552**	**.0798**	**.1188**	**.1301**	**.1429**	**.1633**
4	.02452087	.07588518	.14856824	**.1551**	**.2092**	**.2652**	**.2889**	**.3138**	**.3509**
5	.07173157	.19401742	.33670156	**.3232**	**.4033**	**.4612**	**.4946**	**.5270**	**.5739**
6	.16615295	.38172568	.57730876	**.5290**	**.6124**	**.6555**	**.6873**	**.7159**	**.7582**
7	.31452942	.60173626	.79143888	**.7187**	**.7815**	**.8054**	**.8280**	**.8476**	**.8780**
8	.50000000	.79300171	.92388493	**.8554**	**.8915**	**.9032**	**.9170**	**.9287**	**.9467**
9	.68547058	.91614070	**.9240**	**.9354**	**.9526**	**.9590**	**.9663**	**.9722**	**.9807**
10	.83384705	.97437702	**.9669**	**.9747**	**.9823**	**.9857**	**.9889**	**.9913**	**.9943**
11	.92826843	.99430215	**.9868**	**.9914**	**.9944**	**.9960**	**.9971**	**.9979**	**.9987**
12	.97547913	.99911709	**.9956**	**.9975**	**.9986**	**.9991**	**.9994**	**.9996**	**.9997**
13	.99363708	.99991002	**.9989**	**.9994**	**.9997**	**.9998**	**.9999**	**.9999**	**1.0000**
14	.99882507	.99999444	**.9998**	**.9999**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
15	.99986267	.99999982	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
3 18 0	.00000381	.00001513	.00003752	.00007441	.00012912	.00020486	*.00030465*	*.00043141*	*.00058782*
1	.00007248	.00028196	.00068569	.00133413	.00227189	**.0042**	**.0047**	**.0064**	**.0067**
2	.00065613	.00245919	.00576671	.01082496	**.0130**	**.0260**	**.0286**	**.0355**	**.0372**
3	.00376892	.01329391	.02940623	**.0358**	**.0536**	**.0848**	**.0924**	**.1077**	**.1138**
4	.01544189	.04984945	.10145007	**.1109**	**.1525**	**.2012**	**.2187**	**.2449**	**.2603**
5	.04812622	.13798212	.25202757	**.2509**	**.3182**	**.3733**	**.4016**	**.4359**	**.4600**
6	.11894226	.29424450	.47280322	**.4414**	**.5199**	**.5655**	**.5968**	**.6290**	**.6532**
7	.24034119	.50122546	.70212752	**.6367**	**.7044**	**.7317**	**.7565**	**.7796**	**.7976**
8	.40726471	.70754406	.87110448	**.7934**	**.8386**	**.8516**	**.8680**	**.8824**	**.8943**
9	.59273529	.86245873	**.8856**	**.8961**	**.9215**	**.9282**	**.9380**	**.9462**	**.9530**
10	.75965881	.94966914	**.9473**	**.9539**	**.9667**	**.9708**	**.9759**	**.9798**	**.9829**
11	.88105774	.98609865	**.9771**	**.9822**	**.9880**	**.9903**	**.9925**	**.9940**	**.9951**
12	.95187378	.99719986	**.9912**	**.9942**	**.9964**	**.9974**	**.9981**	**.9986**	**.9989**
13	.98455811	.99960593	**.9972**	**.9984**	**.9991**	**.9994**	**.9996**	**.9997**	**.9998**
14	.99623108	.99996342	**.9993**	**.9997**	**.9998**	**.9999**	**.9999**	**1.0000**	**1.0000**
15	.99934387	.99999794	**.9999**	**.9999**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
16	.99992752	.99999994	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
3 19 0	.00000191	.00000758	.00001884	.00003744	.00006510	.00010350	*.00015426*	*.00021894*	*.00029906*

Table 33: Sheet D. Exact (in roman), approximate *(in italics)* and interpolated **(boldfaced)** probabilities of perfect ($i = 0$) and i-imperfect column triplets ($k = 3$) in Bernoulli ($m \times n$)-matrices. The rows with all 1's are omitted

k	m	i	Width n of Bernoulli ($m \times n$)-matrix								
			3	4	5	6	7	8	9	10	11
3	19	1	.00003815	.00014932	.00036535	.00071516	.00122520	**.0036**	**.0038**	**.0042**	**.0046**
		2	.00036430	.00138341	.00328561	.00624514	**.0082**	**.0225**	**.0239**	**.0255**	**.0274**
		3	.00221252	.00798602	.01805778	**.0238**	**.0374**	**.0739**	**.0783**	**.0833**	**.0888**
		4	.00960541	.03213805	.06761938	**.0823**	**.1151**	**.1748**	**.1864**	**.1986**	**.2116**
		5	.03178406	.09585259	.18319989	**.2024**	**.2575**	**.3290**	**.3512**	**.3731**	**.3948**
		6	.08353424	.22071433	.37503531	**.3807**	**.4486**	**.5150**	**.5436**	**.5699**	**.5941**
		7	.17964172	.40555826	.60419083	**.5785**	**.6412**	**.6891**	**.7148**	**.7368**	**.7558**
		8	.32380295	.61407289	.80182857	**.7480**	**.7930**	**.8211**	**.8393**	**.8542**	**.8668**
		9	.50000000	.79381246	**.8368**	**.8648**	**.8924**	**.9079**	**.9195**	**.9289**	**.9366**
		10	.67619705	.91195431	**.9208**	**.9349**	**.9502**	**.9589**	**.9656**	**.9708**	**.9748**
		11	.82035828	.97076087	**.9634**	**.9719**	**.9798**	**.9844**	**.9877**	**.9901**	**.9919**
		12	.91646576	.99266203	**.9844**	**.9894**	**.9930**	**.9950**	**.9963**	**.9972**	**.9979**
		13	.96821594	.99865413	**.9942**	**.9966**	**.9979**	**.9986**	**.9991**	**.9994**	**.9995**
		14	.99039459	.99982711	**.9983**	**.9991**	**.9995**	**.9997**	**.9998**	**.9999**	**.9999**
		15	.99778748	.99998531	**.9996**	**.9998**	**.9999**	**.9999**	**1.0000**	**1.0000**	**1.0000**
		16	.99963570	.99999924	**.9999**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
3	20	0	.00000095	.00000380	.00000945	.00001881	.00003275	.00005216	*.00007786*	*.00011069*	*.00015146*
		1	.00002003	.00007877	.00019366	.00038089	.00065563	**.0032**	**.0033**	**.0034**	**.0035**
		2	.00020123	.00077223	.00185305	.00355816	**.0049**	**.0199**	**.0203**	**.0208**	**.0212**
		3	.00128841	.00473981	.01091540	**.0149**	**.0247**	**.0630**	**.0649**	**.0669**	**.0689**
		4	.00590897	.02037708	.04411094	**.0558**	**.0819**	**.1434**	**.1500**	**.1567**	**.1636**
		5	.02069473	.06519308	.12966458	**.1485**	**.1962**	**.2670**	**.2826**	**.2978**	**.3126**
		6	.05765915	.16146934	.28865227	**.3006**	**.3657**	**.4269**	**.4513**	**.4736**	**.4939**
		7	.13158798	.31925423	.50430452	**.4887**	**.5564**	**.5947**	**.6203**	**.6418**	**.6601**
		8	.25172234	.51846155	.71878919	**.6690**	**.7240**	**.7390**	**.7588**	**.7747**	**.7875**
		9	.41190147	.71302968	**.7750**	**.8069**	**.8443**	**.8470**	**.8607**	**.8713**	**.8798**
		10	.58809853	.86001228	**.8841**	**.8993**	**.9214**	**.9216**	**.9301**	**.9367**	**.9419**
		11	.74827766	.94552215	**.9439**	**.9525**	**.9647**	**.9658**	**.9706**	**.9742**	**.9769**
		12	.86841202	.98350763	**.9742**	**.9802**	**.9862**	**.9877**	**.9899**	**.9915**	**.9927**
		13	.94234085	.99622127	**.9893**	**.9929**	**.9954**	**.9963**	**.9972**	**.9978**	**.9982**
		14	.97930527	.99936595	**.9962**	**.9978**	**.9987**	**.9991**	**.9994**	**.9995**	**.9997**
		15	.99409103	.99992531	**.9990**	**.9995**	**.9997**	**.9998**	**.9999**	**.9999**	**.9999**
		16	.99871159	.99999417	**.9998**	**.9999**	**.9999**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
		17	.99979877	.99999972	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
3	21	0	.00000048	.00000190	.00000473	.00000944	.00001645	.00002623	*.00003921*	*.00005581*	*.00007646*
		1	.00001049	.00004142	.00010221	.00020177	.00034860	**.0019**	**.0019**	**.0019**	**.0020**
		2	.00011063	.00042816	.00103598	.00200567	**.0031**	**.0133**	**.0135**	**.0138**	**.0141**
		3	.00074482	.00278329	.00650762	**.0092**	**.0168**	**.0458**	**.0469**	**.0481**	**.0494**
		4	.00359869	.01272983	.02823079	**.0374**	**.0583**	**.1102**	**.1146**	**.1191**	**.1238**
		5	.01330185	.04350458	.08958455	**.1072**	**.1452**	**.2132**	**.2249**	**.2365**	**.2480**
		6	.03917694	.11545746	.21601872	**.2328**	**.2838**	**.3548**	**.3760**	**.3958**	**.4143**
		7	.09462357	.24493198	.40861888	**.4042**	**.4564**	**.5166**	**.5424**	**.5649**	**.5845**
		8	.19165516	.42615122	.62672510	**.5865**	**.6271**	**.6683**	**.6907**	**.7090**	**.7239**
		9	.33181190	.62449347	**.7026**	**.7399**	**.7653**	**.7894**	**.8058**	**.8188**	**.8291**
		10	.50000000	.79445316	**.8368**	**.8539**	**.8678**	**.8808**	**.8915**	**.8999**	**.9066**
		11	.66818810	.90820664	**.9172**	**.9252**	**.9336**	**.9411**	**.9477**	**.9527**	**.9566**
		12	.80834484	.96732206	**.9598**	**.9657**	**.9711**	**.9755**	**.9790**	**.9816**	**.9835**
		13	.90537643	.99094344	**.9819**	**.9862**	**.9894**	**.9916**	**.9931**	**.9942**	**.9950**
		14	.96082306	.99809732	**.9928**	**.9952**	**.9967**	**.9976**	**.9982**	**.9986**	**.9988**
		15	.98669815	.99970667	**.9976**	**.9986**	**.9991**	**.9994**	**.9996**	**.9997**	**.9998**
		16	.99640131	.99996819	**.9994**	**.9997**	**.9998**	**.9999**	**.9999**	**1.0000**	**1.0000**
		17	.99925518	.99999771	**.9999**	**.9999**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
		18	.99988937	.99999990	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
3	22	0	.00000024	.00000095	.00000237	.00000473	.00000826	.00001317	*.00001971*	*.00002808*	*.00003850*
		1	.00000548	.00002171	.00005374	.00010640	.00018437	**.0011**	**.0011**	**.0011**	**.0012**
		2	.00006056	.00023597	.00057480	.00112022	**.0020**	**.0088**	**.0089**	**.0091**	**.0093**

Table 33: Sheet E. Exact (in roman), approximate *(in italics)* and interpolated **(boldfaced)** probabilities of perfect ($i = 0$) and i-imperfect column triplets ($k = 3$) in Bernoulli ($m \times n$)-matrices. The rows with all 1's are omitted

k	m	i	Width n of Bernoulli ($m \times n$)-matrix 3	4	5	6	7	8	9	10	11
3	22	3	.00042772	.00161903	.00383313	**.0060**	**.0111**	**.0330**	**.0337**	**.0345**	**.0352**
		4	.00217175	.00784796	.01776367	**.0258**	**.0410**	**.0842**	**.0870**	**.0900**	**.0930**
		5	.00845027	.02853836	.06056000	**.0786**	**.1080**	**.1694**	**.1778**	**.1863**	**.1947**
		6	.02623940	.08085445	.15752951	**.1814**	**.2226**	**.2921**	**.3094**	**.3260**	**.3419**
		7	.06690025	.18347338	.32182707	**.3341**	**.3778**	**.4427**	**.4672**	**.4893**	**.5090**
		8	.14313936	.34139704	.53134019	**.5118**	**.5461**	**.5960**	**.6204**	**.6406**	**.6576**
		9	.26173353	.53325307	**.6226**	**.6735**	**.6933**	**.7257**	**.7451**	**.7605**	**.7729**
		10	.41590595	.71768104	**.7784**	**.8052**	**.8135**	**.8317**	**.8448**	**.8551**	**.8633**
		11	.58409405	.85783675	**.8817**	**.8934**	**.8971**	**.9076**	**.9160**	**.9226**	**.9278**
		12	.73826647	.94170751	**.9402**	**.9468**	**.9500**	**.9562**	**.9612**	**.9650**	**.9678**
		13	.85686064	.98094709	**.9712**	**.9763**	**.9792**	**.9826**	**.9851**	**.9869**	**.9883**
		14	.93309975	.99514675	**.9873**	**.9908**	**.9927**	**.9943**	**.9954**	**.9961**	**.9966**
		15	.97376060	.99906134	**.9951**	**.9969**	**.9978**	**.9984**	**.9988**	**.9991**	**.9992**
		16	.99154973	.99986653	**.9985**	**.9991**	**.9995**	**.9997**	**.9998**	**.9998**	**.9999**
		17	.99782825	.99998662	**.9996**	**.9998**	**.9999**	**.9999**	**1.0000**	**1.0000**	**1.0000**
		18	.99957228	.99999911	**.9999**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
		19	.99993944	.99999996	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
3	23	0	.00000012	.00000048	.00000119	.00000237	.00000414	.00000661	*.00000989*	*.00001411*	*.00001936*
		1	.00000286	.00001135	.00002817	.00005589	.00009707	**.0011**	**.0011**	**.0011**	**.0012**
		2	.00003302	.00012937	.00031683	.00062075	**.0015**	**.0084**	**.0085**	**.0086**	**.0087**
		3	.00024414	.00093391	.00223395	**.0037**	**.0094**	**.0320**	**.0324**	**.0329**	**.0334**
		4	.00129974	.00478137	.01101033	**.0174**	**.0349**	**.0794**	**.0812**	**.0831**	**.0851**
		5	.00531101	.01843401	.04014421	**.0573**	**.0928**	**.1556**	**.1615**	**.1674**	**.1734**
		6	.01734483	.05555802	.11218082	**.1413**	**.1940**	**.2643**	**.2778**	**.2910**	**.3039**
		7	.04656982	.13443287	.24677971	**.2758**	**.3369**	**.4034**	**.4253**	**.4456**	**.4644**
		8	.10501981	.26693490	.43819724	**.4449**	**.5016**	**.5555**	**.5805**	**.6020**	**.6206**
		9	.20243645	.44414517	**.5391**	**.6094**	**.6523**	**.6904**	**.7124**	**.7303**	**.7449**
		10	.33881974	.63343803	**.7114**	**.7551**	**.7825**	**.8061**	**.8218**	**.8341**	**.8440**
		11	.50000000	.79497252	**.8379**	**.8590**	**.8753**	**.8895**	**.9000**	**.9082**	**.9148**
		12	.66118026	.90482724	**.9148**	**.9252**	**.9356**	**.9443**	**.9510**	**.9561**	**.9601**
		13	.79756355	.96405953	**.9568**	**.9640**	**.9706**	**.9758**	**.9795**	**.9823**	**.9844**
		14	.89498019	.98917661	**.9794**	**.9846**	**.9883**	**.9910**	**.9928**	**.9941**	**.9950**
		15	.95343018	.99745673	**.9912**	**.9942**	**.9960**	**.9971**	**.9978**	**.9983**	**.9987**
		16	.98265517	.99954551	**.9968**	**.9981**	**.9988**	**.9992**	**.9994**	**.9996**	**.9997**
		17	.99468899	.99994018	**.9991**	**.9995**	**.9997**	**.9998**	**.9999**	**.9999**	**.9999**
		18	.99870026	.99999444	**.9998**	**.9999**	**.9999**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
		19	.99975586	.99999966	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
3	24	0	.00000006	.00000024	.00000059	.00000119	.00000207	.00000331	*.00000496*	*.00000708*	*.00000972*
		1	.00000149	.00000592	.00001472	.00002927	.00005092	**.0006**	**.0006**	**.0007**	**.0007**
		2	.00001794	.00007060	.00017363	.00034164	**.0015**	**.0055**	**.0055**	**.0056**	**.0057**
		3	.00013858	.00053469	.00128984	**.0023**	**.0089**	**.0227**	**.0229**	**.0232**	**.0235**
		4	.00077194	.00288231	.00673371	**.0118**	**.0317**	**.0603**	**.0614**	**.0626**	**.0638**
		5	.00330538	.01174258	.02614587	**.0405**	**.0802**	**.1228**	**.1267**	**.1308**	**.1349**
		6	.01132792	.03752253	.07817300	**.1034**	**.1624**	**.2130**	**.2231**	**.2332**	**.2431**
		7	.03195733	.09651584	.18456463	**.2097**	**.2798**	**.3323**	**.3509**	**.3685**	**.3852**
		8	.07579482	.20400939	.35186062	**.3538**	**.4228**	**.4712**	**.4956**	**.5172**	**.5363**
		9	.15372813	.36113677	**.4548**	**.5062**	**.5630**	**.6023**	**.6262**	**.6462**	**.6630**
		10	.27062809	.54611192	**.6351**	**.6597**	**.6999**	**.7277**	**.7461**	**.7607**	**.7724**
		11	.41940987	.72168767	**.7822**	**.7844**	**.8095**	**.8273**	**.8399**	**.8498**	**.8578**
		12	.58059013	.85588683	**.8804**	**.8749**	**.8901**	**.9014**	**.9098**	**.9164**	**.9216**
		13	.72937191	.93818643	**.9373**	**.9346**	**.9439**	**.9510**	**.9561**	**.9601**	**.9631**
		14	.84627187	.97843669	**.9685**	**.9698**	**.9753**	**.9793**	**.9820**	**.9841**	**.9856**
		15	.92420518	.99399696	**.9854**	**.9879**	**.9907**	**.9926**	**.9939**	**.9948**	**.9954**
		16	.96804267	.99869432	**.9939**	**.9958**	**.9970**	**.9978**	**.9983**	**.9986**	**.9988**
		17	.98867208	.99978368	**.9979**	**.9987**	**.9992**	**.9994**	**.9996**	**.9997**	**.9998**
		18	.99669462	.99997356	**.9994**	**.9997**	**.9998**	**.9999**	**.9999**	**.9999**	**1.0000**

Table 33: Sheet F. Exact (in roman), approximate *(in italics)* and interpolated **(boldfaced)** probabilities of perfect ($i = 0$) and i-imperfect column triplets ($k = 3$) in Bernoulli ($m \times n$)-matrices. The rows with all 1's are omitted

k m i	Width n of Bernoulli ($m \times n$)-matrix								
	3	4	5	6	7	8	9	10	11
3 24 19	.99922806	.99999771	**.9999**	**.9999**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
20	.99986142	.99999987	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
3 25 0	.00000003	.00000012	.00000030	.00000059	.00000104	.00000166	*.00000249*	*.00000355*	*.00000487*
1	.00000077	.00000308	.00000768	.00001528	.00002662	**.0004**	**.0004**	**.0004**	**.0005**
2	.00000972	.00003836	.00009467	.00018692	**.0018**	**.0035**	**.0036**	**.0036**	**.0037**
3	.00007826	.00030408	.00073860	**.0015**	**.0098**	**.0161**	**.0163**	**.0165**	**.0166**
4	.00045526	.00172103	.00406944	**.0079**	**.0321**	**.0459**	**.0466**	**.0473**	**.0481**
5	.00203866	.00738657	.01676107	**.0288**	**.0764**	**.0980**	**.1005**	**.1032**	**.1060**
6	.00731665	.02494676	.05340958	**.0772**	**.1472**	**.1752**	**.1825**	**.1898**	**.1972**
7	.02164263	.06800884	.13487405	**.1640**	**.2466**	**.2798**	**.2949**	**.3094**	**.3235**
8	.05387607	.15263579	.27543602	**.2896**	**.3715**	**.4081**	**.4305**	**.4510**	**.4697**
9	.11476147	.28697780	**.3743**	**.4307**	**.4995**	**.5347**	**.5593**	**.5805**	**.5988**
10	.21217811	.46002156	**.5541**	**.5853**	**.6364**	**.6657**	**.6867**	**.7035**	**.7173**
11	.34501898	.64121828	**.7168**	**.7211**	**.7542**	**.7750**	**.7899**	**.8017**	**.8112**
12	.50000000	.79540225	**.8367**	**.8269**	**.8469**	**.8607**	**.8709**	**.8789**	**.8854**
13	.65498102	.90175990	**.9113**	**.9021**	**.9143**	**.9232**	**.9299**	**.9351**	**.9393**
14	.78782189	.96096694	**.9536**	**.9505**	**.9579**	**.9634**	**.9674**	**.9703**	**.9726**
15	.88523853	.98738451	**.9770**	**.9781**	**.9823**	**.9852**	**.9872**	**.9887**	**.9897**
16	.94612393	.99674317	**.9897**	**.9916**	**.9936**	**.9950**	**.9958**	**.9964**	**.9969**
17	.97835737	.99934224	**.9959**	**.9972**	**.9981**	**.9986**	**.9989**	**.9991**	**.9992**
18	.99268335	.99989866	**.9987**	**.9992**	**.9995**	**.9997**	**.9998**	**.9998**	**.9999**
19	.99796134	.99998846	**.9996**	**.9998**	**.9999**	**.9999**	**1.0000**	**1.0000**	**1.0000**
20	.99954474	.99999907	**.9999**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
21	.99992174	.99999995	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
3 26 0	.00000001	.00000006	.00000015	.00000030	.00000052	.00000083	*.00000125*	*.00000178*	*.00000244*
1	.00000040	.00000160	.00000399	.00000796	.00001388	**.0004**	**.0004**	**.0004**	**.0004**
2	.00000525	.00002077	.00005139	.00010174	**.0018**	**.0035**	**.0036**	**.0036**	**.0037**
3	.00004399	.00017189	.00041987	**.0012**	**.0096**	**.0159**	**.0161**	**.0162**	**.0163**
4	.00026676	.00101883	.00243334	**.0067**	**.0312**	**.0448**	**.0453**	**.0458**	**.0463**
5	.00124696	.00459383	.01059292	**.0247**	**.0728**	**.0937**	**.0954**	**.0971**	**.0989**
6	.00467765	.01635017	.03584165	**.0670**	**.1374**	**.1634**	**.1685**	**.1737**	**.1789**
7	.01447964	.04710527	.09647773	**.1442**	**.2264**	**.2562**	**.2678**	**.2793**	**.2905**
8	.03775935	.11196323	.21049746	**.2588**	**.3400**	**.3731**	**.3928**	**.4112**	**.4283**
9	.08431877	.22314556	**.3013**	**.3917**	**.4606**	**.4941**	**.5180**	**.5393**	**.5582**
10	.16346979	.37884743	**.4736**	**.5444**	**.5985**	**.6294**	**.6524**	**.6714**	**.6872**
11	.27859855	.55741371	**.6457**	**.6854**	**.7234**	**.7473**	**.7648**	**.7786**	**.7897**
12	.42250949	.72518415	**.7852**	**.7992**	**.8239**	**.8403**	**.8525**	**.8621**	**.8698**
13	.57749051	.85412665	**.8791**	**.8819**	**.8976**	**.9086**	**.9169**	**.9235**	**.9288**
14	.72140145	.93492497	**.9346**	**.9368**	**.9467**	**.9539**	**.9592**	**.9633**	**.9664**
15	.83653021	.97598851	**.9659**	**.9696**	**.9756**	**.9798**	**.9827**	**.9850**	**.9866**
16	.91568123	.99278961	**.9834**	**.9870**	**.9902**	**.9923**	**.9938**	**.9948**	**.9955**
17	.96224065	.99826875	**.9928**	**.9951**	**.9965**	**.9975**	**.9981**	**.9985**	**.9987**
18	.98552036	.99967438	**.9973**	**.9983**	**.9989**	**.9993**	**.9995**	**.9996**	**.9997**
19	.99532235	.99995321	**.9992**	**.9995**	**.9997**	**.9998**	**.9999**	**.9999**	**.9999**
20	.99875304	.99999503	**.9998**	**.9999**	**.9999**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
21	.99973324	.99999962	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
3 27 0	.00000001	.00000003	.00000007	.00000015	.00000026	.00000042	*.00000062*	*.00000089*	*.00000122*
1	.00000021	.00000083	.00000207	.00000414	.00000722	**.0002**	**.0003**	**.0003**	**.0003**
2	.00000282	.00001120	.00002779	.00005513	**.0011**	**.0022**	**.0023**	**.0023**	**.0024**
3	.00002462	.00009664	.00023713	**.0014**	**.0066**	**.0112**	**.0113**	**.0114**	**.0115**
4	.00015537	.00059847	.00144130	**.0069**	**.0230**	**.0337**	**.0340**	**.0342**	**.0345**
5	.00075686	.00282762	.00660948	**.0240**	**.0567**	**.0745**	**.0755**	**.0766**	**.0778**
6	.00296231	.01057709	.02366367	**.0614**	**.1109**	**.1342**	**.1377**	**.1412**	**.1449**
7	.00957865	.03211618	.06766944	**.1260**	**.1862**	**.2135**	**.2222**	**.2308**	**.2394**
8	.02611949	.08063684	.15729483	**.2203**	**.2834**	**.3143**	**.3307**	**.3463**	**.3612**
9	.06103906	.17000071	**.2365**	**.3304**	**.3883**	**.4206**	**.4425**	**.4626**	**.4810**

Table 33: Sheet G. Exact (in roman), approximate *(in italics)* and interpolated **(boldfaced)** probabilities of perfect ($i = 0$) and i-imperfect column triplets ($k = 3$) in Bernoulli ($m \times n$)-matrices. The rows with all 1's are omitted

k	m	i	Width n of Bernoulli ($m \times n$)-matrix								
			3	4	5	6	7	8	9	10	11
3	27	10	.12389428	.30529188	**.3943**	**.4650**	**.5161**	**.5484**	**.5724**	**.5928**	**.6103**
		11	.22103417	.47414896	**.5674**	**.6007**	**.6416**	**.6686**	**.6887**	**.7047**	**.7177**
		12	.35055402	.64806180	**.7222**	**.7223**	**.7515**	**.7711**	**.7855**	**.7967**	**.8058**
		13	.50000000	.79576385	**.8365**	**.8212**	**.8407**	**.8540**	**.8640**	**.8719**	**.8783**
		14	.64944598	.89895937	**.9089**	**.8950**	**.9077**	**.9166**	**.9233**	**.9286**	**.9328**
		15	.77896583	.95803535	**.9509**	**.9448**	**.9528**	**.9584**	**.9625**	**.9657**	**.9681**
		16	.87610572	.98558441	**.9748**	**.9744**	**.9791**	**.9822**	**.9844**	**.9860**	**.9872**
		17	.93896094	.99596719	**.9881**	**.9897**	**.9920**	**.9935**	**.9945**	**.9953**	**.9958**
		18	.97388051	.99909695	**.9950**	**.9963**	**.9974**	**.9980**	**.9984**	**.9987**	**.9989**
		19	.99042135	.99984140	**.9983**	**.9989**	**.9992**	**.9995**	**.9996**	**.9997**	**.9998**
		20	.99703769	.99997869	**.9995**	**.9997**	**.9998**	**.9999**	**.9999**	**.9999**	**1.0000**
		21	.99924314	.99999788	**.9999**	**.9999**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
		22	.99984463	.99999985	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
3	28	0	.00000000	.00000001	.00000004	.00000007	.00000013	.00000021	*.00000031*	*.00000045*	*.00000061*
		1	.00000011	.00000043	.00000108	.00000215	.00000375	**.0002**	**.0002**	**.0002**	**.0002**
		2	.00000152	.00000603	.00001497	.00002975	**.0014**	**.0014**	**.0015**	**.0015**	**.0016**
		3	.00001372	.00005406	.00013315	**.0010**	**.0077**	**.0078**	**.0079**	**.0079**	**.0080**
		4	.00009000	.00034907	.00084648	**.0049**	**.0250**	**.0252**	**.0254**	**.0256**	**.0258**
		5	.00045612	.00172420	.00407673	**.0179**	**.0584**	**.0591**	**.0597**	**.0605**	**.0612**
		6	.00185958	.00676147	.01539420	**.0476**	**.1085**	**.1108**	**.1131**	**.1155**	**.1179**
		7	.00627048	.02158193	.04661570	**.1011**	**.1744**	**.1806**	**.1869**	**.1932**	**.1996**
		8	.01784907	.05709831	.11510578	**.1818**	**.2569**	**.2703**	**.2834**	**.2961**	**.3084**
		9	.04357928	.12705679	**.1814**	**.2791**	**.3463**	**.3667**	**.3858**	**.4038**	**.4205**
		10	.09246667	.24097393	**.3201**	**.4033**	**.4615**	**.4885**	**.5122**	**.5331**	**.5514**
		11	.17246423	.39483178	**.4865**	**.5357**	**.5830**	**.6103**	**.6323**	**.6504**	**.6654**
		12	.28579409	.56744068	**.6502**	**.6615**	**.6965**	**.7185**	**.7353**	**.7484**	**.7591**
		13	.42527701	.72826893	**.7834**	**.7697**	**.7935**	**.8093**	**.8210**	**.8304**	**.8379**
		14	.57472299	.85252756	**.8750**	**.8554**	**.8711**	**.8819**	**.8901**	**.8966**	**.9019**
		15	.71420591	.93189374	**.9309**	**.9177**	**.9278**	**.9350**	**.9404**	**.9446**	**.9479**
		16	.82753577	.97360948	**.9632**	**.9583**	**.9645**	**.9689**	**.9721**	**.9745**	**.9763**
		17	.90753333	.99153956	**.9816**	**.9815**	**.9849**	**.9872**	**.9888**	**.9900**	**.9909**
		18	.95642072	.99778940	**.9916**	**.9928**	**.9945**	**.9955**	**.9963**	**.9968**	**.9971**
		19	.98215093	.99953716	**.9966**	**.9975**	**.9982**	**.9987**	**.9989**	**.9991**	**.9993**
		20	.99372952	.99992390	**.9989**	**.9993**	**.9995**	**.9997**	**.9998**	**.9998**	**.9998**
		21	.99814042	.99999042	**.9997**	**.9998**	**.9999**	**.9999**	**.9999**	**1.0000**	**1.0000**
		22	.99954388	.99999910	**.9999**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
		23	.99991000	.99999994	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
3	29	0	.00000000	.00000001	.00000002	.00000004	.00000007	.00000010	*.00000016*	*.00000022*	*.00000031*
		1	.00000006	.00000022	.00000056	.00000111	.00000194	**.0002**	**.0002**	**.0002**	**.0002**
		2	.00000081	.00000323	.00000804	.00001600	**.0014**	**.0014**	**.0015**	**.0015**	**.0016**
		3	.00000762	.00003011	.00007438	**.0009**	**.0077**	**.0077**	**.0078**	**.0079**	**.0080**
		4	.00005186	.00020231	.00049336	**.0048**	**.0249**	**.0250**	**.0251**	**.0253**	**.0254**
		5	.00027306	.00104240	.00248852	**.0171**	**.0576**	**.0580**	**.0585**	**.0590**	**.0595**
		6	.00115785	.00427554	.00988116	**.0449**	**.1055**	**.1070**	**.1085**	**.1101**	**.1118**
		7	.00406503	.01431141	.03158684	**.0942**	**.1663**	**.1706**	**.1750**	**.1795**	**.1841**
		8	.01205977	.03980152	.08261454	**.1677**	**.2400**	**.2502**	**.2602**	**.2701**	**.2798**
		9	.03071417	.09327814	**.1318**	**.2567**	**.3199**	**.3365**	**.3524**	**.3677**	**.3822**
		10	.06802297	.18650996	**.2425**	**.3714**	**.4263**	**.4509**	**.4731**	**.4932**	**.5114**
		11	.13246545	.32208120	**.3878**	**.4993**	**.5461**	**.5739**	**.5972**	**.6168**	**.6335**
		12	.22912916	.48681418	**.5474**	**.6271**	**.6645**	**.6891**	**.7083**	**.7235**	**.7359**
		13	.35553555	.65413898	**.6954**	**.7412**	**.7685**	**.7870**	**.8008**	**.8116**	**.8204**
		14	.50000000	.79607242	**.8122**	**.8335**	**.8522**	**.8651**	**.8749**	**.8827**	**.8890**
		15	.64446445	.89638892	**.8925**	**.9017**	**.9142**	**.9230**	**.9298**	**.9351**	**.9394**
		16	.77087084	.95525481	**.9420**	**.9474**	**.9554**	**.9612**	**.9654**	**.9687**	**.9713**
		17	.86753455	.98378918	**.9705**	**.9747**	**.9795**	**.9829**	**.9853**	**.9870**	**.9884**
		18	.93197703	.99513864	**.9860**	**.9891**	**.9917**	**.9934**	**.9946**	**.9954**	**.9960**

Table 33: Sheet H. Exact (in roman), approximate *(in italics)* and interpolated **(boldfaced)** probabilities of perfect ($i = 0$) and i-imperfect column triplets ($k = 3$) in Bernoulli ($m \times n$)-matrices. The rows with all 1's are omitted

k m i	Width n of Bernoulli $(m \times n)$-matrix								
	3	4	5	6	7	8	9	10	11
3 29 19	.96928583	.99881079	**.9939**	**.9958**	**.9970**	**.9978**	**.9983**	**.9986**	**.9988**
20	.98794023	.99976664	**.9977**	**.9986**	**.9990**	**.9993**	**.9995**	**.9996**	**.9997**
21	.99593497	.99996400	**.9993**	**.9996**	**.9997**	**.9998**	**.9999**	**.9999**	**.9999**
22	.99884215	.99999574	**.9998**	**.9999**	**.9999**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
23	.99972694	.99999963	**.9999**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
24	.99994814	.99999998	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
3 30 0	.00000000	.00000000	.00000001	.00000002	.00000003	.00000005	*.00000008*	*.00000011*	*.00000015*
1	.00000003	.00000012	.00000029	.00000058	.00000101	**.0001**	**.0001**	**.0002**	**.0002**
2	.00000043	.00000173	.00000431	.00000858	**.0009**	**.0009**	**.0010**	**.0010**	**.0010**
3	.00000422	.00001670	.00004135	**.0007**	**.0053**	**.0053**	**.0054**	**.0055**	**.0056**
4	.00002974	.00011656	.00028558	**.0034**	**.0184**	**.0185**	**.0186**	**.0187**	**.0188**
5	.00016246	.00062530	.00150486	**.0127**	**.0453**	**.0456**	**.0458**	**.0461**	**.0464**
6	.00071545	.00267680	.00626572	**.0350**	**.0874**	**.0883**	**.0893**	**.0903**	**.0914**
7	.00261144	.00937491	.02108268	**.0761**	**.1423**	**.1453**	**.1483**	**.1514**	**.1546**
8	.00806240	.02734502	.05824100	**.1389**	**.2090**	**.2164**	**.2239**	**.2314**	**.2388**
9	.02138697	.06734812	**.0995**	**.2172**	**.2808**	**.2938**	**.3065**	**.3189**	**.3309**
10	.04936857	.14170889	**.1948**	**.3198**	**.3760**	**.3974**	**.4172**	**.4356**	**.4526**
11	.10024421	.25759453	**.3292**	**.4389**	**.4880**	**.5150**	**.5384**	**.5588**	**.5765**
12	.18079730	.40933639	**.4876**	**.5645**	**.6057**	**.6322**	**.6534**	**.6707**	**.6849**
13	.29233236	.57640928	**.6439**	**.6834**	**.7151**	**.7363**	**.7524**	**.7650**	**.7752**
14	.42776778	.73101591	**.7740**	**.7853**	**.8077**	**.8230**	**.8344**	**.8435**	**.8509**
15	.57223222	.85106648	**.8670**	**.8653**	**.8805**	**.8911**	**.8993**	**.9057**	**.9110**
16	.70766764	.92906742	**.9257**	**.9227**	**.9329**	**.9401**	**.9456**	**.9499**	**.9533**
17	.81920270	.97130313	**.9601**	**.9600**	**.9664**	**.9709**	**.9743**	**.9768**	**.9787**
18	.89975579	.99025911	**.9796**	**.9815**	**.9852**	**.9877**	**.9894**	**.9907**	**.9917**
19	.95063143	.99726142	**.9903**	**.9923**	**.9942**	**.9954**	**.9963**	**.9968**	**.9973**
20	.97861303	.99937137	**.9959**	**.9971**	**.9980**	**.9985**	**.9988**	**.9991**	**.9992**
21	.99193760	.99988412	**.9985**	**.9991**	**.9994**	**.9996**	**.9997**	**.9998**	**.9998**
22	.99738856	.99998319	**.9995**	**.9997**	**.9998**	**.9999**	**.9999**	**.9999**	**1.0000**
23	.99928455	.99999813	**.9999**	**.9999**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
24	.99983754	.99999984	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
3 31 0	.00000000	.00000000	.00000000	.00000001	.00000002	.00000003	*.00000004*	*.00000006*	*.00000008*
1	.00000001	.00000006	.00000015	.00000030	.00000052	**.0001**	**.0001**	**.0002**	**.0002**
2	.00000023	.00000092	.00000230	.00000459	**.0006**	**.0006**	**.0006**	**.0007**	**.0007**
3	.00000232	.00000923	.00002289	**.0009**	**.0036**	**.0036**	**.0037**	**.0038**	**.0039**
4	.00001698	.00006680	.00016429	**.0042**	**.0134**	**.0135**	**.0136**	**.0137**	**.0138**
5	.00009610	.00037242	.00090232	**.0147**	**.0353**	**.0355**	**.0357**	**.0358**	**.0361**
6	.00043896	.00166063	.00392944	**.0377**	**.0716**	**.0723**	**.0729**	**.0736**	**.0743**
7	.00166345	.00607253	.01387883	**.0770**	**.1211**	**.1231**	**.1251**	**.1273**	**.1294**
8	.00533692	.01853684	.04038477	**.1335**	**.1817**	**.1870**	**.1925**	**.1979**	**.2033**
9	.01472469	.04787803	**.0703**	**.2014**	**.2470**	**.2568**	**.2666**	**.2763**	**.2857**
10	.03537777	.10581301	**.1434**	**.2886**	**.3318**	**.3496**	**.3665**	**.3825**	**.3976**
11	.07480639	.20217488	**.2538**	**.3925**	**.4344**	**.4594**	**.4818**	**.5018**	**.5197**
12	.14052076	.33752354	**.3951**	**.5084**	**.5481**	**.5754**	**.5980**	**.6169**	**.6328**
13	.23656483	.49824378	**.5487**	**.6258**	**.6599**	**.6836**	**.7020**	**.7166**	**.7284**
14	.36005007	.65958015	**.6915**	**.7333**	**.7592**	**.7770**	**.7903**	**.8008**	**.8093**
15	.50000000	.79633890	**.8060**	**.8230**	**.8412**	**.8538**	**.8634**	**.8710**	**.8773**
16	.63994993	.89401846	**.8866**	**.8918**	**.9042**	**.9130**	**.9197**	**.9251**	**.9294**
17	.76343517	.95261518	**.9377**	**.9399**	**.9480**	**.9539**	**.9582**	**.9616**	**.9643**
18	.85947924	.98200836	**.9677**	**.9699**	**.9750**	**.9785**	**.9810**	**.9829**	**.9844**
19	.92519361	.99426647	**.9843**	**.9866**	**.9893**	**.9912**	**.9925**	**.9934**	**.9941**
20	.96462223	.99848560	**.9929**	**.9946**	**.9960**	**.9969**	**.9974**	**.9978**	**.9981**
21	.98527531	.99967311	**.9971**	**.9981**	**.9987**	**.9990**	**.9992**	**.9994**	**.9995**
22	.99466308	.99994328	**.9990**	**.9994**	**.9996**	**.9997**	**.9998**	**.9998**	**.9999**
23	.99833655	.99999224	**.9997**	**.9998**	**.9999**	**.9999**	**1.0000**	**1.0000**	**1.0000**
24	.99956104	.99999919	**.9999**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**

Table 33: Sheet I. Exact (in roman), approximate *(in italics)* and interpolated **(boldfaced)** probabilities of perfect ($i = 0$) and i-imperfect column triplets ($k = 3$) in Bernoulli ($m \times n$)-matrices. The rows with all 1's are omitted

k m i	Width n of Bernoulli $(m \times n)$-matrix								
	3	4	5	6	7	8	9	10	11
3 31 25	.99990390	.99999994	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
3 32 0	.00000000	.00000000	.00000000	.00000000	.00000001	.00000001	*.00000002*	*.00000003*	*.00000004*
1	.00000001	.00000003	.00000008	.00000015	.00000027	**.0001**	**.0001**	**.0001**	**.0002**
2	.00000012	.00000049	.00000123	.00000245	**.0004**	**.0004**	**.0005**	**.0005**	**.0005**
3	.00000128	.00000508	.00001263	**.0012**	**.0024**	**.0025**	**.0026**	**.0026**	**.0027**
4	.00000965	.00003809	.00009398	**.0054**	**.0097**	**.0098**	**.0098**	**.0099**	**.0100**
5	.00005654	.00022035	.00053688	**.0173**	**.0271**	**.0273**	**.0274**	**.0275**	**.0277**
6	.00026753	.00102160	.00243961	**.0415**	**.0582**	**.0586**	**.0590**	**.0595**	**.0599**
7	.00105120	.00389292	.00902195	**.0801**	**.1023**	**.1037**	**.1050**	**.1065**	**.1079**
8	.00350018	.01241110	.02757964	**.1321**	**.1575**	**.1613**	**.1651**	**.1690**	**.1729**
9	.01003080	.03354935	**.0508**	**.1917**	**.2176**	**.2249**	**.2322**	**.2395**	**.2467**
10	.02505123	.07773196	**.1094**	**.2652**	**.2933**	**.3075**	**.3213**	**.3346**	**.3474**
11	.05509208	.15586830	**.2039**	**.3539**	**.3860**	**.4081**	**.4285**	**.4472**	**.4643**
12	.10766357	.27310697	**.3327**	**.4577**	**.4931**	**.5200**	**.5430**	**.5629**	**.5800**
13	.18854279	.42256361	**.4820**	**.5698**	**.6043**	**.6301**	**.6506**	**.6671**	**.6806**
14	.29830745	.58448854	**.6298**	**.6792**	**.7081**	**.7286**	**.7440**	**.7561**	**.7659**
15	.43002503	.73348176	**.7561**	**.7761**	**.7975**	**.8123**	**.8234**	**.8322**	**.8395**
16	.56997497	.84972465	**.8505**	**.8549**	**.8698**	**.8803**	**.8883**	**.8948**	**.9000**
17	.70169255	.92642420	**.9139**	**.9138**	**.9239**	**.9312**	**.9367**	**.9410**	**.9445**
18	.81145721	.96907070	**.9531**	**.9536**	**.9602**	**.9648**	**.9683**	**.9709**	**.9730**
19	.89233643	.98895838	**.9760**	**.9776**	**.9815**	**.9842**	**.9861**	**.9875**	**.9886**
20	.94490792	.99669007	**.9886**	**.9903**	**.9924**	**.9938**	**.9947**	**.9954**	**.9959**
21	.97494877	.99917700	**.9949**	**.9963**	**.9972**	**.9979**	**.9983**	**.9985**	**.9987**
22	.98996920	.99983261	**.9981**	**.9987**	**.9991**	**.9993**	**.9995**	**.9996**	**.9997**
23	.99649982	.99997261	**.9994**	**.9996**	**.9997**	**.9998**	**.9999**	**.9999**	**.9999**
24	.99894880	.99999646	**.9998**	**.9999**	**.9999**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
25	.99973247	.99999965	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
26	.99994346	.99999997	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
3 33 0	.00000000	.00000000	.00000000	.00000000	.00000000	.00000001	*.00000001*	*.00000001*	*.00000002*
1	.00000000	.00000002	.00000004	.00000008	.00000014	**.0001**	**.0001**	**.0001**	**.0002**
2	.00000007	.00000026	.00000065	.00000130	**.0004**	**.0004**	**.0005**	**.0005**	**.0005**
3	.00000070	.00000279	.00000694	**.0012**	**.0024**	**.0025**	**.0026**	**.0026**	**.0027**
4	.00000546	.00002162	.00005348	**.0053**	**.0097**	**.0097**	**.0098**	**.0099**	**.0100**
5	.00003309	.00012959	.00031721	**.0172**	**.0270**	**.0272**	**.0273**	**.0274**	**.0275**
6	.00016203	.00062364	.00150082	**.0411**	**.0576**	**.0578**	**.0581**	**.0583**	**.0586**
7	.00065936	.00247191	.00579744	**.0787**	**.1005**	**.1014**	**.1023**	**.1032**	**.1041**
8	.00227569	.00821494	.01857241	**.1283**	**.1526**	**.1552**	**.1579**	**.1606**	**.1633**
9	.00676549	.02319581	**.0372**	**.1842**	**.2080**	**.2133**	**.2186**	**.2239**	**.2293**
10	.01754102	.05623828	**.0863**	**.2506**	**.2753**	**.2864**	**.2973**	**.3079**	**.3183**
11	.04007166	.11815564	**.1704**	**.3303**	**.3583**	**.3771**	**.3948**	**.4114**	**.4270**
12	.08137783	.21702379	**.2911**	**.4266**	**.4591**	**.4844**	**.5068**	**.5266**	**.5442**
13	.14810318	.35177278	**.4371**	**.5358**	**.5704**	**.5972**	**.6192**	**.6374**	**.6526**
14	.24342512	.50861914	**.5866**	**.6476**	**.6789**	**.7020**	**.7196**	**.7336**	**.7449**
15	.36416624	.66448691	**.7182**	**.7498**	**.7744**	**.7916**	**.8045**	**.8147**	**.8229**
16	.50000000	.79657138	**.8197**	**.8343**	**.8520**	**.8643**	**.8737**	**.8812**	**.8873**
17	.63583376	.89182309	**.8906**	**.8983**	**.9105**	**.9193**	**.9260**	**.9313**	**.9357**
18	.75657488	.95010662	**.9369**	**.9426**	**.9508**	**.9567**	**.9612**	**.9647**	**.9675**
19	.85189682	.98024891	**.9657**	**.9704**	**.9756**	**.9793**	**.9820**	**.9840**	**.9856**
20	.91862217	.99335863	**.9827**	**.9862**	**.9891**	**.9911**	**.9926**	**.9936**	**.9944**
21	.95992834	.99812374	**.9919**	**.9941**	**.9956**	**.9966**	**.9973**	**.9978**	**.9981**
22	.98245898	.99955999	**.9965**	**.9977**	**.9984**	**.9988**	**.9991**	**.9993**	**.9994**
23	.99323451	.99991552	**.9987**	**.9992**	**.9995**	**.9996**	**.9997**	**.9998**	**.9998**
24	.99772431	.99998694	**.9996**	**.9998**	**.9998**	**.9999**	**.9999**	**.9999**	**1.0000**
25	.99934064	.99999840	**.9999**	**.9999**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
26	.99983797	.99999985	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
3 34 0	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	*.00000000*	*.00000001*	*.00000001*

Table 33: Sheet J. Exact (in roman), approximate *(in italics)* and interpolated **(boldfaced)** probabilities of perfect ($i = 0$) and i-imperfect column triplets ($k = 3$) in Bernoulli ($m \times n$)-matrices. The rows with all 1's are omitted

k	m	i	Width n of Bernoulli $(m \times n)$-matrix								
			3	4	5	6	7	8	9	10	11
3	34	1	.00000000	.00000001	.00000002	.00000004	.00000007	**.0001**	**.0001**	**.0001**	**.0002**
		2	.00000003	.00000014	.00000035	.00000069	**.0003**	**.0003**	**.0003**	**.0004**	**.0004**
		3	.00000038	.00000153	.00000380	**.0008**	**.0016**	**.0017**	**.0018**	**.0018**	**.0019**
		4	.00000308	.00001222	.00003030	**.0038**	**.0069**	**.0070**	**.0070**	**.0071**	**.0072**
		5	.00001928	.00007579	.00018623	**.0129**	**.0205**	**.0206**	**.0207**	**.0208**	**.0209**
		6	.00009756	.00037800	.00091560	**.0325**	**.0462**	**.0464**	**.0466**	**.0468**	**.0470**
		7	.00041070	.00155581	.00368624	**.0651**	**.0844**	**.0850**	**.0855**	**.0862**	**.0868**
		8	.00146753	.00538005	.01234648	**.1095**	**.1321**	**.1339**	**.1357**	**.1375**	**.1394**
		9	.00452059	.01583879	**.0262**	**.1607**	**.1836**	**.1873**	**.1911**	**.1950**	**.1989**
		10	.01215326	.04011135	**.0639**	**.2198**	**.2430**	**.2513**	**.2597**	**.2679**	**.2760**
		11	.02880634	.08815322	**.1329**	**.2885**	**.3141**	**.3296**	**.3443**	**.3585**	**.3719**
		12	.06072474	.16950898	**.2382**	**.3712**	**.4015**	**.4243**	**.4450**	**.4638**	**.4810**
		13	.11474051	.28760613	**.3736**	**.4685**	**.5024**	**.5291**	**.5518**	**.5711**	**.5876**
		14	.19576415	.43468067	**.5215**	**.5742**	**.6074**	**.6325**	**.6522**	**.6681**	**.6810**
		15	.30379568	.59181246	**.6601**	**.6783**	**.7062**	**.7260**	**.7409**	**.7526**	**.7620**
		16	.43208312	.73571077	**.7738**	**.7718**	**.7926**	**.8070**	**.8179**	**.8265**	**.8335**
		17	.56791688	.84848664	**.8576**	**.8493**	**.8639**	**.8743**	**.8822**	**.8885**	**.8938**
		18	.69620432	.92394523	**.9147**	**.9084**	**.9185**	**.9257**	**.9313**	**.9357**	**.9392**
		19	.80423585	.96691188	**.9516**	**.9494**	**.9560**	**.9608**	**.9644**	**.9671**	**.9693**
		20	.88525949	.98764560	**.9742**	**.9748**	**.9788**	**.9817**	**.9837**	**.9853**	**.9864**
		21	.93927526	.99608040	**.9872**	**.9887**	**.9909**	**.9924**	**.9935**	**.9943**	**.9948**
		22	.97119366	.99895455	**.9942**	**.9954**	**.9965**	**.9973**	**.9977**	**.9981**	**.9983**
		23	.98784674	.99976835	**.9976**	**.9983**	**.9988**	**.9991**	**.9993**	**.9994**	**.9995**
		24	.99547941	.99995794	**.9992**	**.9995**	**.9996**	**.9997**	**.9998**	**.9999**	**.9999**
		25	.99853247	.99999384	**.9997**	**.9998**	**.9999**	**.9999**	**1.0000**	**1.0000**	**1.0000**
		26	.99958930	.99999929	**.9999**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
		27	.99990244	.99999994	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
3	35	0	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	*.00000000*	*.00000000*	*.00000000*
		1	.00000000	.00000000	.00000001	.00000002	.00000004	**.0001**	**.0001**	**.0001**	**.0002**
		2	.00000002	.00000007	.00000018	.00000037	**.0002**	**.0002**	**.0003**	**.0003**	**.0004**
		3	.00000021	.00000083	.00000208	**.0011**	**.0011**	**.0012**	**.0013**	**.0013**	**.0014**
		4	.00000173	.00000688	.00001709	**.0048**	**.0049**	**.0049**	**.0050**	**.0051**	**.0051**
		5	.00001118	.00004409	.00010869	**.0153**	**.0154**	**.0155**	**.0156**	**.0157**	**.0158**
		6	.00005842	.00022760	.00055431	**.0366**	**.0367**	**.0369**	**.0370**	**.0371**	**.0373**
		7	.00025413	.00097126	.00232128	**.0699**	**.0703**	**.0707**	**.0711**	**.0715**	**.0720**
		8	.00093911	.00348893	.00811075	**.1130**	**.1141**	**.1153**	**.1165**	**.1177**	**.1190**
		9	.00299406	.01069057	**.0180**	**.1603**	**.1628**	**.1654**	**.1681**	**.1708**	**.1735**
		10	.00833692	.02823006	**.0464**	**.2118**	**.2180**	**.2241**	**.2303**	**.2365**	**.2426**
		11	.02047980	.06479158	**.1014**	**.2696**	**.2821**	**.2944**	**.3063**	**.3178**	**.3290**
		12	.04476554	.13024598	**.1907**	**.3393**	**.3605**	**.3802**	**.3986**	**.4157**	**.4315**
		13	.08773262	.23109445	**.3128**	**.4248**	**.4539**	**.4794**	**.5017**	**.5213**	**.5385**
		14	.15525233	.36496192	**.4551**	**.5233**	**.5559**	**.5822**	**.6036**	**.6211**	**.6357**
		15	.24977992	.51808723	**.5975**	**.6261**	**.6562**	**.6786**	**.6956**	**.7090**	**.7198**
		16	.36793940	.66893969	**.7217**	**.7235**	**.7473**	**.7641**	**.7766**	**.7865**	**.7945**
		17	.50000000	.79677600	**.8184**	**.8082**	**.8255**	**.8376**	**.8468**	**.8541**	**.8602**
		18	.63206060	.88978202	**.8873**	**.8765**	**.8887**	**.8973**	**.9040**	**.9093**	**.9137**
		19	.75022008	.94771978	**.9336**	**.9271**	**.9354**	**.9414**	**.9459**	**.9495**	**.9524**
		20	.84474767	.97851588	**.9631**	**.9610**	**.9664**	**.9701**	**.9730**	**.9751**	**.9768**
		21	.91226738	.99242210	**.9808**	**.9812**	**.9844**	**.9865**	**.9881**	**.9892**	**.9901**
		22	.95523446	.99772781	**.9907**	**.9919**	**.9935**	**.9946**	**.9954**	**.9960**	**.9964**
		23	.97952020	.99942685	**.9959**	**.9968**	**.9976**	**.9981**	**.9985**	**.9987**	**.9988**
		24	.99166308	.99987980	**.9984**	**.9989**	**.9992**	**.9994**	**.9995**	**.9996**	**.9997**
		25	.99700594	.99997932	**.9994**	**.9996**	**.9998**	**.9998**	**.9999**	**.9999**	**.9999**
		26	.99906089	.99999713	**.9998**	**.9999**	**.9999**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
		27	.99974587	.99999969	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
		28	.99994158	.99999997	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**

Table 33: Sheet K. Exact (in roman), approximate *(in italics)* and interpolated **(boldfaced)** probabilities of perfect ($i = 0$) and i-imperfect column triplets ($k = 3$) in Bernoulli ($m \times n$)-matrices. The rows with all 1's are omitted

k	m	i	Width n of Bernoulli $(m \times n)$-matrix								
			3	4	5	6	7	8	9	10	11
3	36	0	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	*.00000000*	*.00000000*	*.00000000*
		1	.00000000	.00000000	.00000001	.00000001	.00000002	**.0001**	**.0001**	**.0001**	**.0002**
		2	.00000001	.00000004	.00000010	.00000019	**.0002**	**.0002**	**.0003**	**.0003**	**.0004**
		3	.00000011	.00000045	.00000113	**.0011**	**.0011**	**.0012**	**.0013**	**.0013**	**.0014**
		4	.00000097	.00000386	.00000960	**.0048**	**.0049**	**.0049**	**.0050**	**.0051**	**.0051**
		5	.00000646	.00002553	.00006310	**.0153**	**.0154**	**.0155**	**.0156**	**.0156**	**.0157**
		6	.00003480	.00013621	.00033324	**.0365**	**.0366**	**.0367**	**.0368**	**.0370**	**.0371**
		7	.00015628	.00060176	.00144881	**.0695**	**.0697**	**.0700**	**.0702**	**.0705**	**.0708**
		8	.00059662	.00224197	.00527026	**.1118**	**.1126**	**.1133**	**.1141**	**.1150**	**.1158**
		9	.00196659	.00713831	**.0122**	**.1577**	**.1595**	**.1613**	**.1631**	**.1650**	**.1669**
		10	.00566549	.01962217	**.0330**	**.2056**	**.2101**	**.2145**	**.2190**	**.2235**	**.2281**
		11	.01440836	.04695576	**.0757**	**.2573**	**.2668**	**.2763**	**.2855**	**.2947**	**.3036**
		12	.03262267	.09853681	**.1495**	**.3190**	**.3363**	**.3528**	**.3684**	**.3833**	**.3973**
		13	.06624908	.18262210	**.2573**	**.3969**	**.4228**	**.4461**	**.4671**	**.4860**	**.5030**
		14	.12149248	.30118015	**.3915**	**.4912**	**.5228**	**.5493**	**.5715**	**.5903**	**.6063**
		15	.20251612	.44582690	**.5349**	**.5945**	**.6263**	**.6507**	**.6698**	**.6850**	**.6974**
		16	.30885966	.59848878	**.6679**	**.6959**	**.7228**	**.7420**	**.7564**	**.7677**	**.7768**
		17	.43396970	.73773807	**.7767**	**.7858**	**.8061**	**.8201**	**.8307**	**.8391**	**.8460**
		18	.56603030	.84733966	**.8573**	**.8591**	**.8735**	**.8837**	**.8915**	**.8978**	**.9030**
		19	.69114034	.92161420	**.9131**	**.9141**	**.9241**	**.9314**	**.9370**	**.9414**	**.9450**
		20	.79748388	.96482537	**.9497**	**.9519**	**.9586**	**.9635**	**.9671**	**.9700**	**.9723**
		21	.87850752	.98632749	**.9725**	**.9753**	**.9796**	**.9825**	**.9847**	**.9864**	**.9876**
		22	.93375092	.99543725	**.9860**	**.9885**	**.9909**	**.9925**	**.9937**	**.9945**	**.9951**
		23	.96737733	.99870493	**.9933**	**.9950**	**.9963**	**.9971**	**.9976**	**.9980**	**.9983**
		24	.98559164	.99969055	**.9972**	**.9981**	**.9986**	**.9990**	**.9992**	**.9994**	**.9995**
		25	.99433451	.99993848	**.9989**	**.9993**	**.9995**	**.9997**	**.9998**	**.9998**	**.9999**
		26	.99803341	.99998996	**.9996**	**.9998**	**.9999**	**.9999**	**.9999**	**1.0000**	**1.0000**
		27	.99940338	.99999868	**.9999**	**.9999**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
		28	.99984372	.99999986	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
3	37	0	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	*.00000000*	*.00000000*	*.00000000*
		1	.00000000	.00000000	.00000000	.00000001	.00000001	**.0001**	**.0001**	**.0001**	**.0002**
		2	.00000001	.00000002	.00000005	.00000010	**.0002**	**.0002**	**.0002**	**.0003**	**.0003**
		3	.00000006	.00000025	.00000061	**.0007**	**.0008**	**.0009**	**.0009**	**.0010**	**.0011**
		4	.00000054	.00000216	.00000537	**.0033**	**.0034**	**.0035**	**.0035**	**.0036**	**.0037**
		5	.00000371	.00001472	.00003645	**.0114**	**.0114**	**.0115**	**.0116**	**.0117**	**.0117**
		6	.00002063	.00008105	.00019905	**.0287**	**.0288**	**.0289**	**.0290**	**.0290**	**.0291**
		7	.00009554	.00037022	.00089691	**.0574**	**.0576**	**.0577**	**.0579**	**.0581**	**.0583**
		8	.00037645	.00142849	.00339023	**.0962**	**.0967**	**.0972**	**.0978**	**.0984**	**.0989**
		9	.00128160	.00471877	**.0084**	**.1404**	**.1416**	**.1429**	**.1442**	**.1455**	**.1469**
		10	.00381604	.01348120	**.0238**	**.1867**	**.1899**	**.1931**	**.1963**	**.1996**	**.2029**
		11	.01003693	.03358334	**.0571**	**.2351**	**.2422**	**.2493**	**.2564**	**.2634**	**.2703**
		12	.02351551	.07346243	**.1180**	**.2904**	**.3042**	**.3175**	**.3303**	**.3427**	**.3546**
		13	.04943587	.14204339	**.2117**	**.3592**	**.3813**	**.4018**	**.4208**	**.4382**	**.4543**
		14	.09387078	.24442919	**.3349**	**.4445**	**.4740**	**.4996**	**.5217**	**.5410**	**.5578**
		15	.16200430	.37720592	**.4739**	**.5427**	**.5749**	**.6006**	**.6214**	**.6384**	**.6524**
		16	.25568789	.52676843	**.6101**	**.6442**	**.6736**	**.6953**	**.7117**	**.7247**	**.7351**
		17	.37141468	.67300313	**.7280**	**.7390**	**.7623**	**.7786**	**.7908**	**.8004**	**.8082**
		18	.50000000	.79695752	**.8200**	**.8203**	**.8373**	**.8491**	**.8581**	**.8654**	**.8714**
		19	.62858532	.88787777	**.8864**	**.8846**	**.8966**	**.9052**	**.9118**	**.9172**	**.9216**
		20	.74431211	.94544596	**.9317**	**.9316**	**.9399**	**.9459**	**.9505**	**.9541**	**.9571**
		21	.83799570	.97681283	**.9611**	**.9628**	**.9682**	**.9721**	**.9751**	**.9773**	**.9791**
		22	.90612922	.99146296	**.9793**	**.9815**	**.9848**	**.9871**	**.9888**	**.9900**	**.9910**
		23	.95056413	.99730053	**.9897**	**.9916**	**.9934**	**.9946**	**.9955**	**.9961**	**.9966**
		24	.97648449	.99927359	**.9952**	**.9965**	**.9974**	**.9980**	**.9984**	**.9987**	**.9989**
		25	.98996307	.99983533	**.9981**	**.9987**	**.9991**	**.9993**	**.9995**	**.9996**	**.9997**
		26	.99618396	.99996892	**.9993**	**.9995**	**.9997**	**.9998**	**.9998**	**.9999**	**.9999**

Table 33: Sheet L. Exact (in roman), approximate *(in italics)* and interpolated **(boldfaced)** probabilities of perfect ($i = 0$) and i-imperfect column triplets ($k = 3$) in Bernoulli ($m \times n$)-matrices. The rows with all 1's are omitted

k m i	Width n of Bernoulli ($m \times n$)-matrix								
	3	4	5	6	7	8	9	10	11
3 37 27	.99871840	.99999518	**.9998**	**.9999**	**.9999**	**.9999**	**1.0000**	**1.0000**	**1.0000**
28	.99962355	.99999940	**.9999**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
29	.99990446	.99999994	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
3 38 0	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	*.00000000*	*.00000000*	*.00000000*
1	.00000000	.00000000	.00000000	.00000000	.00000000	**.0001**	**.0001**	**.0001**	**.0002**
2	.00000000	.00000001	.00000003	.00000005	**.0001**	**.0002**	**.0002**	**.0002**	**.0003**
3	.00000003	.00000013	.00000033	**.0005**	**.0006**	**.0006**	**.0007**	**.0008**	**.0008**
4	.00000030	.00000120	.00000300	**.0023**	**.0024**	**.0024**	**.0025**	**.0026**	**.0027**
5	.00000213	.00000845	.00002096	**.0083**	**.0084**	**.0085**	**.0085**	**.0086**	**.0087**
6	.00001217	.00004797	.00011819	**.0222**	**.0223**	**.0224**	**.0225**	**.0226**	**.0227**
7	.00005808	.00022628	.00055109	**.0468**	**.0470**	**.0471**	**.0472**	**.0474**	**.0475**
8	.00023599	.00090301	.00216070	**.0820**	**.0823**	**.0827**	**.0831**	**.0835**	**.0839**
9	.00082903	.00309027	**.0056**	**.1238**	**.1246**	**.1254**	**.1262**	**.1271**	**.1280**
10	.00254882	.00916195	**.0167**	**.1687**	**.1709**	**.1731**	**.1754**	**.1777**	**.1800**
11	.00692648	.02372346	**.0421**	**.2150**	**.2202**	**.2254**	**.2306**	**.2359**	**.2411**
12	.01677622	.05401617	**.0909**	**.2655**	**.2762**	**.2866**	**.2968**	**.3068**	**.3165**
13	.03647569	.10882512	**.1703**	**.3264**	**.3448**	**.3622**	**.3786**	**.3941**	**.4085**
14	.07165333	.19521054	**.2806**	**.4026**	**.4292**	**.4529**	**.4740**	**.4929**	**.5098**
15	.12793754	.31390977	**.4124**	**.4937**	**.5253**	**.5515**	**.5733**	**.5916**	**.6070**
16	.20884610	.45611916	**.5494**	**.5928**	**.6240**	**.6478**	**.6663**	**.6810**	**.6929**
17	.31355129	.60460519	**.6747**	**.6901**	**.7164**	**.7351**	**.7491**	**.7600**	**.7688**
18	.43570734	.73959197	**.7776**	**.7776**	**.7974**	**.8111**	**.8214**	**.8297**	**.8364**
19	.56429266	.84627301	**.8552**	**.8503**	**.8645**	**.8745**	**.8822**	**.8884**	**.8936**
20	.68644871	.91941692	**.9100**	**.9063**	**.9163**	**.9235**	**.9291**	**.9336**	**.9373**
21	.79115390	.96280925	**.9468**	**.9460**	**.9528**	**.9577**	**.9615**	**.9645**	**.9668**
22	.87206246	.98500945	**.9703**	**.9715**	**.9759**	**.9790**	**.9813**	**.9830**	**.9844**
23	.92834667	.99476507	**.9845**	**.9863**	**.9888**	**.9906**	**.9919**	**.9928**	**.9935**
24	.96352431	.99842928	**.9925**	**.9940**	**.9953**	**.9962**	**.9968**	**.9973**	**.9976**
25	.98322378	.99959869	**.9966**	**.9976**	**.9982**	**.9986**	**.9989**	**.9991**	**.9992**
26	.99307352	.99991357	**.9987**	**.9991**	**.9994**	**.9995**	**.9997**	**.9997**	**.9998**
27	.99745118	.99998449	**.9995**	**.9997**	**.9998**	**.9999**	**.9999**	**.9999**	**.9999**
28	.99917097	.99999771	**.9998**	**.9999**	**.9999**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
29	.99976401	.99999973	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
30	.99994192	.99999997	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
3 39 0	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	*.00000000*	*.00000000*	*.00000000*
1	.00000000	.00000000	.00000000	.00000000	.00000000	**.0001**	**.0001**	**.0001**	**.0002**
2	.00000000	.00000001	.00000001	.00000003	**.0001**	**.0002**	**.0002**	**.0002**	**.0003**
3	.00000002	.00000007	.00000018	**.0005**	**.0006**	**.0006**	**.0007**	**.0008**	**.0008**
4	.00000017	.00000067	.00000167	**.0023**	**.0024**	**.0024**	**.0025**	**.0026**	**.0027**
5	.00000121	.00000483	.00001200	**.0083**	**.0084**	**.0085**	**.0085**	**.0086**	**.0087**
6	.00000715	.00002825	.00006979	**.0222**	**.0223**	**.0224**	**.0225**	**.0225**	**.0226**
7	.00003513	.00013746	.00033626	**.0468**	**.0469**	**.0470**	**.0471**	**.0472**	**.0473**
8	.00014704	.00056664	.00136533	**.0816**	**.0818**	**.0820**	**.0823**	**.0825**	**.0828**
9	.00053251	.00200619	**.0033**	**.1230**	**.1236**	**.1242**	**.1248**	**.1254**	**.1260**
10	.00168892	.00616357	**.0107**	**.1666**	**.1681**	**.1696**	**.1712**	**.1729**	**.1745**
11	.00473765	.01656473	**.0289**	**.2099**	**.2136**	**.2174**	**.2212**	**.2250**	**.2289**
12	.01185135	.03920298	**.0664**	**.2553**	**.2633**	**.2713**	**.2792**	**.2870**	**.2947**
13	.02662596	.08218875	**.1312**	**.3090**	**.3239**	**.3382**	**.3519**	**.3650**	**.3776**
14	.05406451	.15352174	**.2264**	**.3779**	**.4010**	**.4222**	**.4415**	**.4593**	**.4755**
15	.09979543	.25707192	**.3466**	**.4638**	**.4936**	**.5191**	**.5411**	**.5601**	**.5765**
16	.16839182	.38860433	**.4788**	**.5618**	**.5937**	**.6189**	**.6391**	**.6556**	**.6690**
17	.26119869	.53476234	**.6074**	**.6618**	**.6905**	**.7116**	**.7276**	**.7401**	**.7501**
18	.37462931	.67672976	**.7198**	**.7537**	**.7765**	**.7924**	**.8042**	**.8135**	**.8211**
19	.50000000	.79711964	**.8101**	**.8311**	**.8478**	**.8595**	**.8683**	**.8754**	**.8813**
20	.62537069	.88609551	**.8778**	**.8914**	**.9033**	**.9118**	**.9184**	**.9238**	**.9282**
21	.73880131	.94327708	**.9256**	**.9350**	**.9433**	**.9493**	**.9540**	**.9577**	**.9607**

Table 33: Sheet M. Exact (in roman), approximate *(in italics)* and interpolated **(boldfaced)** probabilities of perfect ($i = 0$) and i-imperfect column triplets ($k = 3$) in Bernoulli ($m \times n$)-matrices. The rows with all 1's are omitted

k m i	Width n of Bernoulli $(m \times n)$-matrix								
	3	4	5	6	7	8	9	10	11
3 39 22	.83160818	.97514217	**.9572**	**.9639**	**.9694**	**.9734**	**.9765**	**.9788**	**.9807**
23	.90020457	.99048646	**.9769**	**.9814**	**.9849**	**.9873**	**.9891**	**.9904**	**.9915**
24	.94593549	.99684465	**.9884**	**.9912**	**.9931**	**.9945**	**.9954**	**.9961**	**.9966**
25	.97337404	.99910039	**.9945**	**.9961**	**.9971**	**.9978**	**.9982**	**.9986**	**.9988**
26	.98814865	.99978147	**.9977**	**.9984**	**.9989**	**.9992**	**.9994**	**.9995**	**.9996**
27	.99526235	.99995522	**.9991**	**.9994**	**.9996**	**.9997**	**.9998**	**.9998**	**.9999**
28	.99831108	.99999235	**.9997**	**.9998**	**.9999**	**.9999**	**.9999**	**1.0000**	**1.0000**
29	.99946749	.99999892	**.9999**	**.9999**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
30	.99985296	.99999988	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
3 40 0	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	*.00000000*	*.00000000*	*.00000000*
1	.00000000	.00000000	.00000000	.00000000	.00000000	**.0001**	**.0001**	**.0001**	**.0002**
2	.00000000	.00000000	.00000001	.00000001	**.0001**	**.0002**	**.0002**	**.0002**	**.0003**
3	.00000001	.00000004	.00000010	**.0004**	**.0004**	**.0005**	**.0006**	**.0006**	**.0007**
4	.00000009	.00000037	.00000092	**.0016**	**.0017**	**.0017**	**.0018**	**.0019**	**.0019**
5	.00000069	.00000275	.00000684	**.0060**	**.0061**	**.0062**	**.0062**	**.0063**	**.0064**
6	.00000418	.00001656	.00004100	**.0170**	**.0171**	**.0172**	**.0173**	**.0173**	**.0174**
7	.00002114	.00008303	.00020387	**.0377**	**.0378**	**.0379**	**.0380**	**.0381**	**.0382**
8	.00009108	.00035313	.00085593	**.0688**	**.0690**	**.0691**	**.0693**	**.0695**	**.0697**
9	.00033977	.00129182	**.0023**	**.1079**	**.1083**	**.1087**	**.1091**	**.1096**	**.1101**
10	.00111072	.00410720	**.0074**	**.1504**	**.1514**	**.1525**	**.1537**	**.1549**	**.1561**
11	.00321329	.01144078	**.0207**	**.1927**	**.1953**	**.1980**	**.2007**	**.2035**	**.2062**
12	.00829450	.02810478	**.0496**	**.2357**	**.2417**	**.2477**	**.2537**	**.2596**	**.2655**
13	.01923865	.06123484	**.1021**	**.2840**	**.2958**	**.3072**	**.3183**	**.3290**	**.3395**
14	.04034523	.11897459	**.1833**	**.3443**	**.3637**	**.3820**	**.3990**	**.4149**	**.4298**
15	.07692997	.20728490	**.2916**	**.4210**	**.4481**	**.4721**	**.4933**	**.5121**	**.5288**
16	.13409363	.32586824	**.4175**	**.5125**	**.5439**	**.5698**	**.5911**	**.6089**	**.6239**
17	.21479525	.46565618	**.5467**	**.6109**	**.6415**	**.6647**	**.6826**	**.6968**	**.7083**
18	.31791400	.61023384	**.6657**	**.7062**	**.7319**	**.7501**	**.7636**	**.7742**	**.7828**
19	.43731466	.74129554	**.7659**	**.7905**	**.8099**	**.8233**	**.8334**	**.8415**	**.8481**
20	.56268534	.84527767	**.8441**	**.8594**	**.8734**	**.8833**	**.8909**	**.8971**	**.9022**
21	.68208600	.91734100	**.9015**	**.9117**	**.9217**	**.9289**	**.9345**	**.9390**	**.9427**
22	.78520475	.96086116	**.9412**	**.9485**	**.9554**	**.9604**	**.9642**	**.9672**	**.9697**
23	.86590637	.98369587	**.9669**	**.9722**	**.9767**	**.9799**	**.9823**	**.9841**	**.9856**
24	.92307003	.99406797	**.9826**	**.9861**	**.9888**	**.9907**	**.9920**	**.9931**	**.9938**
25	.95965477	.99812895	**.9914**	**.9936**	**.9951**	**.9960**	**.9967**	**.9972**	**.9976**
26	.98076135	.99949244	**.9960**	**.9973**	**.9980**	**.9985**	**.9988**	**.9990**	**.9992**
27	.99170550	.99988262	**.9984**	**.9989**	**.9992**	**.9994**	**.9996**	**.9997**	**.9997**
28	.99678671	.99997708	**.9994**	**.9996**	**.9997**	**.9998**	**.9999**	**.9999**	**.9999**
29	.99888928	.99999627	**.9998**	**.9999**	**.9999**	**.9999**	**1.0000**	**1.0000**	**1.0000**
30	.99966023	.99999950	**.9999**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
31	.99990892	.99999995	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
3 41 0	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	*.00000000*	*.00000000*	*.00000000*
1	.00000000	.00000000	.00000000	.00000000	.00000000	**.0001**	**.0001**	**.0001**	**.0002**
2	.00000000	.00000000	.00000000	.00000001	**.0001**	**.0001**	**.0002**	**.0002**	**.0003**
3	.00000001	.00000002	.00000005	**.0003**	**.0003**	**.0004**	**.0005**	**.0005**	**.0006**
4	.00000005	.00000020	.00000051	**.0011**	**.0012**	**.0012**	**.0013**	**.0014**	**.0014**
5	.00000039	.00000156	.00000389	**.0043**	**.0044**	**.0045**	**.0045**	**.0046**	**.0047**
6	.00000244	.00000967	.00002398	**.0129**	**.0130**	**.0131**	**.0131**	**.0132**	**.0133**
7	.00001266	.00004989	.00012287	**.0301**	**.0302**	**.0303**	**.0303**	**.0304**	**.0305**
8	.00005611	.00021865	.00053266	**.0574**	**.0575**	**.0577**	**.0578**	**.0579**	**.0581**
9	.00021543	.00082550	**.0016**	**.0936**	**.0940**	**.0943**	**.0947**	**.0950**	**.0954**
10	.00072525	.00271263	**.0050**	**.1344**	**.1351**	**.1358**	**.1366**	**.1374**	**.1381**
11	.00216200	.00782138	**.0148**	**.1763**	**.1782**	**.1801**	**.1820**	**.1839**	**.1859**
12	.00575389	.01991677	**.0369**	**.2180**	**.2223**	**.2267**	**.2311**	**.2355**	**.2400**
13	.01376658	.04504074	**.0789**	**.2622**	**.2713**	**.2801**	**.2889**	**.2974**	**.3058**
14	.02979194	.09091995	**.1472**	**.3154**	**.3313**	**.3465**	**.3610**	**.3748**	**.3878**

Table 33: Sheet N. Exact (in roman), approximate *(in italics)* and interpolated **(boldfaced)** probabilities of perfect ($i = 0$) and i-imperfect column triplets ($k = 3$) in Bernoulli ($m \times n$)-matrices. The rows with all 1's are omitted

k m i	Width n of Bernoulli $(m \times n)$-matrix								
	3	4	5	6	7	8	9	10	11
3 41 15	.05863760	.16466569	**.2428**	**.3831**	**.4070**	**.4287**	**.4484**	**.4663**	**.4825**
16	.10551180	.26906627	**.3598**	**.4667**	**.4966**	**.5221**	**.5438**	**.5625**	**.5784**
17	.17444444	.39924361	**.4864**	**.5611**	**.5926**	**.6173**	**.6370**	**.6529**	**.6659**
18	.26635463	.54215209	**.6091**	**.6573**	**.6855**	**.7060**	**.7215**	**.7336**	**.7433**
19	.37761433	.68016284	**.7172**	**.7465**	**.7688**	**.7843**	**.7958**	**.8049**	**.8123**
20	.50000000	.79726532	**.8055**	**.8229**	**.8393**	**.8507**	**.8594**	**.8664**	**.8722**
21	.62238567	.88442257	**.8728**	**.8837**	**.8955**	**.9039**	**.9105**	**.9158**	**.9202**
22	.73364537	.94120574	**.9212**	**.9288**	**.9371**	**.9432**	**.9479**	**.9516**	**.9547**
23	.82555556	.97350550	**.9539**	**.9595**	**.9652**	**.9693**	**.9724**	**.9748**	**.9768**
24	.89448820	.98949714	**.9747**	**.9788**	**.9823**	**.9849**	**.9868**	**.9882**	**.9893**
25	.94136240	.99636285	**.9869**	**.9897**	**.9918**	**.9932**	**.9942**	**.9950**	**.9956**
26	.97020806	.99890765	**.9937**	**.9954**	**.9965**	**.9972**	**.9977**	**.9981**	**.9983**
27	.98623342	.99971770	**.9972**	**.9981**	**.9986**	**.9989**	**.9992**	**.9993**	**.9994**
28	.99424611	.99993776	**.9989**	**.9993**	**.9995**	**.9996**	**.9997**	**.9998**	**.9998**
29	.99783800	.99998841	**.9996**	**.9997**	**.9998**	**.9999**	**.9999**	**.9999**	**.9999**
30	.99927475	.99999820	**.9999**	**.9999**	**.9999**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
31	.99978457	.99999977	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
32	.99994389	.99999998	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
3 42 0	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	*.00000000*	*.00000000*	*.00000000*
1	.00000000	.00000000	.00000000	.00000000	.00000000	**.0001**	**.0001**	**.0001**	**.0002**
2	.00000000	.00000000	.00000000	.00000000	**.0001**	**.0001**	**.0002**	**.0002**	**.0003**
3	.00000000	.00000001	.00000003	**.0003**	**.0003**	**.0004**	**.0005**	**.0005**	**.0006**
4	.00000003	.00000011	.00000028	**.0011**	**.0012**	**.0012**	**.0013**	**.0014**	**.0014**
5	.00000022	.00000088	.00000220	**.0043**	**.0044**	**.0045**	**.0045**	**.0046**	**.0047**
6	.00000141	.00000562	.00001396	**.0129**	**.0130**	**.0130**	**.0131**	**.0132**	**.0133**
7	.00000755	.00002982	.00007364	**.0301**	**.0301**	**.0302**	**.0303**	**.0304**	**.0305**
8	.00003439	.00013457	.00032923	**.0574**	**.0575**	**.0576**	**.0577**	**.0578**	**.0579**
9	.00013577	.00052375	**.0012**	**.0935**	**.0937**	**.0940**	**.0943**	**.0946**	**.0949**
10	.00047034	.00177669	**.0037**	**.1338**	**.1343**	**.1348**	**.1354**	**.1359**	**.1365**
11	.00144362	.00529590	**.0113**	**.1746**	**.1759**	**.1772**	**.1786**	**.1800**	**.1814**
12	.00395795	.01396141	**.0292**	**.2137**	**.2169**	**.2201**	**.2233**	**.2266**	**.2299**
13	.00976024	.03272943	**.0645**	**.2537**	**.2605**	**.2673**	**.2740**	**.2806**	**.2872**
14	.02177926	.06856213	**.1237**	**.3006**	**.3133**	**.3257**	**.3376**	**.3491**	**.3602**
15	.04421477	.12895286	**.2095**	**.3613**	**.3817**	**.4007**	**.4183**	**.4346**	**.4497**
16	.08207470	.21886079	**.3180**	**.4392**	**.4668**	**.4909**	**.5122**	**.5309**	**.5473**
17	.13997812	.33712177	**.4394**	**.5311**	**.5624**	**.5879**	**.6089**	**.6262**	**.6406**
18	.22039953	.47452193	**.5616**	**.6286**	**.6587**	**.6813**	**.6987**	**.7125**	**.7235**
19	.32198448	.61543465	**.6737**	**.7215**	**.7466**	**.7643**	**.7775**	**.7878**	**.7961**
20	.43880716	.74286779	**.7687**	**.8023**	**.8213**	**.8345**	**.8444**	**.8522**	**.8588**
21	.56119284	.84434597	**.8441**	**.8673**	**.8811**	**.8909**	**.8984**	**.9045**	**.9096**
22	.67801552	.91537558	**.9002**	**.9161**	**.9260**	**.9332**	**.9388**	**.9433**	**.9470**
23	.77960047	.95897855	**.9395**	**.9502**	**.9572**	**.9622**	**.9661**	**.9692**	**.9717**
24	.86002188	.98239023	**.9654**	**.9724**	**.9770**	**.9803**	**.9828**	**.9847**	**.9863**
25	.91792530	.99334967	**.9814**	**.9857**	**.9885**	**.9905**	**.9920**	**.9931**	**.9939**
26	.95578523	.99780541	**.9905**	**.9931**	**.9947**	**.9958**	**.9965**	**.9971**	**.9975**
27	.97822074	.99937165	**.9955**	**.9969**	**.9977**	**.9983**	**.9986**	**.9989**	**.9991**
28	.99023976	.99984511	**.9981**	**.9987**	**.9991**	**.9993**	**.9995**	**.9996**	**.9997**
29	.99604205	.99996741	**.9992**	**.9995**	**.9997**	**.9998**	**.9998**	**.9999**	**.9999**
30	.99855638	.99999420	**.9997**	**.9998**	**.9999**	**.9999**	**.9999**	**1.0000**	**1.0000**
31	.99952966	.99999914	**.9999**	**.9999**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
32	.99986423	.99999989	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
3 43 0	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	*.00000000*	*.00000000*	*.00000000*
1	.00000000	.00000000	.00000000	.00000000	.00000000	**.0001**	**.0001**	**.0001**	**.0002**
2	.00000000	.00000000	.00000000	.00000000	**.0001**	**.0001**	**.0002**	**.0002**	**.0003**
3	.00000000	.00000001	.00000002	**.0002**	**.0003**	**.0004**	**.0004**	**.0005**	**.0006**
4	.00000002	.00000006	.00000016	**.0008**	**.0008**	**.0009**	**.0010**	**.0010**	**.0011**

Table 33: Sheet O. Exact (in roman), approximate *(in italics)* and interpolated **(boldfaced)** probabilities of perfect ($i = 0$) and i-imperfect column triplets ($k = 3$) in Bernoulli ($m \times n$)-matrices. The rows with all 1's are omitted

k	m	i	Width n of Bernoulli ($m \times n$)-matrix								
			3	4	5	6	7	8	9	10	11
3	43	5	.00000012	.00000050	.00000124	**.0031**	**.0031**	**.0032**	**.0033**	**.0034**	**.0034**
		6	.00000082	.00000325	.00000809	**.0097**	**.0097**	**.0098**	**.0099**	**.0100**	**.0100**
		7	.00000448	.00001774	.00004391	**.0237**	**.0238**	**.0239**	**.0239**	**.0240**	**.0241**
		8	.00002097	.00008236	.00020221	**.0473**	**.0474**	**.0475**	**.0476**	**.0477**	**.0478**
		9	.00008508	.00033008	**.0009**	**.0804**	**.0806**	**.0809**	**.0811**	**.0814**	**.0816**
		10	.00030305	.00115459	**.0027**	**.1190**	**.1194**	**.1198**	**.1202**	**.1206**	**.1210**
		11	.00095698	.00355367	**.0083**	**.1594**	**.1604**	**.1614**	**.1624**	**.1634**	**.1645**
		12	.00270079	.00968697	**.0218**	**.1982**	**.2004**	**.2027**	**.2049**	**.2073**	**.2096**
		13	.00685909	.02351195	**.0498**	**.2364**	**.2415**	**.2465**	**.2516**	**.2566**	**.2616**
		14	.01576975	.05105310	**.0986**	**.2786**	**.2886**	**.2983**	**.3079**	**.3172**	**.3262**
		15	.03299702	.09961520	**.1721**	**.3315**	**.3484**	**.3645**	**.3797**	**.3940**	**.4075**
		16	.06314474	.17546788	**.2693**	**.4001**	**.4247**	**.4469**	**.4668**	**.4848**	**.5010**
		17	.11102641	.28045449	**.3831**	**.4846**	**.5147**	**.5401**	**.5615**	**.5798**	**.5954**
		18	.18018883	.40919905	**.5030**	**.5789**	**.6101**	**.6343**	**.6535**	**.6689**	**.6815**
		19	.27119201	.54900768	**.6183**	**.6737**	**.7013**	**.7213**	**.7364**	**.7481**	**.7575**
		20	.38039582	.68333826	**.7206**	**.7602**	**.7820**	**.7972**	**.8084**	**.8172**	**.8245**
		21	.50000000	.79739695	**.8052**	**.8330**	**.8491**	**.8604**	**.8689**	**.8758**	**.8815**
		22	.61960418	.88284805	**.8710**	**.8902**	**.9018**	**.9102**	**.9167**	**.9220**	**.9264**
		23	.72880799	.93922513	**.9190**	**.9321**	**.9404**	**.9465**	**.9512**	**.9550**	**.9582**
		24	.81981117	.97190374	**.9520**	**.9607**	**.9664**	**.9706**	**.9738**	**.9763**	**.9783**
		25	.88897359	.98849889	**.9731**	**.9788**	**.9825**	**.9851**	**.9871**	**.9886**	**.9898**
		26	.93685526	.99585773	**.9859**	**.9893**	**.9915**	**.9930**	**.9941**	**.9950**	**.9956**
		27	.96700298	.99869593	**.9929**	**.9949**	**.9962**	**.9970**	**.9975**	**.9980**	**.9983**
		28	.98423025	.99964363	**.9968**	**.9978**	**.9984**	**.9988**	**.9990**	**.9992**	**.9994**
		29	.99314091	.99991611	**.9987**	**.9991**	**.9994**	**.9995**	**.9997**	**.9997**	**.9998**
		30	.99729921	.99998313	**.9995**	**.9997**	**.9998**	**.9998**	**.9999**	**.9999**	**.9999**
		31	.99904302	.99999713	**.9998**	**.9999**	**.9999**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
		32	.99969695	.99999959	**.9999**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
		33	.99991492	.99999995	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
3	44	0	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	*.00000000*	*.00000000*	*.00000000*
		1	.00000000	.00000000	.00000000	.00000000	.00000000	**.0001**	**.0001**	**.0001**	**.0002**
		2	.00000000	.00000000	.00000000	.00000000	**.0001**	**.0001**	**.0002**	**.0002**	**.0003**
		3	.00000000	.00000000	.00000001	**.0002**	**.0003**	**.0003**	**.0004**	**.0005**	**.0005**
		4	.00000001	.00000003	.00000009	**.0005**	**.0006**	**.0007**	**.0008**	**.0008**	**.0009**
		5	.00000007	.00000028	.00000070	**.0022**	**.0022**	**.0023**	**.0024**	**.0025**	**.0025**
		6	.00000047	.00000188	.00000467	**.0072**	**.0072**	**.0073**	**.0074**	**.0075**	**.0075**
		7	.00000265	.00001051	.00002606	**.0185**	**.0186**	**.0186**	**.0187**	**.0188**	**.0189**
		8	.00001272	.00005013	.00012346	**.0387**	**.0387**	**.0388**	**.0389**	**.0390**	**.0391**
		9	.00005302	.00020672	**.0009**	**.0684**	**.0686**	**.0688**	**.0691**	**.0693**	**.0695**
		10	.00019407	.00074482	**.0028**	**.1049**	**.1052**	**.1055**	**.1058**	**.1062**	**.1065**
		11	.00063002	.00236445	**.0082**	**.1443**	**.1449**	**.1455**	**.1462**	**.1468**	**.1475**
		12	.00182888	.00665662	**.0212**	**.1832**	**.1848**	**.1864**	**.1881**	**.1897**	**.1914**
		13	.00477994	.01670832	**.0469**	**.2207**	**.2244**	**.2281**	**.2318**	**.2355**	**.2393**
		14	.01131442	.03756262	**.0906**	**.2596**	**.2672**	**.2748**	**.2822**	**.2895**	**.2967**
		15	.02438338	.07595578	**.1557**	**.3060**	**.3197**	**.3330**	**.3456**	**.3578**	**.3694**
		16	.04807088	.13873642	**.2416**	**.3660**	**.3873**	**.4069**	**.4249**	**.4416**	**.4569**
		17	.08708557	.22995686	**.3440**	**.4421**	**.4701**	**.4944**	**.5156**	**.5341**	**.5503**
		18	.14560762	.34772996	**.4554**	**.5311**	**.5623**	**.5874**	**.6079**	**.6248**	**.6388**
		19	.22569042	.48278822	**.5673**	**.6252**	**.6548**	**.6769**	**.6938**	**.7072**	**.7178**
		20	.32579391	.62025788	**.6717**	**.7153**	**.7399**	**.7573**	**.7701**	**.7801**	**.7882**
		21	.44019791	.74432455	**.7627**	**.7947**	**.8134**	**.8263**	**.8360**	**.8437**	**.8501**
		22	.55980209	.84347136	**.8371**	**.8598**	**.8735**	**.8831**	**.8905**	**.8966**	**.9016**
		23	.67420609	.91351110	**.8940**	**.9097**	**.9196**	**.9268**	**.9323**	**.9369**	**.9406**
		24	.77430958	.95715876	**.9347**	**.9455**	**.9525**	**.9576**	**.9615**	**.9647**	**.9672**
		25	.85439238	.98109533	**.9621**	**.9692**	**.9739**	**.9773**	**.9799**	**.9819**	**.9835**
		26	.91291443	.99261352	**.9793**	**.9838**	**.9868**	**.9888**	**.9904**	**.9916**	**.9925**

Table 33: Sheet P. Exact (in roman), approximate *(in italics)* and interpolated **(boldfaced)** probabilities of perfect ($i = 0$) and i-imperfect column triplets ($k = 3$) in Bernoulli ($m \times n$)-matrices. The rows with all 1's are omitted

k m i	Width n of Bernoulli $(m \times n)$-matrix								
	3	4	5	6	7	8	9	10	11
3 44 27	.95192912	.99746016	**.9893**	**.9920**	**.9937**	**.9949**	**.9958**	**.9964**	**.9968**
28	.97561662	.99923634	**.9948**	**.9963**	**.9973**	**.9979**	**.9983**	**.9986**	**.9988**
29	.98868558	.99980059	**.9977**	**.9985**	**.9989**	**.9992**	**.9993**	**.9995**	**.9996**
30	.99522006	.99995512	**.9991**	**.9994**	**.9996**	**.9997**	**.9998**	**.9998**	**.9999**
31	.99817112	.99999137	**.9997**	**.9998**	**.9999**	**.9999**	**.9999**	**.9999**	**1.0000**
32	.99936998	.99999860	**.9999**	**.9999**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
33	.99980593	.99999981	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
34	.99994698	.99999998	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
3 45 0	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	*.00000000*	*.00000000*	*.00000000*
1	.00000000	.00000000	.00000000	.00000000	.00000000	**.0001**	**.0001**	**.0001**	**.0002**
2	.00000000	.00000000	.00000000	.00000000	**.0001**	**.0001**	**.0002**	**.0002**	**.0003**
3	.00000000	.00000000	.00000000	**.0002**	**.0002**	**.0003**	**.0004**	**.0004**	**.0005**
4	.00000000	.00000002	.00000005	**.0004**	**.0005**	**.0005**	**.0006**	**.0007**	**.0007**
5	.00000004	.00000016	.00000039	**.0015**	**.0016**	**.0017**	**.0017**	**.0018**	**.0019**
6	.00000027	.00000108	.00000269	**.0053**	**.0053**	**.0054**	**.0055**	**.0056**	**.0056**
7	.00000156	.00000620	.00001539	**.0143**	**.0143**	**.0144**	**.0145**	**.0146**	**.0146**
8	.00000769	.00003036	.00007497	**.0312**	**.0313**	**.0314**	**.0315**	**.0315**	**.0316**
9	.00003287	.00012869	**.0008**	**.0576**	**.0578**	**.0580**	**.0582**	**.0584**	**.0587**
10	.00012354	.00047716	**.0022**	**.0916**	**.0918**	**.0921**	**.0924**	**.0926**	**.0929**
11	.00041204	.00156069	**.0064**	**.1298**	**.1302**	**.1307**	**.1312**	**.1317**	**.1322**
12	.00122945	.00453283	**.0169**	**.1686**	**.1698**	**.1709**	**.1721**	**.1734**	**.1746**
13	.00330441	.01175256	**.0379**	**.2057**	**.2083**	**.2110**	**.2137**	**.2164**	**.2191**
14	.00804718	.02732485	**.0747**	**.2427**	**.2484**	**.2541**	**.2598**	**.2654**	**.2710**
15	.01784890	.05720133	**.1307**	**.2842**	**.2951**	**.3058**	**.3161**	**.3261**	**.3358**
16	.03622713	.10824186	**.2066**	**.3365**	**.3544**	**.3713**	**.3871**	**.4019**	**.4158**
17	.06757823	.18592682	**.2997**	**.4040**	**.4292**	**.4518**	**.4719**	**.4899**	**.5061**
18	.11634660	.29127680	**.4045**	**.4862**	**.5164**	**.5416**	**.5628**	**.5807**	**.5959**
19	.18564902	.41853645	**.5137**	**.5773**	**.6080**	**.6318**	**.6506**	**.6655**	**.6776**
20	.27574216	.55538843	**.6199**	**.6689**	**.6960**	**.7156**	**.7302**	**.7415**	**.7507**
21	.38299591	.68628614	**.7163**	**.7532**	**.7746**	**.7895**	**.8005**	**.8091**	**.8162**
22	.50000000	.79751647	**.7985**	**.8253**	**.8412**	**.8523**	**.8607**	**.8675**	**.8731**
23	.61700409	.88136254	**.8642**	**.8831**	**.8947**	**.9029**	**.9094**	**.9146**	**.9190**
24	.72425784	.93732903	**.9133**	**.9264**	**.9347**	**.9408**	**.9456**	**.9494**	**.9526**
25	.81435098	.97033732	**.9478**	**.9566**	**.9624**	**.9667**	**.9699**	**.9725**	**.9746**
26	.88365340	.98749502	**.9703**	**.9761**	**.9799**	**.9827**	**.9848**	**.9863**	**.9876**
27	.93242177	.99533174	**.9842**	**.9878**	**.9901**	**.9917**	**.9929**	**.9938**	**.9945**
28	.96377287	.99846593	**.9920**	**.9941**	**.9954**	**.9963**	**.9969**	**.9974**	**.9978**
29	.98215110	.99955900	**.9962**	**.9974**	**.9980**	**.9985**	**.9988**	**.9990**	**.9992**
30	.99195282	.99988985	**.9984**	**.9989**	**.9992**	**.9994**	**.9996**	**.9996**	**.9997**
31	.99669559	.99997627	**.9994**	**.9996**	**.9997**	**.9998**	**.9998**	**.9999**	**.9999**
32	.99877055	.99999563	**.9998**	**.9999**	**.9999**	**.9999**	**1.0000**	**1.0000**	**1.0000**
33	.99958796	.99999932	**.9999**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
34	.99987646	.99999991	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
3 46 0	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	*.00000000*	*.00000000*	*.00000000*
1	.00000000	.00000000	.00000000	.00000000	.00000000	**.0001**	**.0001**	**.0001**	**.0002**
2	.00000000	.00000000	.00000000	.00000000	**.0001**	**.0001**	**.0002**	**.0002**	**.0003**
3	.00000000	.00000000	.00000000	**.0002**	**.0002**	**.0003**	**.0004**	**.0004**	**.0005**
4	.00000000	.00000001	.00000003	**.0004**	**.0005**	**.0005**	**.0006**	**.0007**	**.0007**
5	.00000002	.00000009	.00000022	**.0015**	**.0016**	**.0017**	**.0017**	**.0018**	**.0019**
6	.00000016	.00000062	.00000154	**.0053**	**.0053**	**.0054**	**.0055**	**.0056**	**.0056**
7	.00000092	.00000364	.00000905	**.0143**	**.0143**	**.0144**	**.0145**	**.0146**	**.0146**
8	.00000462	.00001830	.00004529	**.0312**	**.0313**	**.0314**	**.0314**	**.0315**	**.0316**
9	.00002028	.00007967	**.0007**	**.0576**	**.0578**	**.0580**	**.0582**	**.0584**	**.0586**
10	.00007821	.00030370	**.0021**	**.0915**	**.0917**	**.0919**	**.0922**	**.0924**	**.0927**
11	.00026779	.00102244	**.0060**	**.1294**	**.1298**	**.1302**	**.1305**	**.1309**	**.1313**
12	.00082075	.00306028	**.0157**	**.1674**	**.1682**	**.1689**	**.1697**	**.1705**	**.1714**

Table 33: Sheet Q. Exact (in roman), approximate *(in italics)* and interpolated **(boldfaced)** probabilities of perfect ($i = 0$) and i-imperfect column triplets ($k = 3$) in Bernoulli ($m \times n$)-matrices. The rows with all 1's are omitted

k m i	Width n of Bernoulli $(m \times n)$-matrix								
	3	4	5	6	7	8	9	10	11
3 46 13	.00226693	.00818718	**.0354**	**.2031**	**.2050**	**.2069**	**.2088**	**.2108**	**.2128**
14	.00567580	.01966471	**.0698**	**.2372**	**.2415**	**.2457**	**.2500**	**.2542**	**.2584**
15	.01294804	.04257202	**.1220**	**.2742**	**.2826**	**.2909**	**.2991**	**.3071**	**.3149**
16	.02703802	.08338008	**.1929**	**.3205**	**.3352**	**.3492**	**.3626**	**.3754**	**.3875**
17	.05190268	.14830878	**.2804**	**.3817**	**.4037**	**.4239**	**.4424**	**.4593**	**.4748**
18	.09196241	.24059351	**.3800**	**.4592**	**.4875**	**.5119**	**.5330**	**.5513**	**.5672**
19	.15099781	.35774646	**.4859**	**.5487**	**.5796**	**.6045**	**.6245**	**.6410**	**.6545**
20	.23069559	.49051687	**.5914**	**.6418**	**.6709**	**.6925**	**.7090**	**.7219**	**.7323**
21	.32936904	.62474600	**.6896**	**.7296**	**.7537**	**.7706**	**.7832**	**.7929**	**.8008**
22	.44149796	.74567913	**.7752**	**.8057**	**.8241**	**.8368**	**.8462**	**.8538**	**.8601**
23	.55850204	.84264817	**.8450**	**.8672**	**.8807**	**.8902**	**.8976**	**.9035**	**.9085**
24	.67063096	.91173911	**.8983**	**.9139**	**.9238**	**.9309**	**.9364**	**.9410**	**.9447**
25	.76930441	.95539909	**.9365**	**.9473**	**.9543**	**.9594**	**.9634**	**.9666**	**.9692**
26	.84900219	.97981338	**.9623**	**.9695**	**.9743**	**.9778**	**.9805**	**.9826**	**.9842**
27	.90803759	.99186254	**.9788**	**.9835**	**.9865**	**.9887**	**.9903**	**.9916**	**.9926**
28	.94809732	.99709474	**.9887**	**.9916**	**.9934**	**.9946**	**.9955**	**.9962**	**.9967**
29	.97296198	.99908665	**.9943**	**.9960**	**.9970**	**.9976**	**.9981**	**.9984**	**.9987**
30	.98705196	.99974871	**.9974**	**.9982**	**.9987**	**.9990**	**.9992**	**.9994**	**.9995**
31	.99432420	.99993990	**.9989**	**.9993**	**.9995**	**.9996**	**.9997**	**.9998**	**.9998**
32	.99773307	.99998760	**.9996**	**.9997**	**.9998**	**.9999**	**.9999**	**.9999**	**.9999**
33	.99917925	.99999781	**.9998**	**.9999**	**.9999**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
34	.99973221	.99999967	**.9999**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
35	.99992179	.99999996	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
3 47 0	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	*.00000000*	*.00000000*	*.00000000*
1	.00000000	.00000000	.00000000	.00000000	.00000000	**.0001**	**.0001**	**.0001**	**.0002**
2	.00000000	.00000000	.00000000	.00000000	**.0001**	**.0001**	**.0002**	**.0002**	**.0003**
3	.00000000	.00000000	.00000000	**.0002**	**.0002**	**.0003**	**.0004**	**.0004**	**.0005**
4	.00000000	.00000001	.00000001	**.0003**	**.0004**	**.0004**	**.0005**	**.0006**	**.0007**
5	.00000001	.00000005	.00000012	**.0011**	**.0012**	**.0012**	**.0013**	**.0014**	**.0014**
6	.00000009	.00000035	.00000088	**.0038**	**.0039**	**.0040**	**.0041**	**.0041**	**.0042**
7	.00000054	.00000213	.00000531	**.0109**	**.0110**	**.0110**	**.0111**	**.0112**	**.0113**
8	.00000277	.00001098	.00002723	**.0250**	**.0250**	**.0251**	**.0252**	**.0252**	**.0253**
9	.00001245	.00004906	**.0006**	**.0480**	**.0482**	**.0484**	**.0486**	**.0488**	**.0490**
10	.00004924	.00019211	**.0017**	**.0791**	**.0793**	**.0795**	**.0797**	**.0800**	**.0802**
11	.00017300	.00066508	**.0049**	**.1156**	**.1159**	**.1162**	**.1165**	**.1168**	**.1171**
12	.00054427	.00204945	**.0127**	**.1533**	**.1539**	**.1545**	**.1550**	**.1556**	**.1562**
13	.00154384	.00565158	**.0292**	**.1895**	**.1908**	**.1922**	**.1936**	**.1951**	**.1965**
14	.00397136	.01400852	**.0583**	**.2232**	**.2263**	**.2295**	**.2326**	**.2358**	**.2390**
15	.00931192	.03133064	**.1034**	**.2575**	**.2639**	**.2703**	**.2767**	**.2829**	**.2891**
16	.01999303	.06345124	**.1657**	**.2981**	**.3099**	**.3213**	**.3323**	**.3430**	**.3532**
17	.03947035	.11677409	**.2442**	**.3511**	**.3698**	**.3874**	**.4037**	**.4190**	**.4332**
18	.07193254	.19604526	**.3359**	**.4200**	**.4457**	**.4685**	**.4888**	**.5069**	**.5229**
19	.12148011	.30157107	**.4363**	**.5031**	**.5334**	**.5584**	**.5793**	**.5968**	**.6116**
20	.19084670	.42731350	**.5398**	**.5941**	**.6245**	**.6479**	**.6661**	**.6806**	**.6924**
21	.28003231	.56134501	**.6401**	**.6842**	**.7109**	**.7300**	**.7442**	**.7553**	**.7641**
22	.38543350	.68903193	**.7313**	**.7659**	**.7870**	**.8015**	**.8123**	**.8207**	**.8276**
23	.50000000	.79762548	**.8088**	**.8348**	**.8505**	**.8613**	**.8696**	**.8763**	**.8819**
24	.61456650	.87995782	**.8705**	**.8892**	**.9006**	**.9088**	**.9152**	**.9205**	**.9248**
25	.71996769	.93551174	**.9165**	**.9296**	**.9379**	**.9440**	**.9488**	**.9527**	**.9558**
26	.80915330	.96880630	**.9489**	**.9578**	**.9637**	**.9680**	**.9713**	**.9740**	**.9761**
27	.87851989	.98648838	**.9703**	**.9762**	**.9801**	**.9830**	**.9851**	**.9868**	**.9881**
28	.92806746	.99478719	**.9837**	**.9874**	**.9898**	**.9915**	**.9928**	**.9938**	**.9945**
29	.96052965	.99821842	**.9915**	**.9937**	**.9951**	**.9961**	**.9968**	**.9973**	**.9977**
30	.98000697	.99946365	**.9958**	**.9971**	**.9978**	**.9983**	**.9986**	**.9989**	**.9991**
31	.99068808	.99985863	**.9982**	**.9987**	**.9991**	**.9993**	**.9995**	**.9996**	**.9997**
32	.99602864	.99996759	**.9992**	**.9995**	**.9996**	**.9997**	**.9998**	**.9998**	**.9999**

Table 33: Sheet R. Exact (in roman), approximate *(in italics)* and interpolated **(boldfaced)** probabilities of perfect ($i = 0$) and i-imperfect column triplets ($k = 3$) in Bernoulli ($m \times n$)-matrices. The rows with all 1's are omitted

k m i	Width n of Bernoulli $(m \times n)$-matrix								
	3	4	5	6	7	8	9	10	11
3 47 33	.99845616	.99999359	**.9997**	**.9998**	**.9999**	**.9999**	**.9999**	**1.0000**	**1.0000**
34	.99945573	.99999891	**.9999**	**.9999**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
35	.99982700	.99999984	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
3 48 0	.00000000	.00000000	.00000000	.00000000	.00000000	.00000000	*.00000000*	*.00000000*	*.00000000*
1	.00000000	.00000000	.00000000	.00000000	.00000000	**.0001**	**.0001**	**.0001**	**.0002**
2	.00000000	.00000000	.00000000	.00000000	**.0001**	**.0001**	**.0002**	**.0002**	**.0003**
3	.00000000	.00000000	.00000000	**.0001**	**.0002**	**.0003**	**.0004**	**.0004**	**.0005**
4	.00000000	.00000000	.00000001	**.0002**	**.0003**	**.0004**	**.0004**	**.0005**	**.0006**
5	.00000001	.00000003	.00000007	**.0008**	**.0008**	**.0009**	**.0010**	**.0011**	**.0011**
6	.00000005	.00000020	.00000050	**.0028**	**.0029**	**.0029**	**.0030**	**.0031**	**.0031**
7	.00000031	.00000124	.00000310	**.0082**	**.0083**	**.0084**	**.0085**	**.0085**	**.0086**
8	.00000165	.00000656	.00001629	**.0197**	**.0198**	**.0199**	**.0200**	**.0200**	**.0201**
9	.00000761	.00003006	**.0005**	**.0397**	**.0399**	**.0400**	**.0402**	**.0404**	**.0406**
10	.00003085	.00012082	**.0014**	**.0678**	**.0680**	**.0682**	**.0684**	**.0686**	**.0688**
11	.00011112	.00042974	**.0039**	**.1023**	**.1025**	**.1028**	**.1030**	**.1033**	**.1036**
12	.00035863	.00136206	**.0104**	**.1395**	**.1399**	**.1403**	**.1407**	**.1412**	**.1416**
13	.00104405	.00386773	**.0242**	**.1760**	**.1770**	**.1780**	**.1791**	**.1801**	**.1812**
14	.00275760	.00988340	**.0488**	**.2097**	**.2119**	**.2142**	**.2164**	**.2187**	**.2210**
15	.00664164	.02281315	**.0878**	**.2425**	**.2474**	**.2522**	**.2570**	**.2617**	**.2665**
16	.01465247	.04772791	**.1426**	**.2789**	**.2882**	**.2972**	**.3061**	**.3147**	**.3231**
17	.02973169	.09080506	**.2128**	**.3249**	**.3405**	**.3553**	**.3693**	**.3826**	**.3952**
18	.05570145	.15765892	**.2964**	**.3853**	**.4081**	**.4288**	**.4476**	**.4647**	**.4802**
19	.09670633	.25079189	**.3902**	**.4610**	**.4896**	**.5140**	**.5350**	**.5531**	**.5687**
20	.15616340	.36721950	**.4898**	**.5478**	**.5785**	**.6030**	**.6227**	**.6387**	**.6518**
21	.23543951	.49776135	**.5900**	**.6379**	**.6666**	**.6878**	**.7038**	**.7164**	**.7264**
22	.33273291	.62893516	**.6847**	**.7235**	**.7472**	**.7637**	**.7760**	**.7855**	**.7932**
23	.44271675	.74694278	**.7687**	**.7986**	**.8167**	**.8291**	**.8384**	**.8459**	**.8520**
24	.55728325	.84187153	**.8383**	**.8603**	**.8737**	**.8830**	**.8903**	**.8962**	**.9011**
25	.66726709	.91005213	**.8925**	**.9081**	**.9179**	**.9249**	**.9305**	**.9350**	**.9388**
26	.76456049	.95369687	**.9320**	**.9429**	**.9499**	**.9551**	**.9591**	**.9624**	**.9650**
27	.84383660	.97854610	**.9592**	**.9665**	**.9714**	**.9750**	**.9777**	**.9799**	**.9816**
28	.90329367	.99109943	**.9768**	**.9816**	**.9847**	**.9870**	**.9887**	**.9900**	**.9911**
29	.94429855	.99671069	**.9875**	**.9905**	**.9924**	**.9937**	**.9947**	**.9954**	**.9960**
30	.97026831	.99892285	**.9937**	**.9954**	**.9964**	**.9972**	**.9977**	**.9980**	**.9983**
31	.98534753	.99968917	**.9970**	**.9979**	**.9984**	**.9988**	**.9990**	**.9992**	**.9994**
32	.99335836	.99992144	**.9987**	**.9991**	**.9994**	**.9995**	**.9996**	**.9997**	**.9998**
33	.99724240	.99998272	**.9995**	**.9997**	**.9998**	**.9998**	**.9999**	**.9999**	**.9999**
34	.99895595	.99999672	**.9998**	**.9999**	**.9999**	**.9999**	**1.0000**	**1.0000**	**1.0000**
35	.99964137	.99999947	**.9999**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
36	.99988888	.99999993	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**

Table 33: Sheet S. Exact (in roman), approximate *(in italics)* and interpolated **(boldfaced)** probabilities of perfect ($i = 0$) and i-imperfect column triplets ($k = 3$) in Bernoulli ($m \times n$)-matrices. The rows with all 1's are omitted

k m i	Width n of Bernoulli ($m \times n$)-matrix								
	12	13	14	15	16	17	18	19	20
3 1 0	.99682617	.99829102	.99908447	.99951172	.99974060	.99986267	.99992752	.99996185	.99997997
3 2 0	.96539468	.97465602	.98132761	.98617879	.98973327	.99235335	.99429363	.99573548	.99680971
1	.99998993	.99999708	.99999916	.99999976	.99999993	.99999998	.99999999	1.00000000	1.00000000
3 3 0	.91254806	.93371826	.94980919	.96202954	.97130156	.97832924	.98365005	.98767421	.99071455
1	.99956436	.99980078	.99990801	.99995710	.99997981	.99999041	.99999541	.99999779	.99999893
2	.99999997	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
3 4 0	.82603278	.86231031	.89154963	.91495750	.93358279	.94832058	.95992353	.96901663	.97611321
1	.99618725	.99790958	.99885017	.99936571	.99964924	.99980563	.99989211	.99994002	.99996662
2	.99999627	.99999899	.99999972	.99999992	.99999998	.99999999	1.00000000	1.00000000	1.00000000
3	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
3 5 0	.70130374	.75102324	.79372946	.83006016	.86070366	.88635254	.90767282	.92528447	.93975047
1	.98399378	.99036205	.99423307	.99656933	.99797011	.99880490	.99929960	.99959125	.99976237
2	.99989521	.99996060	.99998508	.99999431	.99999782	.99999916	.99999967	.99999987	.99999995
3	.99999995	.99999999	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
4	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
3 6 0	.55277591	.60876972	.65981562	.70584123	.74694029	.78332525	.81528880	.84317313	.86734566
1	.95083194	.96719323	.97829572	.98575085	.99071011	.99398140	.99612290	.99751524	.99841491
2	.99902826	.99956838	.99980918	.99991590	.99996299	.99998371	.99999281	.99999681	.99999858
3	.99999748	.99999929	.99999979	.99999993	.99999998	.99999999	1.00000000	1.00000000	1.00000000
4	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
5	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
3 7 0	.40436962	.45779233	.50907133	.55771770	.60339607	.64590094	.68513444	.72108600	.75381401
1	.88374790	.91490092	.93824812	.95554074	.96821698	.97742419	.98405703	.98880025	.99216981
2	.99460465	.99720448	.99856614	.99927112	.99963238	.99981583	.99990825	.99995448	.99997748
3	.99995633	.99998592	.99999541	.99999847	.99999947	.99999981	.99999993	.99999997	.99999999
4	.99999993	.99999998	.99999999	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
5	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
6	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000
3 8 0	**.2962**	**.3397**	**.3823**	**.4244**	**.4651**	**.5044**	**.5419**	**.5775**	**.6110**
1	**.6761**	**.7101**	**.7402**	**.7671**	**.7870**	**.8069**	**.8251**	**.8418**	**.8570**
2	**.8979**	**.9176**	**.9330**	**.9455**	**.9549**	**.9630**	**.9697**	**.9752**	**.9798**
3	**.9761**	**.9823**	**.9869**	**.9903**	**.9928**	**.9946**	**.9960**	**.9971**	**.9979**
4	**.9957**	**.9971**	**.9981**	**.9987**	**.9992**	**.9994**	**.9996**	**.9998**	**.9998**
5	**.9995**	**.9997**	**.9998**	**.9999**	**.9999**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
6	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
7	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
3 9 0	**.1980**	**.2321**	**.2659**	**.3009**	**.3358**	**.3703**	**.4043**	**.4374**	**.4697**
1	**.5711**	**.6054**	**.6366**	**.6662**	**.6934**	**.7181**	**.7407**	**.7612**	**.7756**
2	**.8247**	**.8536**	**.8768**	**.8965**	**.9128**	**.9263**	**.9376**	**.9471**	**.9543**
3	**.9493**	**.9611**	**.9697**	**.9764**	**.9815**	**.9855**	**.9886**	**.9911**	**.9930**
4	**.9881**	**.9915**	**.9939**	**.9956**	**.9968**	**.9977**	**.9984**	**.9988**	**.9992**
5	**.9979**	**.9986**	**.9991**	**.9994**	**.9996**	**.9998**	**.9998**	**.9999**	**.9999**
6	**.9997**	**.9998**	**.9999**	**.9999**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
7	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
8	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
3 10 0	**.1263**	**.1512**	**.1758**	**.2028**	**.2301**	**.2578**	**.2857**	**.3136**	**.3414**
1	**.4881**	**.5192**	**.5481**	**.5770**	**.6041**	**.6294**	**.6448**	**.6631**	**.6806**
2	**.7436**	**.7781**	**.8067**	**.8322**	**.8539**	**.8723**	**.8853**	**.8983**	**.9099**
3	**.9089**	**.9274**	**.9410**	**.9522**	**.9610**	**.9681**	**.9735**	**.9782**	**.9820**
4	**.9738**	**.9803**	**.9848**	**.9884**	**.9911**	**.9932**	**.9948**	**.9960**	**.9969**
5	**.9937**	**.9956**	**.9969**	**.9978**	**.9984**	**.9989**	**.9992**	**.9995**	**.9996**
6	**.9989**	**.9993**	**.9995**	**.9997**	**.9998**	**.9999**	**.9999**	**.9999**	**1.0000**
7	**.9999**	**.9999**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
8	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
9	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
3 11 0	**.0774**	**.0945**	**.1116**	**.1308**	**.1506**	**.1711**	**.1924**	**.2141**	**.2361**

Table 33: Sheet T. Exact (in roman), approximate *(in italics)* and interpolated **(boldfaced)** probabilities of perfect ($i = 0$) and i-imperfect column triplets ($k = 3$) in Bernoulli ($m \times n$)-matrices. The rows with all 1's are omitted

k m i	Width n of Bernoulli ($m \times n$)-matrix								
	12	13	14	15	16	17	18	19	20
3 11 1	**.3823**	**.4324**	**.4717**	**.4983**	**.4859**	**.5040**	**.5218**	**.5392**	**.5562**
2	**.6568**	**.6941**	**.7256**	**.7553**	**.7437**	**.7648**	**.7844**	**.8025**	**.8192**
3	**.8530**	**.8786**	**.8981**	**.9146**	**.9130**	**.9249**	**.9351**	**.9440**	**.9516**
4	**.9504**	**.9612**	**.9688**	**.9751**	**.9768**	**.9811**	**.9845**	**.9873**	**.9896**
5	**.9852**	**.9891**	**.9917**	**.9938**	**.9950**	**.9962**	**.9971**	**.9978**	**.9983**
6	**.9965**	**.9976**	**.9983**	**.9988**	**.9992**	**.9994**	**.9996**	**.9997**	**.9998**
7	**.9994**	**.9996**	**.9997**	**.9998**	**.9999**	**.9999**	**1.0000**	**1.0000**	**1.0000**
8	**.9999**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
9	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
10	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
3 12 0	**.0456**	**.0569**	**.0681**	**.0813**	**.0949**	**.1093**	**.1242**	**.1400**	**.1561**
1	**.1523**	**.1803**	**.1749**	**.3306**	**.3994**	**.4252**	**.4407**	**.4551**	**.4693**
2	**.4058**	**.4533**	**.4395**	**.6093**	**.6354**	**.6630**	**.6855**	**.7051**	**.7238**
3	**.6773**	**.7201**	**.7146**	**.8187**	**.8409**	**.8629**	**.8793**	**.8925**	**.9043**
4	**.8636**	**.8877**	**.8896**	**.9359**	**.9471**	**.9575**	**.9645**	**.9697**	**.9742**
5	**.9511**	**.9615**	**.9657**	**.9808**	**.9851**	**.9888**	**.9912**	**.9929**	**.9942**
6	**.9861**	**.9897**	**.9921**	**.9957**	**.9969**	**.9978**	**.9984**	**.9988**	**.9991**
7	**.9971**	**.9980**	**.9987**	**.9993**	**.9995**	**.9997**	**.9998**	**.9998**	**.9999**
8	**.9996**	**.9997**	**.9998**	**.9999**	**.9999**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
9	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
10	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
11	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
3 13 0	**.0262**	**.0334**	**.0407**	**.0489**	**.0578**	**.0675**	**.0775**	**.0882**	**.0995**
1	**.0924**	**.1108**	**.1290**	**.1504**	**.1729**	**.2369**	**.3403**	**.3899**	**.4037**
2	**.2845**	**.3247**	**.3618**	**.4018**	**.4398**	**.5670**	**.5867**	**.6063**	**.6291**
3	**.5458**	**.5945**	**.6358**	**.6764**	**.7111**	**.7848**	**.8036**	**.8214**	**.8412**
4	**.7773**	**.8136**	**.8414**	**.8669**	**.8864**	**.9219**	**.9319**	**.9408**	**.9506**
5	**.9079**	**.9271**	**.9407**	**.9527**	**.9611**	**.9753**	**.9793**	**.9828**	**.9864**
6	**.9683**	**.9764**	**.9819**	**.9865**	**.9894**	**.9938**	**.9951**	**.9961**	**.9971**
7	**.9915**	**.9941**	**.9958**	**.9971**	**.9978**	**.9988**	**.9991**	**.9993**	**.9995**
8	**.9983**	**.9989**	**.9993**	**.9995**	**.9997**	**.9998**	**.9999**	**.9999**	**.9999**
9	**.9997**	**.9999**	**.9999**	**.9999**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
10	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
11	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
12	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
3 14 0	**.0147**	**.0191**	**.0237**	**.0289**	**.0345**	**.0405**	**.0469**	**.0538**	**.0611**
1	**.0592**	**.0716**	**.0847**	**.1002**	**.0952**	**.1071**	**.1190**	**.1314**	**.1458**
2	**.2054**	**.2368**	**.2684**	**.3020**	**.2835**	**.3079**	**.3310**	**.3540**	**.3806**
3	**.4364**	**.4801**	**.5222**	**.5613**	**.5449**	**.5756**	**.6030**	**.6292**	**.6581**
4	**.6852**	**.7231**	**.7586**	**.7866**	**.7795**	**.8032**	**.8227**	**.8406**	**.8596**
5	**.8518**	**.8736**	**.8949**	**.9088**	**.9082**	**.9215**	**.9314**	**.9403**	**.9494**
6	**.9397**	**.9497**	**.9604**	**.9662**	**.9679**	**.9739**	**.9780**	**.9816**	**.9850**
7	**.9799**	**.9837**	**.9881**	**.9900**	**.9911**	**.9932**	**.9945**	**.9956**	**.9965**
8	**.9948**	**.9959**	**.9973**	**.9977**	**.9981**	**.9986**	**.9989**	**.9992**	**.9994**
9	**.9990**	**.9992**	**.9995**	**.9996**	**.9997**	**.9998**	**.9998**	**.9999**	**.9999**
10	**.9999**	**.9999**	**.9999**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
11	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
12	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
13	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
3 15 0	**.0080**	**.0108**	**.0136**	**.0167**	**.0202**	**.0239**	**.0279**	**.0322**	**.0368**
1	**.0372**	**.0457**	**.0548**	**.0533**	**.0609**	**.0690**	**.0770**	**.0859**	**.0956**
2	**.1447**	**.1701**	**.1956**	**.1842**	**.2028**	**.2221**	**.2395**	**.2586**	**.2788**
3	**.3365**	**.3776**	**.4165**	**.4036**	**.4325**	**.4615**	**.4853**	**.5117**	**.5377**
4	**.5808**	**.6245**	**.6635**	**.6584**	**.6873**	**.7153**	**.7354**	**.7582**	**.7790**
5	**.7758**	**.8071**	**.8341**	**.8352**	**.8545**	**.8730**	**.8837**	**.8973**	**.9085**
6	**.8930**	**.9108**	**.9265**	**.9304**	**.9410**	**.9509**	**.9555**	**.9622**	**.9672**

Table 33: Sheet U. Exact (in roman), approximate *(in italics)* and interpolated **(boldfaced)** probabilities of perfect ($i = 0$) and i-imperfect column triplets ($k = 3$) in Bernoulli ($m \times n$)-matrices. The rows with all 1's are omitted

k m i	Width n of Bernoulli ($m \times n$)-matrix								
	12	13	14	15	16	17	18	19	20
3 15 7	**.9559**	**.9647**	**.9725**	**.9758**	**.9804**	**.9847**	**.9862**	**.9888**	**.9905**
8	**.9851**	**.9887**	**.9918**	**.9933**	**.9949**	**.9963**	**.9966**	**.9974**	**.9978**
9	**.9960**	**.9972**	**.9981**	**.9986**	**.9990**	**.9993**	**.9994**	**.9995**	**.9996**
10	**.9992**	**.9995**	**.9997**	**.9998**	**.9998**	**.9999**	**.9999**	**.9999**	**.9999**
11	**.9999**	**.9999**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
12	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
13	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
3 16 0	**.0043**	**.0060**	**.0077**	**.0096**	**.0117**	**.0139**	**.0163**	**.0189**	**.0218**
1	**.0216**	**.0261**	**.0308**	**.0370**	**.0440**	**.0523**	**.0495**	**.0556**	**.0622**
2	**.0938**	**.1085**	**.1240**	**.1430**	**.1635**	**.1866**	**.1719**	**.1866**	**.2025**
3	**.2437**	**.2716**	**.3002**	**.3325**	**.3657**	**.4008**	**.3827**	**.4059**	**.4304**
4	**.4724**	**.5086**	**.5439**	**.5805**	**.6163**	**.6522**	**.6426**	**.6667**	**.6918**
5	**.6941**	**.7244**	**.7528**	**.7799**	**.8057**	**.8312**	**.8292**	**.8453**	**.8622**
6	**.8453**	**.8636**	**.8805**	**.8957**	**.9105**	**.9254**	**.9275**	**.9362**	**.9454**
7	**.9318**	**.9409**	**.9495**	**.9568**	**.9642**	**.9718**	**.9743**	**.9781**	**.9822**
8	**.9750**	**.9787**	**.9824**	**.9853**	**.9883**	**.9914**	**.9927**	**.9940**	**.9954**
9	**.9926**	**.9939**	**.9951**	**.9960**	**.9970**	**.9980**	**.9983**	**.9987**	**.9991**
10	**.9983**	**.9986**	**.9989**	**.9992**	**.9994**	**.9996**	**.9997**	**.9998**	**.9999**
11	**.9997**	**.9998**	**.9998**	**.9999**	**.9999**	**.9999**	**1.0000**	**1.0000**	**1.0000**
12	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
13	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
14	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
3 17 0	*.00150730*	**.0033**	**.0044**	**.0055**	**.0067**	**.0080**	**.0095**	**.0110**	**.0127**
1	**.0133**	**.0163**	**.0199**	**.0235**	**.0280**	**.0271**	**.0307**	**.0347**	**.0391**
2	**.0640**	**.0754**	**.0884**	**.1015**	**.1168**	**.1079**	**.1183**	**.1294**	**.1413**
3	**.1798**	**.2038**	**.2292**	**.2551**	**.2820**	**.2677**	**.2867**	**.3063**	**.3280**
4	**.3784**	**.4138**	**.4482**	**.4832**	**.5150**	**.5061**	**.5309**	**.5556**	**.5837**
5	**.6044**	**.6396**	**.6711**	**.7027**	**.7274**	**.7258**	**.7467**	**.7668**	**.7913**
6	**.7815**	**.8061**	**.8265**	**.8474**	**.8609**	**.8637**	**.8770**	**.8895**	**.9064**
7	**.8925**	**.9065**	**.9175**	**.9294**	**.9356**	**.9398**	**.9471**	**.9539**	**.9638**
8	**.9545**	**.9614**	**.9666**	**.9724**	**.9748**	**.9777**	**.9810**	**.9839**	**.9886**
9	**.9841**	**.9869**	**.9889**	**.9912**	**.9919**	**.9932**	**.9944**	**.9954**	**.9971**
10	**.9955**	**.9964**	**.9971**	**.9978**	**.9979**	**.9983**	**.9987**	**.9989**	**.9994**
11	**.9990**	**.9992**	**.9994**	**.9996**	**.9996**	**.9997**	**.9997**	**.9998**	**.9999**
12	**.9998**	**.9999**	**.9999**	**.9999**	**.9999**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
13	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
14	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
15	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
3 18 0	*.00077622*	*.00099847*	*.00125561*	**.0033**	**.0039**	**.0047**	**.0055**	**.0064**	**.0073**
1	**.0070**	**.0093**	**.0134**	**.0170**	**.0184**	**.0173**	**.0193**	**.0215**	**.0242**
2	**.0391**	**.0491**	**.0648**	**.0781**	**.0856**	**.0754**	**.0819**	**.0887**	**.0965**
3	**.1204**	**.1428**	**.1780**	**.2035**	**.2211**	**.2009**	**.2137**	**.2271**	**.2420**
4	**.2762**	**.3115**	**.3688**	**.4033**	**.4296**	**.4111**	**.4303**	**.4499**	**.4709**
5	**.4836**	**.5231**	**.5925**	**.6261**	**.6523**	**.6426**	**.6614**	**.6800**	**.6994**
6	**.6756**	**.7073**	**.7712**	**.7947**	**.8128**	**.8102**	**.8227**	**.8349**	**.8476**
7	**.8141**	**.8344**	**.8825**	**.8963**	**.9068**	**.9085**	**.9156**	**.9225**	**.9298**
8	**.9050**	**.9165**	**.9467**	**.9540**	**.9596**	**.9620**	**.9655**	**.9688**	**.9723**
9	**.9589**	**.9645**	**.9797**	**.9830**	**.9854**	**.9869**	**.9883**	**.9896**	**.9909**
10	**.9855**	**.9877**	**.9936**	**.9949**	**.9957**	**.9963**	**.9968**	**.9971**	**.9975**
11	**.9960**	**.9967**	**.9984**	**.9988**	**.9990**	**.9992**	**.9993**	**.9994**	**.9995**
12	**.9991**	**.9993**	**.9997**	**.9998**	**.9998**	**.9999**	**.9999**	**.9999**	**.9999**
13	**.9999**	**.9999**	**.9999**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
14	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
15	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
16	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
3 19 0	*.00039605*	*.00051121*	*.00064570*	*.00080045*	**.0021**	**.0028**	**.0033**	**.0040**	**.0043**

Table 33: Sheet V. Exact (in roman), approximate *(in italics)* and interpolated **(boldfaced)** probabilities of perfect ($i = 0$) and i-imperfect column triplets ($k = 3$) in Bernoulli ($m \times n$)-matrices. The rows with all 1's are omitted

k	m	i	Width n of Bernoulli ($m \times n$)-matrix								
			12	13	14	15	16	17	18	19	20
3	19	1	**.0062**	**.0064**	**.0067**	**.0069**	**.0090**	**.0126**	**.0156**	**.0196**	**.0159**
		2	**.0339**	**.0350**	**.0363**	**.0377**	**.0468**	**.0607**	**.0717**	**.0851**	**.0691**
		3	**.1026**	**.1069**	**.1114**	**.1163**	**.1363**	**.1679**	**.1893**	**.2128**	**.1856**
		4	**.2340**	**.2454**	**.2570**	**.2690**	**.2998**	**.3518**	**.3812**	**.4105**	**.3859**
		5	**.4235**	**.4425**	**.4611**	**.4794**	**.5139**	**.5784**	**.6080**	**.6353**	**.6222**
		6	**.6210**	**.6408**	**.6592**	**.6768**	**.7043**	**.7650**	**.7864**	**.8047**	**.8008**
		7	**.7747**	**.7895**	**.8031**	**.8158**	**.8332**	**.8795**	**.8922**	**.9026**	**.9037**
		8	**.8785**	**.8884**	**.8973**	**.9057**	**.9154**	**.9447**	**.9515**	**.9570**	**.9594**
		9	**.9434**	**.9492**	**.9543**	**.9590**	**.9637**	**.9785**	**.9817**	**.9841**	**.9857**
		10	**.9782**	**.9810**	**.9833**	**.9854**	**.9873**	**.9931**	**.9943**	**.9952**	**.9958**
		11	**.9932**	**.9943**	**.9951**	**.9958**	**.9965**	**.9982**	**.9986**	**.9988**	**.9990**
		12	**.9983**	**.9986**	**.9989**	**.9991**	**.9992**	**.9996**	**.9997**	**.9998**	**.9998**
		13	**.9997**	**.9997**	**.9998**	**.9998**	**.9999**	**.9999**	**.9999**	**1.0000**	**1.0000**
		14	**.9999**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
		15	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
		16	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
3	20	0	*.00020095*	*.00025991*	*.00032905*	*.00040905*	*.00050050*	**.0014**	**.0017**	**.0019**	**.0022**
		1	**.0036**	**.0037**	**.0038**	**.0039**	**.0041**	**.0070**	**.0089**	**.0096**	**.0081**
		2	**.0218**	**.0223**	**.0229**	**.0235**	**.0242**	**.0382**	**.0462**	**.0505**	**.0397**
		3	**.0711**	**.0734**	**.0758**	**.0783**	**.0808**	**.1120**	**.1284**	**.1391**	**.1145**
		4	**.1706**	**.1777**	**.1847**	**.1919**	**.1991**	**.2441**	**.2670**	**.2841**	**.2550**
		5	**.3271**	**.3411**	**.3546**	**.3677**	**.3804**	**.4279**	**.4523**	**.4717**	**.4506**
		6	**.5125**	**.5296**	**.5451**	**.5594**	**.5726**	**.6091**	**.6276**	**.6427**	**.6320**
		7	**.6759**	**.6895**	**.7013**	**.7117**	**.7210**	**.7419**	**.7526**	**.7615**	**.7585**
		8	**.7982**	**.8073**	**.8151**	**.8219**	**.8280**	**.8381**	**.8440**	**.8492**	**.8503**
		9	**.8869**	**.8928**	**.8978**	**.9021**	**.9058**	**.9103**	**.9135**	**.9163**	**.9180**
		10	**.9460**	**.9494**	**.9521**	**.9544**	**.9564**	**.9582**	**.9597**	**.9610**	**.9621**
		11	**.9789**	**.9805**	**.9818**	**.9828**	**.9836**	**.9843**	**.9849**	**.9854**	**.9858**
		12	**.9935**	**.9941**	**.9946**	**.9950**	**.9952**	**.9955**	**.9956**	**.9958**	**.9959**
		13	**.9985**	**.9987**	**.9988**	**.9989**	**.9990**	**.9991**	**.9991**	**.9991**	**.9992**
		14	**.9997**	**.9998**	**.9998**	**.9998**	**.9999**	**.9999**	**.9999**	**.9999**	**.9999**
		15	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
		16	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
		17	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
3	21	0	*.00010157*	*.00013155*	*.00016679*	*.00020767*	*.00025457*	*.00030781*	**.0009**	**.0010**	**.0012**
		1	**.0020**	**.0021**	**.0022**	**.0022**	**.0023**	**.0023**	**.0024**	**.0025**	**.0026**
		2	**.0144**	**.0147**	**.0151**	**.0155**	**.0159**	**.0163**	**.0168**	**.0173**	**.0178**
		3	**.0508**	**.0522**	**.0537**	**.0552**	**.0568**	**.0585**	**.0603**	**.0622**	**.0642**
		4	**.1286**	**.1334**	**.1384**	**.1434**	**.1486**	**.1537**	**.1591**	**.1646**	**.1702**
		5	**.2594**	**.2706**	**.2815**	**.2924**	**.3030**	**.3134**	**.3237**	**.3340**	**.3444**
		6	**.4318**	**.4482**	**.4634**	**.4778**	**.4912**	**.5039**	**.5159**	**.5275**	**.5386**
		7	**.6018**	**.6170**	**.6304**	**.6424**	**.6532**	**.6629**	**.6719**	**.6803**	**.6882**
		8	**.7366**	**.7473**	**.7565**	**.7646**	**.7718**	**.7781**	**.7840**	**.7895**	**.7947**
		9	**.8378**	**.8451**	**.8512**	**.8567**	**.8614**	**.8655**	**.8693**	**.8729**	**.8761**
		10	**.9121**	**.9166**	**.9204**	**.9236**	**.9264**	**.9288**	**.9309**	**.9328**	**.9345**
		11	**.9597**	**.9621**	**.9641**	**.9658**	**.9671**	**.9682**	**.9692**	**.9700**	**.9708**
		12	**.9849**	**.9860**	**.9869**	**.9875**	**.9881**	**.9885**	**.9889**	**.9892**	**.9895**
		13	**.9955**	**.9960**	**.9963**	**.9965**	**.9967**	**.9968**	**.9969**	**.9970**	**.9971**
		14	**.9990**	**.9991**	**.9992**	**.9993**	**.9993**	**.9994**	**.9994**	**.9994**	**.9994**
		15	**.9998**	**.9999**	**.9999**	**.9999**	**.9999**	**.9999**	**.9999**	**.9999**	**.9999**
		16	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
		17	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
		18	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
3	22	0	*.00005120*	*.00006637*	*.00008424*	*.00010500*	*.00012885*	*.00015600*	*.00018662*	**.0006**	**.0007**
		1	**.0012**	**.0013**	**.0013**	**.0013**	**.0014**	**.0014**	**.0015**	**.0015**	**.0016**
		2	**.0095**	**.0097**	**.0100**	**.0102**	**.0105**	**.0107**	**.0110**	**.0113**	**.0117**

Table 33: Sheet W. Exact (in roman), approximate *(in italics)* and interpolated **(boldfaced)** probabilities of perfect ($i = 0$) and i-imperfect column triplets ($k = 3$) in Bernoulli ($m \times n$)-matrices. The rows with all 1's are omitted

k m i	Width n of Bernoulli $(m \times n)$-matrix								
	12	13	14	15	16	17	18	19	20
3 22 3	**.0361**	**.0370**	**.0379**	**.0389**	**.0399**	**.0410**	**.0421**	**.0433**	**.0446**
4	**.0962**	**.0995**	**.1028**	**.1062**	**.1097**	**.1133**	**.1169**	**.1207**	**.1246**
5	**.2032**	**.2117**	**.2201**	**.2285**	**.2369**	**.2452**	**.2534**	**.2616**	**.2700**
6	**.3572**	**.3718**	**.3858**	**.3991**	**.4119**	**.4240**	**.4357**	**.4469**	**.4580**
7	**.5269**	**.5431**	**.5576**	**.5709**	**.5829**	**.5939**	**.6040**	**.6135**	**.6224**
8	**.6722**	**.6847**	**.6954**	**.7049**	**.7133**	**.7208**	**.7276**	**.7339**	**.7398**
9	**.7833**	**.7921**	**.7996**	**.8061**	**.8119**	**.8170**	**.8216**	**.8258**	**.8298**
10	**.8702**	**.8760**	**.8808**	**.8851**	**.8887**	**.8919**	**.8947**	**.8973**	**.8996**
11	**.9321**	**.9356**	**.9384**	**.9408**	**.9428**	**.9446**	**.9460**	**.9473**	**.9485**
12	**.9701**	**.9719**	**.9733**	**.9744**	**.9754**	**.9761**	**.9768**	**.9773**	**.9778**
13	**.9893**	**.9900**	**.9906**	**.9911**	**.9914**	**.9917**	**.9919**	**.9921**	**.9923**
14	**.9970**	**.9972**	**.9974**	**.9976**	**.9977**	**.9978**	**.9978**	**.9979**	**.9979**
15	**.9993**	**.9994**	**.9995**	**.9995**	**.9995**	**.9996**	**.9996**	**.9996**	**.9996**
16	**.9999**	**.9999**	**.9999**	**.9999**	**.9999**	**.9999**	**.9999**	**1.0000**	**1.0000**
17	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
18	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
19	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
3 23 0	*.00002575*	*.00003341*	*.00004244*	*.00005293*	*.00006501*	*.00007877*	*.00009432*	*.00011175*	**.0004**
1	**.0012**	**.0012**	**.0013**	**.0013**	**.0013**	**.0014**	**.0014**	**.0014**	**.0015**
2	**.0089**	**.0090**	**.0091**	**.0092**	**.0094**	**.0095**	**.0097**	**.0099**	**.0101**
3	**.0339**	**.0345**	**.0351**	**.0357**	**.0364**	**.0371**	**.0378**	**.0386**	**.0394**
4	**.0872**	**.0893**	**.0915**	**.0938**	**.0961**	**.0985**	**.1010**	**.1035**	**.1061**
5	**.1795**	**.1857**	**.1918**	**.1980**	**.2043**	**.2105**	**.2167**	**.2230**	**.2293**
6	**.3165**	**.3288**	**.3407**	**.3522**	**.3635**	**.3744**	**.3849**	**.3951**	**.4052**
7	**.4817**	**.4978**	**.5126**	**.5264**	**.5391**	**.5510**	**.5620**	**.5722**	**.5820**
8	**.6367**	**.6508**	**.6632**	**.6742**	**.6840**	**.6928**	**.7007**	**.7079**	**.7146**
9	**.7573**	**.7677**	**.7766**	**.7845**	**.7914**	**.7975**	**.8030**	**.8079**	**.8126**
10	**.8522**	**.8592**	**.8652**	**.8704**	**.8750**	**.8791**	**.8827**	**.8859**	**.8889**
11	**.9203**	**.9248**	**.9286**	**.9319**	**.9347**	**.9372**	**.9393**	**.9411**	**.9428**
12	**.9634**	**.9660**	**.9681**	**.9699**	**.9713**	**.9726**	**.9736**	**.9745**	**.9752**
13	**.9860**	**.9873**	**.9883**	**.9891**	**.9897**	**.9902**	**.9907**	**.9910**	**.9913**
14	**.9957**	**.9962**	**.9966**	**.9969**	**.9971**	**.9973**	**.9974**	**.9976**	**.9977**
15	**.9989**	**.9991**	**.9992**	**.9993**	**.9994**	**.9994**	**.9995**	**.9995**	**.9995**
16	**.9998**	**.9998**	**.9999**	**.9999**	**.9999**	**.9999**	**.9999**	**.9999**	**.9999**
17	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
18	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
19	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
3 24 0	*.00001294*	*.00001679*	*.00002134*	*.00002663*	*.00003272*	*.00003967*	*.00004753*	*.00005635*	*.00006618*
1	**.0007**	**.0008**	**.0008**	**.0008**	**.0009**	**.0009**	**.0009**	**.0010**	**.0010**
2	**.0058**	**.0059**	**.0059**	**.0060**	**.0061**	**.0062**	**.0063**	**.0065**	**.0066**
3	**.0238**	**.0241**	**.0245**	**.0249**	**.0253**	**.0257**	**.0261**	**.0266**	**.0270**
4	**.0651**	**.0665**	**.0679**	**.0694**	**.0709**	**.0725**	**.0741**	**.0757**	**.0774**
5	**.1391**	**.1434**	**.1478**	**.1522**	**.1566**	**.1611**	**.1657**	**.1703**	**.1748**
6	**.2529**	**.2627**	**.2722**	**.2816**	**.2909**	**.3000**	**.3089**	**.3176**	**.3262**
7	**.4010**	**.4159**	**.4300**	**.4433**	**.4559**	**.4678**	**.4790**	**.4896**	**.4997**
8	**.5534**	**.5687**	**.5823**	**.5946**	**.6057**	**.6158**	**.6249**	**.6333**	**.6409**
9	**.6773**	**.6895**	**.7001**	**.7094**	**.7176**	**.7249**	**.7314**	**.7373**	**.7426**
10	**.7822**	**.7905**	**.7976**	**.8038**	**.8094**	**.8143**	**.8186**	**.8226**	**.8262**
11	**.8645**	**.8702**	**.8750**	**.8792**	**.8829**	**.8861**	**.8889**	**.8914**	**.8936**
12	**.9259**	**.9295**	**.9325**	**.9350**	**.9371**	**.9389**	**.9405**	**.9418**	**.9430**
13	**.9656**	**.9675**	**.9690**	**.9703**	**.9714**	**.9722**	**.9730**	**.9736**	**.9741**
14	**.9867**	**.9876**	**.9883**	**.9888**	**.9893**	**.9896**	**.9899**	**.9902**	**.9904**
15	**.9959**	**.9962**	**.9965**	**.9967**	**.9968**	**.9970**	**.9970**	**.9971**	**.9972**
16	**.9990**	**.9991**	**.9992**	**.9992**	**.9993**	**.9993**	**.9994**	**.9994**	**.9994**
17	**.9998**	**.9998**	**.9999**	**.9999**	**.9999**	**.9999**	**.9999**	**.9999**	**.9999**
18	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**

Table 33: Sheet X. Exact (in roman), approximate *(in italics)* and interpolated **(boldfaced)** probabilities of perfect ($i = 0$) and i-imperfect column triplets ($k = 3$) in Bernoulli ($m \times n$)-matrices. The rows with all 1's are omitted

k	m	i	Width n of Bernoulli ($m \times n$)-matrix								
			12	13	14	15	16	17	18	19	20
3	24	19	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
		20	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
3	25	0	*.00000649*	*.00000843*	*.00001071*	*.00001338*	*.00001644*	*.00001994*	*.00002390*	*.00002835*	*.00003331*
		1	**.0005**	**.0005**	**.0005**	**.0006**	**.0006**	**.0006**	**.0007**	**.0007**	**.0007**
		2	**.0038**	**.0038**	**.0039**	**.0040**	**.0040**	**.0041**	**.0042**	**.0043**	**.0044**
		3	**.0169**	**.0171**	**.0173**	**.0175**	**.0178**	**.0181**	**.0184**	**.0187**	**.0190**
		4	**.0490**	**.0498**	**.0507**	**.0517**	**.0527**	**.0537**	**.0547**	**.0558**	**.0569**
		5	**.1088**	**.1118**	**.1147**	**.1178**	**.1208**	**.1240**	**.1272**	**.1304**	**.1336**
		6	**.2045**	**.2119**	**.2191**	**.2264**	**.2337**	**.2408**	**.2479**	**.2550**	**.2619**
		7	**.3370**	**.3500**	**.3625**	**.3745**	**.3861**	**.3972**	**.4079**	**.4181**	**.4279**
		8	**.4867**	**.5024**	**.5166**	**.5297**	**.5417**	**.5528**	**.5630**	**.5724**	**.5812**
		9	**.6148**	**.6288**	**.6410**	**.6518**	**.6615**	**.6702**	**.6779**	**.6850**	**.6914**
		10	**.7289**	**.7387**	**.7470**	**.7544**	**.7609**	**.7667**	**.7719**	**.7766**	**.7809**
		11	**.8191**	**.8259**	**.8317**	**.8368**	**.8413**	**.8453**	**.8488**	**.8520**	**.8549**
		12	**.8909**	**.8955**	**.8993**	**.9027**	**.9056**	**.9081**	**.9102**	**.9121**	**.9138**
		13	**.9426**	**.9454**	**.9476**	**.9495**	**.9511**	**.9524**	**.9536**	**.9546**	**.9554**
		14	**.9744**	**.9758**	**.9769**	**.9778**	**.9786**	**.9792**	**.9797**	**.9801**	**.9805**
		15	**.9905**	**.9911**	**.9916**	**.9920**	**.9923**	**.9925**	**.9927**	**.9929**	**.9930**
		16	**.9972**	**.9974**	**.9976**	**.9977**	**.9978**	**.9979**	**.9979**	**.9980**	**.9980**
		17	**.9993**	**.9994**	**.9995**	**.9995**	**.9995**	**.9996**	**.9996**	**.9996**	**.9996**
		18	**.9999**	**.9999**	**.9999**	**.9999**	**.9999**	**.9999**	**.9999**	**.9999**	**.9999**
		19	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
		20	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
		21	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
3	26	0	*.00000325*	*.00000423*	*.00000537*	*.00000671*	*.00000825*	*.00001001*	*.00001200*	*.00001424*	*.00001674*
		1	**.0005**	**.0005**	**.0005**	**.0006**	**.0006**	**.0006**	**.0006**	**.0007**	**.0007**
		2	**.0037**	**.0038**	**.0038**	**.0039**	**.0039**	**.0040**	**.0040**	**.0041**	**.0042**
		3	**.0165**	**.0166**	**.0168**	**.0170**	**.0171**	**.0173**	**.0175**	**.0177**	**.0180**
		4	**.0468**	**.0474**	**.0480**	**.0486**	**.0493**	**.0499**	**.0506**	**.0514**	**.0521**
		5	**.1008**	**.1028**	**.1048**	**.1068**	**.1089**	**.1110**	**.1132**	**.1154**	**.1177**
		6	**.1842**	**.1895**	**.1949**	**.2002**	**.2056**	**.2110**	**.2164**	**.2218**	**.2271**
		7	**.3015**	**.3122**	**.3227**	**.3329**	**.3428**	**.3525**	**.3619**	**.3711**	**.3800**
		8	**.4443**	**.4593**	**.4733**	**.4864**	**.4986**	**.5101**	**.5209**	**.5309**	**.5404**
		9	**.5751**	**.5902**	**.6037**	**.6160**	**.6270**	**.6370**	**.6461**	**.6544**	**.6620**
		10	**.7006**	**.7121**	**.7219**	**.7306**	**.7382**	**.7450**	**.7511**	**.7567**	**.7617**
		11	**.7990**	**.8070**	**.8138**	**.8198**	**.8251**	**.8299**	**.8342**	**.8380**	**.8415**
		12	**.8764**	**.8820**	**.8867**	**.8909**	**.8946**	**.8978**	**.9007**	**.9032**	**.9055**
		13	**.9331**	**.9368**	**.9398**	**.9424**	**.9447**	**.9466**	**.9482**	**.9497**	**.9509**
		14	**.9690**	**.9711**	**.9728**	**.9742**	**.9753**	**.9763**	**.9771**	**.9778**	**.9784**
		15	**.9879**	**.9889**	**.9897**	**.9904**	**.9909**	**.9913**	**.9917**	**.9920**	**.9922**
		16	**.9961**	**.9965**	**.9968**	**.9971**	**.9973**	**.9974**	**.9976**	**.9977**	**.9978**
		17	**.9990**	**.9991**	**.9992**	**.9993**	**.9994**	**.9994**	**.9995**	**.9995**	**.9995**
		18	**.9998**	**.9998**	**.9998**	**.9999**	**.9999**	**.9999**	**.9999**	**.9999**	**.9999**
		19	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
		20	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
		21	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
3	27	0	*.00000163*	*.00000212*	*.00000269*	*.00000336*	*.00000414*	*.00000502*	*.00000602*	*.00000715*	*.00000840*
		1	**.0003**	**.0004**	**.0004**	**.0004**	**.0004**	**.0005**	**.0005**	**.0005**	**.0006**
		2	**.0024**	**.0025**	**.0025**	**.0026**	**.0026**	**.0027**	**.0027**	**.0028**	**.0028**
		3	**.0116**	**.0117**	**.0118**	**.0120**	**.0121**	**.0123**	**.0124**	**.0125**	**.0127**
		4	**.0349**	**.0352**	**.0356**	**.0359**	**.0363**	**.0367**	**.0372**	**.0376**	**.0381**
		5	**.0790**	**.0803**	**.0816**	**.0829**	**.0843**	**.0857**	**.0871**	**.0886**	**.0901**
		6	**.1486**	**.1523**	**.1561**	**.1600**	**.1638**	**.1677**	**.1717**	**.1756**	**.1796**
		7	**.2479**	**.2563**	**.2646**	**.2728**	**.2809**	**.2888**	**.2967**	**.3044**	**.3119**
		8	**.3754**	**.3890**	**.4018**	**.4141**	**.4258**	**.4369**	**.4475**	**.4575**	**.4671**
		9	**.4978**	**.5132**	**.5273**	**.5403**	**.5523**	**.5633**	**.5734**	**.5828**	**.5915**

Table 33: Sheet Y. Exact (in roman), approximate *(in italics)* and interpolated **(boldfaced)** probabilities of perfect ($i = 0$) and i-imperfect column triplets ($k = 3$) in Bernoulli ($m \times n$)-matrices. The rows with all 1's are omitted

k m i	Width n of Bernoulli ($m \times n$)-matrix								
	12	13	14	15	16	17	18	19	20
3 27 10	**.6254**	**.6385**	**.6499**	**.6600**	**.6690**	**.6770**	**.6842**	**.6907**	**.6966**
11	**.7286**	**.7378**	**.7457**	**.7527**	**.7589**	**.7645**	**.7694**	**.7740**	**.7781**
12	**.8135**	**.8201**	**.8257**	**.8307**	**.8352**	**.8391**	**.8427**	**.8458**	**.8487**
13	**.8837**	**.8883**	**.8922**	**.8956**	**.8985**	**.9011**	**.9033**	**.9053**	**.9071**
14	**.9363**	**.9392**	**.9415**	**.9435**	**.9452**	**.9466**	**.9479**	**.9490**	**.9499**
15	**.9700**	**.9716**	**.9728**	**.9738**	**.9747**	**.9754**	**.9760**	**.9765**	**.9769**
16	**.9882**	**.9889**	**.9894**	**.9899**	**.9902**	**.9905**	**.9908**	**.9910**	**.9911**
17	**.9962**	**.9964**	**.9967**	**.9968**	**.9970**	**.9971**	**.9972**	**.9972**	**.9973**
18	**.9990**	**.9991**	**.9992**	**.9992**	**.9993**	**.9993**	**.9993**	**.9993**	**.9994**
19	**.9998**	**.9998**	**.9998**	**.9999**	**.9999**	**.9999**	**.9999**	**.9999**	**.9999**
20	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
21	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
22	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
3 28 0	*.00000082*	*.00000106*	*.00000135*	*.00000169*	*.00000207*	*.00000252*	*.00000302*	*.00000358*	*.00000421*
1	**.0003**	**.0003**	**.0003**	**.0003**	**.0004**	**.0004**	**.0004**	**.0004**	**.0005**
2	**.0016**	**.0017**	**.0017**	**.0017**	**.0018**	**.0018**	**.0019**	**.0019**	**.0020**
3	**.0081**	**.0082**	**.0083**	**.0084**	**.0085**	**.0087**	**.0088**	**.0089**	**.0090**
4	**.0260**	**.0262**	**.0265**	**.0267**	**.0270**	**.0272**	**.0275**	**.0278**	**.0281**
5	**.0620**	**.0628**	**.0637**	**.0645**	**.0654**	**.0664**	**.0673**	**.0683**	**.0693**
6	**.1204**	**.1230**	**.1256**	**.1283**	**.1310**	**.1337**	**.1365**	**.1393**	**.1421**
7	**.2059**	**.2122**	**.2185**	**.2248**	**.2311**	**.2373**	**.2434**	**.2495**	**.2556**
8	**.3204**	**.3319**	**.3430**	**.3538**	**.3643**	**.3743**	**.3841**	**.3934**	**.4025**
9	**.4363**	**.4510**	**.4648**	**.4777**	**.4898**	**.5012**	**.5118**	**.5219**	**.5313**
10	**.5676**	**.5820**	**.5948**	**.6062**	**.6166**	**.6259**	**.6343**	**.6419**	**.6489**
11	**.6780**	**.6888**	**.6980**	**.7061**	**.7133**	**.7197**	**.7255**	**.7307**	**.7355**
12	**.7680**	**.7756**	**.7821**	**.7879**	**.7931**	**.7977**	**.8019**	**.8057**	**.8091**
13	**.8444**	**.8499**	**.8546**	**.8587**	**.8624**	**.8656**	**.8685**	**.8711**	**.8734**
14	**.9063**	**.9100**	**.9131**	**.9158**	**.9181**	**.9201**	**.9219**	**.9234**	**.9247**
15	**.9506**	**.9528**	**.9546**	**.9561**	**.9574**	**.9585**	**.9594**	**.9602**	**.9609**
16	**.9777**	**.9788**	**.9797**	**.9805**	**.9811**	**.9816**	**.9820**	**.9823**	**.9826**
17	**.9915**	**.9920**	**.9924**	**.9927**	**.9930**	**.9932**	**.9933**	**.9935**	**.9936**
18	**.9974**	**.9976**	**.9977**	**.9978**	**.9979**	**.9980**	**.9980**	**.9981**	**.9981**
19	**.9993**	**.9994**	**.9994**	**.9995**	**.9995**	**.9995**	**.9995**	**.9996**	**.9996**
20	**.9999**	**.9999**	**.9999**	**.9999**	**.9999**	**.9999**	**.9999**	**.9999**	**.9999**
21	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
22	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
23	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
3 29 0	*.00000041*	*.00000053*	*.00000068*	*.00000084*	*.00000104*	*.00000126*	*.00000151*	*.00000179*	*.00000211*
1	**.0003**	**.0003**	**.0003**	**.0003**	**.0004**	**.0004**	**.0004**	**.0004**	**.0005**
2	**.0016**	**.0016**	**.0017**	**.0017**	**.0018**	**.0018**	**.0019**	**.0019**	**.0019**
3	**.0081**	**.0082**	**.0082**	**.0083**	**.0084**	**.0085**	**.0086**	**.0087**	**.0088**
4	**.0256**	**.0257**	**.0259**	**.0261**	**.0263**	**.0265**	**.0267**	**.0269**	**.0271**
5	**.0600**	**.0605**	**.0611**	**.0617**	**.0623**	**.0629**	**.0636**	**.0643**	**.0649**
6	**.1135**	**.1152**	**.1170**	**.1188**	**.1206**	**.1225**	**.1244**	**.1264**	**.1284**
7	**.1886**	**.1932**	**.1978**	**.2025**	**.2071**	**.2118**	**.2164**	**.2211**	**.2257**
8	**.2894**	**.2988**	**.3079**	**.3169**	**.3257**	**.3343**	**.3427**	**.3509**	**.3589**
9	**.3961**	**.4093**	**.4219**	**.4339**	**.4454**	**.4564**	**.4668**	**.4767**	**.4862**
10	**.5279**	**.5429**	**.5564**	**.5689**	**.5802**	**.5906**	**.6001**	**.6089**	**.6170**
11	**.6478**	**.6602**	**.6708**	**.6803**	**.6887**	**.6961**	**.7029**	**.7089**	**.7145**
12	**.7462**	**.7550**	**.7625**	**.7692**	**.7751**	**.7804**	**.7852**	**.7896**	**.7936**
13	**.8279**	**.8343**	**.8398**	**.8447**	**.8490**	**.8529**	**.8564**	**.8596**	**.8625**
14	**.8944**	**.8989**	**.9028**	**.9062**	**.9092**	**.9118**	**.9141**	**.9162**	**.9180**
15	**.9429**	**.9459**	**.9483**	**.9504**	**.9522**	**.9537**	**.9551**	**.9562**	**.9572**
16	**.9734**	**.9750**	**.9764**	**.9775**	**.9784**	**.9792**	**.9799**	**.9804**	**.9809**
17	**.9894**	**.9902**	**.9909**	**.9914**	**.9918**	**.9922**	**.9925**	**.9927**	**.9929**
18	**.9965**	**.9968**	**.9971**	**.9973**	**.9975**	**.9976**	**.9977**	**.9978**	**.9979**

Table 33: Sheet Z. Exact (in roman), approximate *(in italics)* and interpolated **(boldfaced)** probabilities of perfect ($i = 0$) and i-imperfect column triplets ($k = 3$) in Bernoulli ($m \times n$)-matrices. The rows with all 1's are omitted

k m i	Width n of Bernoulli $(m \times n)$-matrix								
	12	13	14	15	16	17	18	19	20
3 29 19	.9990	.9991	.9992	.9993	.9994	.9994	.9994	.9995	.9995
20	.9998	.9998	.9998	.9999	.9999	.9999	.9999	.9999	.9999
21	.9999	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
22	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
23	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
24	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
3 30 0	.00000020	.00000027	.00000034	.00000042	.00000052	.00000063	.00000076	.00000090	.00000106
1	.0002	.0002	.0003	.0003	.0003	.0004	.0004	.0004	.0004
2	.0011	.0011	.0012	.0012	.0013	.0013	.0013	.0014	.0014
3	.0056	.0057	.0058	.0059	.0060	.0060	.0061	.0062	.0063
4	.0190	.0191	.0192	.0193	.0195	.0196	.0198	.0199	.0201
5	.0467	.0471	.0474	.0478	.0482	.0486	.0490	.0494	.0498
6	.0925	.0936	.0948	.0960	.0972	.0985	.0997	.1011	.1024
7	.1578	.1610	.1643	.1676	.1710	.1743	.1777	.1811	.1845
8	.2461	.2534	.2606	.2677	.2747	.2817	.2885	.2953	.3019
9	.3425	.3538	.3646	.3752	.3854	.3953	.4048	.4140	.4229
10	.4684	.4831	.4967	.5094	.5211	.5321	.5423	.5519	.5608
11	.5921	.6058	.6179	.6287	.6383	.6470	.6548	.6619	.6684
12	.6969	.7070	.7157	.7234	.7302	.7363	.7418	.7467	.7513
13	.7837	.7910	.7973	.8029	.8080	.8125	.8166	.8203	.8237
14	.8572	.8626	.8672	.8713	.8750	.8782	.8811	.8837	.8860
15	.9154	.9192	.9223	.9251	.9275	.9295	.9314	.9330	.9344
16	.9561	.9584	.9603	.9619	.9632	.9644	.9654	.9662	.9670
17	.9803	.9815	.9825	.9833	.9840	.9846	.9851	.9855	.9858
18	.9925	.9930	.9935	.9939	.9942	.9944	.9946	.9948	.9949
19	.9976	.9978	.9980	.9981	.9983	.9983	.9984	.9985	.9985
20	.9993	.9994	.9995	.9995	.9996	.9996	.9996	.9997	.9997
21	.9998	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999
22	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
23	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
24	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
3 31 0	.00000010	.00000013	.00000017	.00000021	.00000026	.00000032	.00000038	.00000045	.00000053
1	.0002	.0002	.0003	.0003	.0003	.0003	.0004	.0004	.0004
2	.0008	.0008	.0009	.0009	.0009	.0010	.0010	.0011	.0011
3	.0039	.0040	.0041	.0042	.0042	.0043	.0044	.0045	.0046
4	.0139	.0140	.0141	.0142	.0143	.0144	.0145	.0147	.0148
5	.0363	.0365	.0367	.0370	.0372	.0375	.0378	.0381	.0383
6	.0750	.0758	.0765	.0773	.0781	.0790	.0799	.0807	.0817
7	.1316	.1339	.1361	.1384	.1408	.1431	.1455	.1480	.1504
8	.2088	.2143	.2197	.2251	.2305	.2359	.2412	.2465	.2517
9	.2951	.3042	.3131	.3219	.3304	.3388	.3470	.3550	.3628
10	.4119	.4255	.4383	.4504	.4619	.4727	.4830	.4927	.5020
11	.5358	.5503	.5633	.5752	.5859	.5957	.6047	.6128	.6203
12	.6464	.6581	.6682	.6770	.6849	.6919	.6982	.7039	.7092
13	.7382	.7466	.7538	.7602	.7658	.7709	.7756	.7798	.7837
14	.8166	.8228	.8281	.8329	.8372	.8411	.8445	.8477	.8505
15	.8826	.8871	.8910	.8944	.8974	.9001	.9024	.9045	.9064
16	.9330	.9360	.9385	.9407	.9426	.9442	.9456	.9468	.9479
17	.9665	.9683	.9697	.9709	.9719	.9728	.9735	.9742	.9747
18	.9855	.9864	.9872	.9878	.9883	.9887	.9890	.9893	.9895
19	.9947	.9951	.9954	.9957	.9959	.9960	.9962	.9963	.9964
20	.9983	.9985	.9986	.9987	.9988	.9989	.9989	.9990	.9990
21	.9996	.9996	.9997	.9997	.9997	.9997	.9998	.9998	.9998
22	.9999	.9999	.9999	.9999	.9999	1.0000	1.0000	1.0000	1.0000
23	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
24	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Table 33: Sheet Z1. Exact (in roman), approximate *(in italics)* and interpolated **(boldfaced)** probabilities of perfect ($i = 0$) and i-imperfect column triplets ($k = 3$) in Bernoulli ($m \times n$)-matrices. The rows with all 1's are omitted

k m i	Width n of Bernoulli ($m \times n$)-matrix								
	12	13	14	15	16	17	18	19	20
3 31 25	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
3 32 0	*.00000005*	*.00000007*	*.00000008*	*.00000011*	*.00000013*	*.00000016*	*.00000019*	*.00000023*	*.00000026*
1	**.0002**	**.0002**	**.0002**	**.0003**	**.0003**	**.0003**	**.0003**	**.0004**	**.0004**
2	**.0006**	**.0006**	**.0007**	**.0007**	**.0007**	**.0008**	**.0008**	**.0009**	**.0009**
3	**.0028**	**.0029**	**.0029**	**.0030**	**.0031**	**.0031**	**.0032**	**.0033**	**.0034**
4	**.0101**	**.0102**	**.0103**	**.0104**	**.0105**	**.0106**	**.0107**	**.0108**	**.0109**
5	**.0279**	**.0280**	**.0282**	**.0284**	**.0285**	**.0287**	**.0289**	**.0291**	**.0293**
6	**.0604**	**.0610**	**.0615**	**.0620**	**.0626**	**.0632**	**.0638**	**.0644**	**.0650**
7	**.1094**	**.1110**	**.1125**	**.1141**	**.1158**	**.1174**	**.1191**	**.1208**	**.1225**
8	**.1768**	**.1808**	**.1848**	**.1888**	**.1928**	**.1968**	**.2008**	**.2048**	**.2088**
9	**.2539**	**.2611**	**.2681**	**.2750**	**.2819**	**.2887**	**.2954**	**.3020**	**.3085**
10	**.3598**	**.3716**	**.3829**	**.3939**	**.4044**	**.4145**	**.4241**	**.4334**	**.4424**
11	**.4801**	**.4946**	**.5079**	**.5202**	**.5316**	**.5422**	**.5519**	**.5610**	**.5694**
12	**.5949**	**.6080**	**.6194**	**.6296**	**.6386**	**.6467**	**.6541**	**.6607**	**.6667**
13	**.6920**	**.7016**	**.7099**	**.7172**	**.7236**	**.7294**	**.7346**	**.7394**	**.7438**
14	**.7742**	**.7812**	**.7873**	**.7927**	**.7976**	**.8020**	**.8060**	**.8096**	**.8130**
15	**.8456**	**.8510**	**.8555**	**.8596**	**.8633**	**.8665**	**.8694**	**.8720**	**.8744**
16	**.9044**	**.9082**	**.9114**	**.9142**	**.9166**	**.9188**	**.9207**	**.9223**	**.9238**
17	**.9474**	**.9498**	**.9518**	**.9535**	**.9549**	**.9562**	**.9572**	**.9582**	**.9590**
18	**.9747**	**.9760**	**.9771**	**.9780**	**.9788**	**.9794**	**.9799**	**.9804**	**.9808**
19	**.9895**	**.9901**	**.9906**	**.9911**	**.9914**	**.9917**	**.9919**	**.9922**	**.9923**
20	**.9963**	**.9966**	**.9968**	**.9970**	**.9971**	**.9972**	**.9973**	**.9974**	**.9974**
21	**.9989**	**.9990**	**.9991**	**.9991**	**.9992**	**.9992**	**.9993**	**.9993**	**.9993**
22	**.9997**	**.9998**	**.9998**	**.9998**	**.9998**	**.9998**	**.9998**	**.9998**	**.9999**
23	**.9999**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
24	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
25	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
26	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
3 33 0	*.00000003*	*.00000003*	*.00000004*	*.00000005*	*.00000007*	*.00000008*	*.00000009*	*.00000011*	*.00000013*
1	**.0002**	**.0002**	**.0002**	**.0003**	**.0003**	**.0003**	**.0003**	**.0004**	**.0004**
2	**.0006**	**.0006**	**.0007**	**.0007**	**.0007**	**.0008**	**.0008**	**.0009**	**.0009**
3	**.0028**	**.0028**	**.0029**	**.0030**	**.0031**	**.0031**	**.0032**	**.0033**	**.0033**
4	**.0100**	**.0101**	**.0102**	**.0103**	**.0104**	**.0105**	**.0105**	**.0106**	**.0107**
5	**.0276**	**.0277**	**.0279**	**.0280**	**.0281**	**.0283**	**.0284**	**.0286**	**.0287**
6	**.0589**	**.0592**	**.0596**	**.0599**	**.0603**	**.0606**	**.0610**	**.0614**	**.0618**
7	**.1051**	**.1061**	**.1072**	**.1082**	**.1093**	**.1104**	**.1116**	**.1127**	**.1139**
8	**.1661**	**.1689**	**.1717**	**.1745**	**.1774**	**.1803**	**.1832**	**.1861**	**.1891**
9	**.2347**	**.2400**	**.2453**	**.2507**	**.2560**	**.2612**	**.2665**	**.2717**	**.2768**
10	**.3284**	**.3383**	**.3479**	**.3572**	**.3663**	**.3752**	**.3837**	**.3921**	**.4002**
11	**.4416**	**.4553**	**.4682**	**.4803**	**.4916**	**.5023**	**.5123**	**.5218**	**.5307**
12	**.5599**	**.5739**	**.5864**	**.5977**	**.6079**	**.6171**	**.6255**	**.6331**	**.6401**
13	**.6655**	**.6766**	**.6861**	**.6945**	**.7019**	**.7085**	**.7145**	**.7199**	**.7248**
14	**.7543**	**.7623**	**.7692**	**.7753**	**.7808**	**.7857**	**.7902**	**.7943**	**.7981**
15	**.8300**	**.8360**	**.8413**	**.8460**	**.8502**	**.8540**	**.8574**	**.8605**	**.8633**
16	**.8926**	**.8971**	**.9010**	**.9044**	**.9074**	**.9101**	**.9125**	**.9146**	**.9165**
17	**.9393**	**.9424**	**.9450**	**.9472**	**.9492**	**.9509**	**.9523**	**.9536**	**.9548**
18	**.9698**	**.9716**	**.9732**	**.9745**	**.9756**	**.9765**	**.9773**	**.9780**	**.9786**
19	**.9868**	**.9878**	**.9886**	**.9893**	**.9899**	**.9903**	**.9907**	**.9910**	**.9913**
20	**.9950**	**.9955**	**.9959**	**.9962**	**.9964**	**.9966**	**.9968**	**.9969**	**.9970**
21	**.9984**	**.9986**	**.9987**	**.9988**	**.9989**	**.9990**	**.9991**	**.9991**	**.9992**
22	**.9995**	**.9996**	**.9997**	**.9997**	**.9997**	**.9998**	**.9998**	**.9998**	**.9998**
23	**.9999**	**.9999**	**.9999**	**.9999**	**.9999**	**.9999**	**1.0000**	**1.0000**	**1.0000**
24	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
25	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
26	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
3 34 0	*.00000001*	*.00000002*	*.00000002*	*.00000003*	*.00000003*	*.00000004*	*.00000005*	*.00000006*	*.00000007*

Table 33: Sheet Z2. Exact (in roman), approximate *(in italics)* and interpolated **(boldfaced)** probabilities of perfect ($i = 0$) and i-imperfect column triplets ($k = 3$) in Bernoulli ($m \times n$)-matrices. The rows with all 1's are omitted

k m i	Width n of Bernoulli ($m \times n$)-matrix								
	12	13	14	15	16	17	18	19	20
3 34 1	**.0002**	**.0002**	**.0002**	**.0003**	**.0003**	**.0003**	**.0003**	**.0004**	**.0004**
2	**.0005**	**.0005**	**.0005**	**.0006**	**.0006**	**.0007**	**.0007**	**.0007**	**.0008**
3	**.0020**	**.0021**	**.0021**	**.0022**	**.0023**	**.0023**	**.0024**	**.0025**	**.0026**
4	**.0073**	**.0073**	**.0074**	**.0075**	**.0076**	**.0076**	**.0077**	**.0078**	**.0079**
5	**.0210**	**.0211**	**.0212**	**.0213**	**.0214**	**.0216**	**.0217**	**.0218**	**.0219**
6	**.0472**	**.0474**	**.0476**	**.0479**	**.0481**	**.0483**	**.0486**	**.0489**	**.0491**
7	**.0875**	**.0881**	**.0888**	**.0896**	**.0903**	**.0911**	**.0918**	**.0926**	**.0934**
8	**.1413**	**.1433**	**.1453**	**.1473**	**.1493**	**.1513**	**.1534**	**.1555**	**.1576**
9	**.2028**	**.2067**	**.2107**	**.2147**	**.2186**	**.2226**	**.2266**	**.2305**	**.2345**
10	**.2840**	**.2919**	**.2996**	**.3072**	**.3147**	**.3220**	**.3292**	**.3363**	**.3432**
11	**.3848**	**.3971**	**.4088**	**.4200**	**.4307**	**.4409**	**.4506**	**.4599**	**.4687**
12	**.4966**	**.5109**	**.5239**	**.5359**	**.5468**	**.5569**	**.5662**	**.5747**	**.5827**
13	**.6019**	**.6144**	**.6252**	**.6348**	**.6434**	**.6510**	**.6579**	**.6641**	**.6697**
14	**.6918**	**.7010**	**.7089**	**.7158**	**.7220**	**.7275**	**.7325**	**.7371**	**.7413**
15	**.7700**	**.7768**	**.7827**	**.7880**	**.7927**	**.7970**	**.8009**	**.8045**	**.8078**
16	**.8396**	**.8448**	**.8493**	**.8534**	**.8570**	**.8602**	**.8631**	**.8657**	**.8681**
17	**.8982**	**.9020**	**.9052**	**.9080**	**.9105**	**.9127**	**.9146**	**.9163**	**.9179**
18	**.9422**	**.9447**	**.9467**	**.9485**	**.9500**	**.9513**	**.9525**	**.9535**	**.9543**
19	**.9710**	**.9725**	**.9736**	**.9746**	**.9755**	**.9762**	**.9768**	**.9773**	**.9777**
20	**.9874**	**.9881**	**.9887**	**.9892**	**.9896**	**.9899**	**.9902**	**.9904**	**.9906**
21	**.9953**	**.9956**	**.9959**	**.9961**	**.9962**	**.9964**	**.9965**	**.9966**	**.9967**
22	**.9985**	**.9986**	**.9987**	**.9988**	**.9989**	**.9989**	**.9990**	**.9990**	**.9990**
23	**.9996**	**.9996**	**.9997**	**.9997**	**.9997**	**.9997**	**.9998**	**.9998**	**.9998**
24	**.9999**	**.9999**	**.9999**	**.9999**	**.9999**	**.9999**	**1.0000**	**1.0000**	**1.0000**
25	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
26	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
27	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
3 35 0	*.00000001*	*.00000001*	*.00000001*	*.00000001*	*.00000002*	*.00000002*	*.00000002*	*.00000003*	*.00000003*
1	**.0002**	**.0002**	**.0002**	**.0003**	**.0003**	**.0003**	**.0003**	**.0004**	**.0004**
2	**.0004**	**.0004**	**.0005**	**.0005**	**.0006**	**.0006**	**.0006**	**.0007**	**.0007**
3	**.0015**	**.0015**	**.0016**	**.0017**	**.0018**	**.0018**	**.0019**	**.0020**	**.0020**
4	**.0052**	**.0053**	**.0054**	**.0054**	**.0055**	**.0056**	**.0057**	**.0057**	**.0058**
5	**.0158**	**.0159**	**.0160**	**.0161**	**.0162**	**.0163**	**.0164**	**.0165**	**.0166**
6	**.0374**	**.0376**	**.0378**	**.0379**	**.0381**	**.0383**	**.0385**	**.0387**	**.0389**
7	**.0724**	**.0729**	**.0734**	**.0739**	**.0744**	**.0750**	**.0755**	**.0760**	**.0766**
8	**.1203**	**.1216**	**.1229**	**.1243**	**.1257**	**.1271**	**.1285**	**.1300**	**.1315**
9	**.1763**	**.1791**	**.1819**	**.1848**	**.1877**	**.1906**	**.1935**	**.1964**	**.1993**
10	**.2488**	**.2548**	**.2608**	**.2668**	**.2727**	**.2786**	**.2843**	**.2901**	**.2957**
11	**.3399**	**.3504**	**.3605**	**.3703**	**.3798**	**.3889**	**.3978**	**.4063**	**.4146**
12	**.4463**	**.4601**	**.4729**	**.4849**	**.4961**	**.5065**	**.5163**	**.5254**	**.5339**
13	**.5537**	**.5672**	**.5792**	**.5900**	**.5996**	**.6083**	**.6162**	**.6234**	**.6299**
14	**.6481**	**.6586**	**.6676**	**.6756**	**.6826**	**.6889**	**.6945**	**.6996**	**.7044**
15	**.7289**	**.7366**	**.7432**	**.7491**	**.7544**	**.7592**	**.7635**	**.7675**	**.7712**
16	**.8013**	**.8072**	**.8123**	**.8170**	**.8211**	**.8248**	**.8282**	**.8313**	**.8341**
17	**.8654**	**.8699**	**.8738**	**.8772**	**.8802**	**.8829**	**.8853**	**.8875**	**.8894**
18	**.9174**	**.9205**	**.9232**	**.9255**	**.9275**	**.9292**	**.9307**	**.9321**	**.9333**
19	**.9547**	**.9567**	**.9583**	**.9597**	**.9608**	**.9618**	**.9627**	**.9635**	**.9641**
20	**.9781**	**.9792**	**.9801**	**.9808**	**.9814**	**.9819**	**.9824**	**.9828**	**.9831**
21	**.9908**	**.9913**	**.9918**	**.9921**	**.9924**	**.9926**	**.9928**	**.9930**	**.9931**
22	**.9967**	**.9969**	**.9971**	**.9972**	**.9974**	**.9975**	**.9975**	**.9976**	**.9976**
23	**.9990**	**.9991**	**.9991**	**.9992**	**.9992**	**.9993**	**.9993**	**.9993**	**.9993**
24	**.9997**	**.9998**	**.9998**	**.9998**	**.9998**	**.9998**	**.9998**	**.9998**	**.9999**
25	**.9999**	**.9999**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
26	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
27	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
28	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**

Table 33: Sheet Z3. Exact (in roman), approximate *(in italics)* and interpolated **(boldfaced)** probabilities of perfect ($i = 0$) and i-imperfect column triplets ($k = 3$) in Bernoulli ($m \times n$)-matrices. The rows with all 1's are omitted

k m i	Width n of Bernoulli $(m \times n)$-matrix								
	12	13	14	15	16	17	18	19	20
3 36 0	*.00000000*	*.00000000*	*.00000001*	*.00000001*	*.00000001*	*.00000001*	*.00000001*	*.00000001*	*.00000002*
1	**.0002**	**.0002**	**.0002**	**.0003**	**.0003**	**.0003**	**.0003**	**.0004**	**.0004**
2	**.0004**	**.0004**	**.0005**	**.0005**	**.0006**	**.0006**	**.0006**	**.0007**	**.0007**
3	**.0015**	**.0015**	**.0016**	**.0017**	**.0017**	**.0018**	**.0019**	**.0020**	**.0020**
4	**.0052**	**.0053**	**.0054**	**.0054**	**.0055**	**.0056**	**.0056**	**.0057**	**.0058**
5	**.0158**	**.0159**	**.0160**	**.0160**	**.0161**	**.0162**	**.0163**	**.0164**	**.0165**
6	**.0372**	**.0373**	**.0374**	**.0376**	**.0377**	**.0378**	**.0380**	**.0381**	**.0383**
7	**.0710**	**.0713**	**.0716**	**.0719**	**.0723**	**.0726**	**.0729**	**.0733**	**.0737**
8	**.1167**	**.1176**	**.1185**	**.1194**	**.1204**	**.1213**	**.1223**	**.1233**	**.1243**
9	**.1689**	**.1709**	**.1729**	**.1749**	**.1770**	**.1791**	**.1812**	**.1833**	**.1855**
10	**.2326**	**.2372**	**.2417**	**.2463**	**.2508**	**.2553**	**.2598**	**.2642**	**.2686**
11	**.3124**	**.3210**	**.3293**	**.3375**	**.3455**	**.3533**	**.3610**	**.3684**	**.3757**
12	**.4106**	**.4233**	**.4352**	**.4465**	**.4573**	**.4675**	**.4771**	**.4863**	**.4950**
13	**.5185**	**.5325**	**.5451**	**.5567**	**.5672**	**.5768**	**.5857**	**.5938**	**.6012**
14	**.6200**	**.6319**	**.6422**	**.6513**	**.6594**	**.6666**	**.6730**	**.6789**	**.6842**
15	**.7078**	**.7166**	**.7240**	**.7307**	**.7366**	**.7419**	**.7467**	**.7511**	**.7552**
16	**.7845**	**.7911**	**.7968**	**.8020**	**.8066**	**.8108**	**.8146**	**.8181**	**.8214**
17	**.8519**	**.8571**	**.8615**	**.8656**	**.8691**	**.8723**	**.8752**	**.8778**	**.8802**
18	**.9074**	**.9112**	**.9144**	**.9173**	**.9198**	**.9220**	**.9240**	**.9258**	**.9273**
19	**.9480**	**.9506**	**.9527**	**.9545**	**.9561**	**.9575**	**.9587**	**.9597**	**.9607**
20	**.9741**	**.9756**	**.9769**	**.9779**	**.9788**	**.9796**	**.9802**	**.9808**	**.9813**
21	**.9887**	**.9895**	**.9901**	**.9907**	**.9911**	**.9915**	**.9918**	**.9921**	**.9923**
22	**.9956**	**.9960**	**.9963**	**.9966**	**.9968**	**.9970**	**.9971**	**.9972**	**.9973**
23	**.9985**	**.9987**	**.9988**	**.9989**	**.9990**	**.9991**	**.9991**	**.9992**	**.9992**
24	**.9996**	**.9996**	**.9997**	**.9997**	**.9997**	**.9998**	**.9998**	**.9998**	**.9998**
25	**.9999**	**.9999**	**.9999**	**.9999**	**.9999**	**.9999**	**1.0000**	**1.0000**	**1.0000**
26	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
27	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
28	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
3 37 0	*.00000000*	*.00000000*	*.00000000*	*.00000000*	*.00000000*	*.00000000*	*.00000001*	*.00000001*	*.00000001*
1	**.0002**	**.0002**	**.0002**	**.0003**	**.0003**	**.0003**	**.0003**	**.0004**	**.0004**
2	**.0004**	**.0004**	**.0004**	**.0005**	**.0005**	**.0006**	**.0006**	**.0006**	**.0007**
3	**.0011**	**.0012**	**.0013**	**.0013**	**.0014**	**.0015**	**.0015**	**.0016**	**.0017**
4	**.0038**	**.0038**	**.0039**	**.0040**	**.0040**	**.0041**	**.0042**	**.0043**	**.0043**
5	**.0118**	**.0119**	**.0120**	**.0120**	**.0121**	**.0122**	**.0123**	**.0124**	**.0124**
6	**.0292**	**.0293**	**.0295**	**.0296**	**.0297**	**.0298**	**.0299**	**.0300**	**.0301**
7	**.0585**	**.0587**	**.0589**	**.0591**	**.0593**	**.0596**	**.0598**	**.0600**	**.0603**
8	**.0995**	**.1001**	**.1008**	**.1014**	**.1021**	**.1027**	**.1034**	**.1041**	**.1048**
9	**.1483**	**.1497**	**.1512**	**.1527**	**.1542**	**.1557**	**.1572**	**.1588**	**.1603**
10	**.2063**	**.2096**	**.2130**	**.2164**	**.2198**	**.2232**	**.2265**	**.2299**	**.2333**
11	**.2772**	**.2840**	**.2906**	**.2972**	**.3037**	**.3101**	**.3164**	**.3226**	**.3287**
12	**.3660**	**.3771**	**.3876**	**.3978**	**.4076**	**.4170**	**.4260**	**.4347**	**.4430**
13	**.4692**	**.4829**	**.4956**	**.5074**	**.5183**	**.5285**	**.5379**	**.5467**	**.5549**
14	**.5726**	**.5856**	**.5971**	**.6073**	**.6165**	**.6247**	**.6321**	**.6389**	**.6450**
15	**.6642**	**.6743**	**.6829**	**.6904**	**.6971**	**.7031**	**.7084**	**.7133**	**.7178**
16	**.7438**	**.7512**	**.7576**	**.7633**	**.7684**	**.7730**	**.7773**	**.7812**	**.7848**
17	**.8148**	**.8206**	**.8256**	**.8301**	**.8342**	**.8379**	**.8412**	**.8443**	**.8471**
18	**.8765**	**.8809**	**.8848**	**.8882**	**.8912**	**.8939**	**.8964**	**.8986**	**.9005**
19	**.9253**	**.9285**	**.9311**	**.9335**	**.9355**	**.9373**	**.9389**	**.9403**	**.9416**
20	**.9595**	**.9615**	**.9632**	**.9647**	**.9659**	**.9670**	**.9679**	**.9687**	**.9694**
21	**.9805**	**.9817**	**.9826**	**.9834**	**.9841**	**.9847**	**.9852**	**.9856**	**.9859**
22	**.9918**	**.9924**	**.9928**	**.9932**	**.9936**	**.9938**	**.9941**	**.9942**	**.9944**
23	**.9969**	**.9972**	**.9974**	**.9976**	**.9977**	**.9979**	**.9980**	**.9980**	**.9981**
24	**.9990**	**.9991**	**.9992**	**.9993**	**.9993**	**.9994**	**.9994**	**.9994**	**.9995**
25	**.9997**	**.9998**	**.9998**	**.9998**	**.9998**	**.9998**	**.9999**	**.9999**	**.9999**
26	**.9999**	**.9999**	**.9999**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**

Table 33: Sheet Z4. Exact (in roman), approximate *(in italics)* and interpolated **(boldfaced)** probabilities of perfect ($i = 0$) and i-imperfect column triplets ($k = 3$) in Bernoulli ($m \times n$)-matrices. The rows with all 1's are omitted

k m i	Width n of Bernoulli ($m \times n$)-matrix								
	12	13	14	15	16	17	18	19	20
3 37 27	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
28	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
29	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
3 38 0	*.00000000*	*.00000000*	*.00000000*	*.00000000*	*.00000000*	*.00000000*	*.00000000*	*.00000000*	*.00000000*
1	**.0002**	**.0002**	**.0002**	**.0003**	**.0003**	**.0003**	**.0003**	**.0004**	**.0004**
2	**.0003**	**.0004**	**.0004**	**.0005**	**.0005**	**.0005**	**.0006**	**.0006**	**.0007**
3	**.0009**	**.0010**	**.0011**	**.0011**	**.0012**	**.0013**	**.0013**	**.0014**	**.0015**
4	**.0027**	**.0028**	**.0029**	**.0029**	**.0030**	**.0031**	**.0031**	**.0032**	**.0033**
5	**.0087**	**.0088**	**.0089**	**.0090**	**.0090**	**.0091**	**.0092**	**.0093**	**.0093**
6	**.0227**	**.0228**	**.0229**	**.0230**	**.0231**	**.0232**	**.0233**	**.0234**	**.0235**
7	**.0476**	**.0478**	**.0480**	**.0481**	**.0483**	**.0485**	**.0486**	**.0488**	**.0490**
8	**.0843**	**.0847**	**.0852**	**.0856**	**.0861**	**.0866**	**.0871**	**.0876**	**.0881**
9	**.1289**	**.1299**	**.1309**	**.1319**	**.1329**	**.1339**	**.1349**	**.1360**	**.1371**
10	**.1823**	**.1847**	**.1871**	**.1896**	**.1920**	**.1945**	**.1969**	**.1994**	**.2019**
11	**.2463**	**.2514**	**.2566**	**.2617**	**.2667**	**.2718**	**.2767**	**.2817**	**.2866**
12	**.3260**	**.3352**	**.3441**	**.3528**	**.3613**	**.3695**	**.3775**	**.3852**	**.3928**
13	**.4222**	**.4351**	**.4471**	**.4585**	**.4693**	**.4794**	**.4889**	**.4980**	**.5065**
14	**.5250**	**.5387**	**.5509**	**.5621**	**.5722**	**.5813**	**.5897**	**.5974**	**.6044**
15	**.6202**	**.6316**	**.6414**	**.6500**	**.6577**	**.6645**	**.6706**	**.6762**	**.6812**
16	**.7028**	**.7112**	**.7184**	**.7248**	**.7304**	**.7355**	**.7402**	**.7445**	**.7484**
17	**.7762**	**.7827**	**.7882**	**.7933**	**.7978**	**.8019**	**.8056**	**.8091**	**.8123**
18	**.8423**	**.8473**	**.8517**	**.8557**	**.8592**	**.8624**	**.8653**	**.8679**	**.8703**
19	**.8980**	**.9018**	**.9050**	**.9079**	**.9105**	**.9127**	**.9147**	**.9165**	**.9181**
20	**.9403**	**.9429**	**.9451**	**.9470**	**.9486**	**.9501**	**.9513**	**.9524**	**.9534**
21	**.9687**	**.9703**	**.9716**	**.9728**	**.9737**	**.9745**	**.9752**	**.9759**	**.9764**
22	**.9855**	**.9863**	**.9871**	**.9877**	**.9882**	**.9886**	**.9889**	**.9892**	**.9895**
23	**.9941**	**.9945**	**.9948**	**.9951**	**.9954**	**.9955**	**.9957**	**.9959**	**.9960**
24	**.9979**	**.9981**	**.9982**	**.9983**	**.9984**	**.9985**	**.9986**	**.9986**	**.9987**
25	**.9993**	**.9994**	**.9995**	**.9995**	**.9995**	**.9996**	**.9996**	**.9996**	**.9996**
26	**.9998**	**.9998**	**.9999**	**.9999**	**.9999**	**.9999**	**.9999**	**.9999**	**.9999**
27	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
28	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
29	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
30	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
3 39 0	*.00000000*	*.00000000*	*.00000000*	*.00000000*	*.00000000*	*.00000000*	*.00000000*	*.00000000*	*.00000000*
1	**.0002**	**.0002**	**.0002**	**.0003**	**.0003**	**.0003**	**.0003**	**.0004**	**.0004**
2	**.0003**	**.0004**	**.0004**	**.0005**	**.0005**	**.0005**	**.0006**	**.0006**	**.0007**
3	**.0009**	**.0010**	**.0011**	**.0011**	**.0012**	**.0013**	**.0013**	**.0014**	**.0015**
4	**.0027**	**.0028**	**.0029**	**.0029**	**.0030**	**.0031**	**.0031**	**.0032**	**.0033**
5	**.0087**	**.0088**	**.0089**	**.0090**	**.0090**	**.0091**	**.0092**	**.0092**	**.0093**
6	**.0227**	**.0228**	**.0229**	**.0229**	**.0230**	**.0231**	**.0232**	**.0233**	**.0234**
7	**.0474**	**.0475**	**.0477**	**.0478**	**.0479**	**.0481**	**.0482**	**.0483**	**.0485**
8	**.0830**	**.0833**	**.0836**	**.0838**	**.0841**	**.0844**	**.0847**	**.0851**	**.0854**
9	**.1267**	**.1274**	**.1281**	**.1288**	**.1295**	**.1302**	**.1310**	**.1317**	**.1325**
10	**.1762**	**.1779**	**.1796**	**.1814**	**.1832**	**.1850**	**.1868**	**.1886**	**.1904**
11	**.2328**	**.2366**	**.2405**	**.2444**	**.2482**	**.2521**	**.2559**	**.2597**	**.2635**
12	**.3022**	**.3097**	**.3169**	**.3241**	**.3311**	**.3380**	**.3447**	**.3513**	**.3578**
13	**.3895**	**.4010**	**.4119**	**.4224**	**.4324**	**.4419**	**.4510**	**.4598**	**.4681**
14	**.4903**	**.5040**	**.5165**	**.5281**	**.5387**	**.5485**	**.5576**	**.5661**	**.5739**
15	**.5908**	**.6033**	**.6143**	**.6241**	**.6328**	**.6406**	**.6476**	**.6540**	**.6598**
16	**.6804**	**.6900**	**.6981**	**.7054**	**.7117**	**.7174**	**.7226**	**.7273**	**.7316**
17	**.7585**	**.7657**	**.7718**	**.7774**	**.7823**	**.7868**	**.7909**	**.7947**	**.7983**
18	**.8276**	**.8332**	**.8382**	**.8426**	**.8466**	**.8502**	**.8535**	**.8566**	**.8593**
19	**.8864**	**.8908**	**.8946**	**.8980**	**.9011**	**.9038**	**.9062**	**.9084**	**.9104**
20	**.9319**	**.9351**	**.9378**	**.9402**	**.9423**	**.9441**	**.9457**	**.9472**	**.9485**
21	**.9632**	**.9653**	**.9670**	**.9685**	**.9698**	**.9710**	**.9719**	**.9728**	**.9736**

Table 33: Sheet Z5. Exact (in roman), approximate *(in italics)* and interpolated **(boldfaced)** probabilities of perfect ($i = 0$) and i-imperfect column triplets ($k = 3$) in Bernoulli ($m \times n$)-matrices. The rows with all 1's are omitted

k m i	Width n of Bernoulli ($m \times n$)-matrix								
	12	13	14	15	16	17	18	19	20
3 39 22	**.9822**	**.9834**	**.9844**	**.9853**	**.9860**	**.9866**	**.9872**	**.9876**	**.9880**
23	**.9923**	**.9930**	**.9935**	**.9939**	**.9943**	**.9946**	**.9949**	**.9951**	**.9953**
24	**.9970**	**.9973**	**.9976**	**.9978**	**.9980**	**.9981**	**.9982**	**.9983**	**.9984**
25	**.9990**	**.9991**	**.9992**	**.9993**	**.9994**	**.9994**	**.9995**	**.9995**	**.9995**
26	**.9997**	**.9997**	**.9998**	**.9998**	**.9998**	**.9998**	**.9999**	**.9999**	**.9999**
27	**.9999**	**.9999**	**.9999**	**.9999**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
28	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
29	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
30	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
3 40 0	*.00000000*	*.00000000*	*.00000000*	*.00000000*	*.00000000*	*.00000000*	*.00000000*	*.00000000*	*.00000000*
1	**.0002**	**.0002**	**.0002**	**.0003**	**.0003**	**.0003**	**.0003**	**.0004**	**.0004**
2	**.0003**	**.0004**	**.0004**	**.0004**	**.0005**	**.0005**	**.0006**	**.0006**	**.0006**
3	**.0008**	**.0008**	**.0009**	**.0010**	**.0011**	**.0011**	**.0012**	**.0013**	**.0013**
4	**.0020**	**.0021**	**.0021**	**.0022**	**.0023**	**.0024**	**.0024**	**.0025**	**.0026**
5	**.0064**	**.0065**	**.0066**	**.0067**	**.0067**	**.0068**	**.0069**	**.0069**	**.0070**
6	**.0175**	**.0176**	**.0176**	**.0177**	**.0178**	**.0179**	**.0180**	**.0180**	**.0181**
7	**.0383**	**.0384**	**.0385**	**.0386**	**.0387**	**.0388**	**.0389**	**.0390**	**.0392**
8	**.0698**	**.0700**	**.0702**	**.0704**	**.0706**	**.0709**	**.0711**	**.0713**	**.0715**
9	**.1106**	**.1111**	**.1116**	**.1121**	**.1126**	**.1132**	**.1137**	**.1143**	**.1149**
10	**.1573**	**.1585**	**.1598**	**.1611**	**.1624**	**.1637**	**.1650**	**.1664**	**.1677**
11	**.2090**	**.2118**	**.2147**	**.2175**	**.2203**	**.2232**	**.2260**	**.2289**	**.2318**
12	**.2713**	**.2771**	**.2828**	**.2885**	**.2941**	**.2996**	**.3051**	**.3104**	**.3158**
13	**.3496**	**.3593**	**.3688**	**.3779**	**.3868**	**.3953**	**.4036**	**.4116**	**.4193**
14	**.4437**	**.4567**	**.4688**	**.4802**	**.4909**	**.5009**	**.5102**	**.5190**	**.5273**
15	**.5437**	**.5571**	**.5689**	**.5797**	**.5893**	**.5981**	**.6061**	**.6133**	**.6200**
16	**.6366**	**.6475**	**.6568**	**.6651**	**.6723**	**.6788**	**.6846**	**.6899**	**.6947**
17	**.7178**	**.7259**	**.7328**	**.7389**	**.7444**	**.7493**	**.7538**	**.7580**	**.7618**
18	**.7900**	**.7962**	**.8017**	**.8066**	**.8110**	**.8150**	**.8187**	**.8221**	**.8252**
19	**.8538**	**.8588**	**.8631**	**.8671**	**.8706**	**.8737**	**.8766**	**.8792**	**.8816**
20	**.9066**	**.9104**	**.9136**	**.9165**	**.9191**	**.9214**	**.9234**	**.9252**	**.9269**
21	**.9458**	**.9484**	**.9507**	**.9526**	**.9543**	**.9558**	**.9571**	**.9582**	**.9593**
22	**.9717**	**.9733**	**.9747**	**.9759**	**.9769**	**.9778**	**.9785**	**.9792**	**.9798**
23	**.9867**	**.9877**	**.9884**	**.9891**	**.9896**	**.9901**	**.9905**	**.9908**	**.9911**
24	**.9944**	**.9949**	**.9953**	**.9956**	**.9959**	**.9961**	**.9963**	**.9965**	**.9966**
25	**.9979**	**.9981**	**.9983**	**.9985**	**.9986**	**.9987**	**.9988**	**.9988**	**.9989**
26	**.9993**	**.9994**	**.9995**	**.9995**	**.9996**	**.9996**	**.9996**	**.9997**	**.9997**
27	**.9998**	**.9998**	**.9998**	**.9999**	**.9999**	**.9999**	**.9999**	**.9999**	**.9999**
28	**.9999**	**.9999**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
29	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
30	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
31	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
3 41 0	*.00000000*	*.00000000*	*.00000000*	*.00000000*	*.00000000*	*.00000000*	*.00000000*	*.00000000*	*.00000000*
1	**.0002**	**.0002**	**.0002**	**.0003**	**.0003**	**.0003**	**.0003**	**.0004**	**.0004**
2	**.0003**	**.0004**	**.0004**	**.0004**	**.0005**	**.0005**	**.0006**	**.0006**	**.0006**
3	**.0007**	**.0008**	**.0008**	**.0009**	**.0010**	**.0010**	**.0011**	**.0012**	**.0012**
4	**.0015**	**.0016**	**.0017**	**.0017**	**.0018**	**.0019**	**.0019**	**.0020**	**.0021**
5	**.0047**	**.0048**	**.0049**	**.0050**	**.0050**	**.0051**	**.0052**	**.0052**	**.0053**
6	**.0133**	**.0134**	**.0135**	**.0136**	**.0136**	**.0137**	**.0138**	**.0139**	**.0139**
7	**.0306**	**.0307**	**.0308**	**.0309**	**.0310**	**.0311**	**.0311**	**.0312**	**.0313**
8	**.0582**	**.0584**	**.0585**	**.0587**	**.0588**	**.0590**	**.0591**	**.0593**	**.0595**
9	**.0958**	**.0961**	**.0965**	**.0969**	**.0973**	**.0978**	**.0982**	**.0986**	**.0991**
10	**.1390**	**.1398**	**.1406**	**.1415**	**.1424**	**.1432**	**.1441**	**.1450**	**.1460**
11	**.1879**	**.1900**	**.1920**	**.1941**	**.1962**	**.1982**	**.2003**	**.2025**	**.2046**
12	**.2444**	**.2488**	**.2531**	**.2575**	**.2618**	**.2661**	**.2704**	**.2747**	**.2789**
13	**.3140**	**.3220**	**.3299**	**.3375**	**.3450**	**.3523**	**.3595**	**.3664**	**.3732**
14	**.4003**	**.4121**	**.4232**	**.4339**	**.4440**	**.4537**	**.4628**	**.4715**	**.4798**

Table 33: Sheet Z6. Exact (in roman), approximate *(in italics)* and interpolated **(boldfaced)** probabilities of perfect ($i = 0$) and i-imperfect column triplets ($k = 3$) in Bernoulli ($m \times n$)-matrices. The rows with all 1's are omitted

k	m	i	Width n of Bernoulli ($m \times n$)-matrix								
			12	13	14	15	16	17	18	19	20
3	41	15	**.4974**	**.5109**	**.5232**	**.5344**	**.5448**	**.5543**	**.5630**	**.5711**	**.5785**
		16	**.5923**	**.6044**	**.6149**	**.6243**	**.6326**	**.6400**	**.6466**	**.6526**	**.6581**
		17	**.6768**	**.6860**	**.6938**	**.7007**	**.7068**	**.7123**	**.7172**	**.7217**	**.7259**
		18	**.7514**	**.7583**	**.7643**	**.7697**	**.7745**	**.7789**	**.7829**	**.7866**	**.7900**
		19	**.8187**	**.8242**	**.8290**	**.8334**	**.8373**	**.8409**	**.8442**	**.8472**	**.8499**
		20	**.8773**	**.8816**	**.8854**	**.8888**	**.8918**	**.8945**	**.8970**	**.8992**	**.9012**
		21	**.9239**	**.9272**	**.9299**	**.9323**	**.9344**	**.9363**	**.9380**	**.9395**	**.9408**
		22	**.9573**	**.9594**	**.9612**	**.9628**	**.9641**	**.9653**	**.9663**	**.9672**	**.9680**
		23	**.9784**	**.9797**	**.9807**	**.9817**	**.9824**	**.9831**	**.9837**	**.9842**	**.9846**
		24	**.9902**	**.9909**	**.9915**	**.9920**	**.9924**	**.9927**	**.9930**	**.9933**	**.9935**
		25	**.9960**	**.9964**	**.9967**	**.9969**	**.9971**	**.9972**	**.9974**	**.9975**	**.9976**
		26	**.9986**	**.9987**	**.9988**	**.9989**	**.9990**	**.9991**	**.9991**	**.9992**	**.9992**
		27	**.9995**	**.9996**	**.9996**	**.9997**	**.9997**	**.9997**	**.9998**	**.9998**	**.9998**
		28	**.9999**	**.9999**	**.9999**	**.9999**	**.9999**	**.9999**	**.9999**	**.9999**	**.9999**
		29	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
		30	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
		31	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
		32	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
3	42	0	*.00000000*	*.00000000*	*.00000000*	*.00000000*	*.00000000*	*.00000000*	*.00000000*	*.00000000*	*.00000000*
		1	**.0002**	**.0002**	**.0002**	**.0003**	**.0003**	**.0003**	**.0003**	**.0004**	**.0004**
		2	**.0003**	**.0004**	**.0004**	**.0004**	**.0005**	**.0005**	**.0006**	**.0006**	**.0006**
		3	**.0007**	**.0008**	**.0008**	**.0009**	**.0010**	**.0010**	**.0011**	**.0012**	**.0012**
		4	**.0015**	**.0016**	**.0017**	**.0017**	**.0018**	**.0019**	**.0019**	**.0020**	**.0021**
		5	**.0047**	**.0048**	**.0049**	**.0049**	**.0050**	**.0051**	**.0052**	**.0052**	**.0053**
		6	**.0133**	**.0134**	**.0135**	**.0136**	**.0136**	**.0137**	**.0138**	**.0138**	**.0139**
		7	**.0305**	**.0306**	**.0307**	**.0308**	**.0309**	**.0310**	**.0310**	**.0311**	**.0312**
		8	**.0580**	**.0581**	**.0582**	**.0584**	**.0585**	**.0586**	**.0587**	**.0589**	**.0590**
		9	**.0952**	**.0955**	**.0958**	**.0962**	**.0965**	**.0968**	**.0972**	**.0975**	**.0979**
		10	**.1371**	**.1377**	**.1383**	**.1389**	**.1395**	**.1402**	**.1408**	**.1415**	**.1422**
		11	**.1829**	**.1844**	**.1858**	**.1874**	**.1889**	**.1904**	**.1920**	**.1935**	**.1951**
		12	**.2332**	**.2364**	**.2397**	**.2430**	**.2463**	**.2496**	**.2529**	**.2562**	**.2594**
		13	**.2937**	**.3001**	**.3063**	**.3125**	**.3187**	**.3247**	**.3306**	**.3364**	**.3421**
		14	**.3708**	**.3811**	**.3909**	**.4004**	**.4096**	**.4184**	**.4269**	**.4350**	**.4429**
		15	**.4638**	**.4769**	**.4890**	**.5003**	**.5108**	**.5207**	**.5298**	**.5384**	**.5464**
		16	**.5619**	**.5749**	**.5864**	**.5967**	**.6060**	**.6143**	**.6219**	**.6288**	**.6351**
		17	**.6529**	**.6633**	**.6723**	**.6801**	**.6871**	**.6932**	**.6988**	**.7038**	**.7084**
		18	**.7327**	**.7405**	**.7472**	**.7531**	**.7584**	**.7631**	**.7675**	**.7715**	**.7752**
		19	**.8032**	**.8092**	**.8145**	**.8193**	**.8237**	**.8276**	**.8312**	**.8345**	**.8376**
		20	**.8644**	**.8693**	**.8736**	**.8774**	**.8809**	**.8840**	**.8869**	**.8895**	**.8919**
		21	**.9139**	**.9177**	**.9210**	**.9239**	**.9265**	**.9288**	**.9308**	**.9327**	**.9343**
		22	**.9502**	**.9528**	**.9551**	**.9571**	**.9588**	**.9604**	**.9617**	**.9629**	**.9640**
		23	**.9738**	**.9755**	**.9769**	**.9782**	**.9792**	**.9802**	**.9810**	**.9817**	**.9823**
		24	**.9875**	**.9885**	**.9893**	**.9900**	**.9906**	**.9911**	**.9916**	**.9919**	**.9923**
		25	**.9946**	**.9951**	**.9956**	**.9959**	**.9962**	**.9965**	**.9967**	**.9969**	**.9970**
		26	**.9979**	**.9981**	**.9983**	**.9985**	**.9986**	**.9987**	**.9988**	**.9989**	**.9990**
		27	**.9992**	**.9993**	**.9994**	**.9995**	**.9996**	**.9996**	**.9996**	**.9997**	**.9997**
		28	**.9997**	**.9998**	**.9998**	**.9998**	**.9999**	**.9999**	**.9999**	**.9999**	**.9999**
		29	**.9999**	**.9999**	**.9999**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
		30	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
		31	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
		32	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
3	43	0	*.00000000*	*.00000000*	*.00000000*	*.00000000*	*.00000000*	*.00000000*	*.00000000*	*.00000000*	*.00000000*
		1	**.0002**	**.0002**	**.0002**	**.0003**	**.0003**	**.0003**	**.0003**	**.0004**	**.0004**
		2	**.0003**	**.0003**	**.0004**	**.0004**	**.0005**	**.0005**	**.0005**	**.0006**	**.0006**
		3	**.0006**	**.0007**	**.0008**	**.0008**	**.0009**	**.0010**	**.0011**	**.0011**	**.0012**
		4	**.0012**	**.0013**	**.0013**	**.0014**	**.0015**	**.0015**	**.0016**	**.0017**	**.0017**

Table 33: Sheet Z7. Exact (in roman), approximate *(in italics)* and interpolated **(boldfaced)** probabilities of perfect ($i = 0$) and i-imperfect column triplets ($k = 3$) in Bernoulli ($m \times n$)-matrices. The rows with all 1's are omitted

k m i	Width n of Bernoulli $(m \times n)$-matrix								
	12	13	14	15	16	17	18	19	20
3 43 5	.0035	.0036	.0036	.0037	.0038	.0038	.0039	.0040	.0041
6	.0101	.0102	.0102	.0103	.0104	.0105	.0105	.0106	.0107
7	.0242	.0242	.0243	.0244	.0245	.0246	.0246	.0247	.0248
8	.0479	.0480	.0481	.0482	.0483	.0484	.0485	.0486	.0487
9	.0819	.0821	.0824	.0827	.0830	.0832	.0835	.0838	.0841
10	.1215	.1219	.1224	.1229	.1233	.1238	.1243	.1248	.1254
11	.1656	.1666	.1678	.1689	.1700	.1712	.1723	.1735	.1747
12	.2120	.2144	.2167	.2192	.2216	.2240	.2264	.2288	.2313
13	.2665	.2715	.2764	.2812	.2860	.2908	.2955	.3002	.3048
14	.3350	.3436	.3520	.3601	.3680	.3757	.3831	.3904	.3975
15	.4203	.4324	.4437	.4545	.4647	.4744	.4835	.4921	.5003
16	.5158	.5291	.5411	.5521	.5621	.5713	.5797	.5874	.5945
17	.6088	.6205	.6306	.6395	.6474	.6545	.6608	.6665	.6717
18	.6919	.7008	.7083	.7149	.7208	.7260	.7308	.7351	.7392
19	.7653	.7721	.7778	.7831	.7878	.7921	.7960	.7996	.8030
20	.8307	.8361	.8408	.8451	.8490	.8526	.8558	.8587	.8615
21	.8865	.8908	.8946	.8980	.9010	.9037	.9061	.9084	.9104
22	.9301	.9333	.9361	.9385	.9407	.9426	.9443	.9458	.9472
23	.9608	.9630	.9648	.9664	.9678	.9690	.9701	.9711	.9719
24	.9800	.9813	.9825	.9834	.9843	.9850	.9856	.9861	.9866
25	.9907	.9915	.9921	.9927	.9931	.9935	.9938	.9941	.9943
26	.9961	.9965	.9968	.9971	.9973	.9975	.9976	.9978	.9979
27	.9985	.9987	.9988	.9990	.9990	.9991	.9992	.9993	.9993
28	.9995	.9995	.9996	.9997	.9997	.9997	.9998	.9998	.9998
29	.9998	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999
30	.9999	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
31	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
32	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
33	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
3 44 0	*.00000000*	*.00000000*	*.00000000*	*.00000000*	*.00000000*	*.00000000*	*.00000000*	*.00000000*	*.00000000*
1	.0002	.0002	.0002	.0003	.0003	.0003	.0003	.0004	.0004
2	.0003	.0003	.0004	.0004	.0005	.0005	.0005	.0006	.0006
3	.0006	.0007	.0007	.0008	.0009	.0010	.0010	.0011	.0012
4	.0010	.0010	.0011	.0012	.0012	.0013	.0014	.0014	.0015
5	.0026	.0027	.0027	.0028	.0029	.0029	.0030	.0031	.0032
6	.0076	.0077	.0077	.0078	.0079	.0080	.0080	.0081	.0082
7	.0189	.0190	.0191	.0192	.0192	.0193	.0194	.0195	.0195
8	.0392	.0392	.0393	.0394	.0395	.0396	.0397	.0398	.0399
9	.0697	.0700	.0702	.0705	.0707	.0710	.0712	.0715	.0717
10	.1068	.1072	.1076	.1079	.1083	.1087	.1091	.1095	.1099
11	.1482	.1490	.1497	.1504	.1512	.1520	.1527	.1535	.1543
12	.1931	.1949	.1966	.1984	.2001	.2019	.2037	.2055	.2073
13	.2430	.2468	.2505	.2542	.2579	.2616	.2653	.2689	.2725
14	.3038	.3108	.3176	.3243	.3308	.3373	.3436	.3498	.3558
15	.3806	.3913	.4015	.4113	.4207	.4297	.4383	.4465	.4544
16	.4711	.4842	.4962	.5074	.5178	.5274	.5363	.5446	.5523
17	.5646	.5772	.5883	.5982	.6071	.6151	.6223	.6289	.6348
18	.6506	.6607	.6692	.6767	.6834	.6893	.6946	.6994	.7038
19	.7267	.7342	.7407	.7464	.7515	.7561	.7604	.7643	.7679
20	.7950	.8010	.8061	.8108	.8151	.8189	.8225	.8258	.8288
21	.8556	.8605	.8647	.8685	.8720	.8751	.8779	.8805	.8829
22	.9059	.9097	.9130	.9159	.9184	.9208	.9228	.9247	.9264
23	.9438	.9465	.9488	.9508	.9526	.9542	.9556	.9568	.9579
24	.9694	.9712	.9726	.9739	.9751	.9760	.9769	.9776	.9783
25	.9848	.9859	.9868	.9875	.9882	.9887	.9892	.9896	.9900
26	.9932	.9938	.9942	.9946	.9950	.9953	.9955	.9957	.9959

Table 33: Sheet Z8. Exact (in roman), approximate *(in italics)* and interpolated **(boldfaced)** probabilities of perfect ($i = 0$) and i-imperfect column triplets ($k = 3$) in Bernoulli ($m \times n$)-matrices. The rows with all 1's are omitted

k m i	Width n of Bernoulli ($m \times n$)-matrix								
	12	13	14	15	16	17	18	19	20
3 44 27	**.9972**	**.9975**	**.9977**	**.9979**	**.9981**	**.9982**	**.9983**	**.9984**	**.9985**
28	**.9990**	**.9991**	**.9992**	**.9993**	**.9993**	**.9994**	**.9994**	**.9995**	**.9995**
29	**.9996**	**.9997**	**.9997**	**.9998**	**.9998**	**.9998**	**.9998**	**.9999**	**.9999**
30	**.9999**	**.9999**	**.9999**	**.9999**	**.9999**	**.9999**	**1.0000**	**1.0000**	**1.0000**
31	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
32	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
33	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
34	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
3 45 0	*.00000000*	*.00000000*	*.00000000*	*.00000000*	*.00000000*	*.00000000*	*.00000000*	*.00000000*	*.00000000*
1	**.0002**	**.0002**	**.0002**	**.0003**	**.0003**	**.0003**	**.0003**	**.0004**	**.0004**
2	**.0003**	**.0003**	**.0004**	**.0004**	**.0005**	**.0005**	**.0005**	**.0006**	**.0006**
3	**.0006**	**.0007**	**.0007**	**.0008**	**.0009**	**.0009**	**.0010**	**.0011**	**.0011**
4	**.0008**	**.0009**	**.0010**	**.0010**	**.0011**	**.0012**	**.0012**	**.0013**	**.0014**
5	**.0020**	**.0020**	**.0021**	**.0022**	**.0022**	**.0023**	**.0024**	**.0024**	**.0025**
6	**.0057**	**.0058**	**.0058**	**.0059**	**.0060**	**.0060**	**.0061**	**.0062**	**.0063**
7	**.0147**	**.0148**	**.0148**	**.0149**	**.0150**	**.0151**	**.0151**	**.0152**	**.0153**
8	**.0317**	**.0318**	**.0319**	**.0319**	**.0320**	**.0321**	**.0322**	**.0323**	**.0324**
9	**.0589**	**.0591**	**.0593**	**.0595**	**.0597**	**.0600**	**.0602**	**.0604**	**.0607**
10	**.0932**	**.0935**	**.0938**	**.0941**	**.0944**	**.0947**	**.0951**	**.0954**	**.0957**
11	**.1327**	**.1332**	**.1338**	**.1343**	**.1349**	**.1355**	**.1360**	**.1366**	**.1372**
12	**.1759**	**.1771**	**.1784**	**.1797**	**.1810**	**.1824**	**.1837**	**.1851**	**.1864**
13	**.2219**	**.2246**	**.2274**	**.2302**	**.2329**	**.2357**	**.2385**	**.2412**	**.2440**
14	**.2765**	**.2820**	**.2873**	**.2927**	**.2979**	**.3031**	**.3083**	**.3133**	**.3183**
15	**.3452**	**.3543**	**.3631**	**.3716**	**.3799**	**.3879**	**.3957**	**.4032**	**.4104**
16	**.4289**	**.4412**	**.4527**	**.4635**	**.4737**	**.4833**	**.4923**	**.5009**	**.5089**
17	**.5206**	**.5337**	**.5455**	**.5562**	**.5659**	**.5747**	**.5828**	**.5902**	**.5969**
18	**.6089**	**.6201**	**.6298**	**.6384**	**.6459**	**.6527**	**.6587**	**.6641**	**.6691**
19	**.6877**	**.6962**	**.7034**	**.7098**	**.7155**	**.7205**	**.7251**	**.7294**	**.7333**
20	**.7583**	**.7648**	**.7704**	**.7755**	**.7801**	**.7843**	**.7881**	**.7917**	**.7950**
21	**.8223**	**.8276**	**.8322**	**.8364**	**.8403**	**.8437**	**.8469**	**.8499**	**.8526**
22	**.8780**	**.8823**	**.8860**	**.8894**	**.8924**	**.8951**	**.8976**	**.8998**	**.9018**
23	**.9228**	**.9260**	**.9288**	**.9313**	**.9334**	**.9354**	**.9371**	**.9386**	**.9401**
24	**.9552**	**.9574**	**.9593**	**.9610**	**.9624**	**.9637**	**.9648**	**.9658**	**.9667**
25	**.9763**	**.9777**	**.9789**	**.9799**	**.9808**	**.9816**	**.9822**	**.9828**	**.9833**
26	**.9886**	**.9894**	**.9901**	**.9907**	**.9912**	**.9916**	**.9919**	**.9922**	**.9925**
27	**.9950**	**.9955**	**.9958**	**.9961**	**.9964**	**.9966**	**.9967**	**.9969**	**.9970**
28	**.9980**	**.9982**	**.9984**	**.9985**	**.9987**	**.9987**	**.9988**	**.9989**	**.9989**
29	**.9993**	**.9994**	**.9994**	**.9995**	**.9996**	**.9996**	**.9996**	**.9996**	**.9997**
30	**.9998**	**.9998**	**.9998**	**.9998**	**.9999**	**.9999**	**.9999**	**.9999**	**.9999**
31	**.9999**	**.9999**	**.9999**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
32	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
33	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
34	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
3 46 0	*.00000000*	*.00000000*	*.00000000*	*.00000000*	*.00000000*	*.00000000*	*.00000000*	*.00000000*	*.00000000*
1	**.0002**	**.0002**	**.0002**	**.0003**	**.0003**	**.0003**	**.0003**	**.0004**	**.0004**
2	**.0003**	**.0003**	**.0004**	**.0004**	**.0005**	**.0005**	**.0005**	**.0006**	**.0006**
3	**.0006**	**.0007**	**.0007**	**.0008**	**.0009**	**.0009**	**.0010**	**.0011**	**.0011**
4	**.0008**	**.0009**	**.0010**	**.0010**	**.0011**	**.0012**	**.0012**	**.0013**	**.0014**
5	**.0020**	**.0020**	**.0021**	**.0022**	**.0022**	**.0023**	**.0024**	**.0024**	**.0025**
6	**.0057**	**.0058**	**.0058**	**.0059**	**.0060**	**.0060**	**.0061**	**.0062**	**.0063**
7	**.0147**	**.0148**	**.0148**	**.0149**	**.0150**	**.0151**	**.0151**	**.0152**	**.0153**
8	**.0317**	**.0317**	**.0318**	**.0319**	**.0320**	**.0321**	**.0321**	**.0322**	**.0323**
9	**.0588**	**.0590**	**.0592**	**.0594**	**.0596**	**.0598**	**.0600**	**.0602**	**.0604**
10	**.0929**	**.0932**	**.0934**	**.0937**	**.0939**	**.0942**	**.0945**	**.0948**	**.0950**
11	**.1317**	**.1321**	**.1325**	**.1330**	**.1334**	**.1338**	**.1343**	**.1347**	**.1352**
12	**.1722**	**.1731**	**.1740**	**.1749**	**.1758**	**.1767**	**.1776**	**.1785**	**.1794**

Table 33: Sheet Z9. Exact (in roman), approximate *(in italics)* and interpolated **(boldfaced)** probabilities of perfect ($i = 0$) and i-imperfect column triplets ($k = 3$) in Bernoulli ($m \times n$)-matrices. The rows with all 1's are omitted

k m i	Width n of Bernoulli $(m \times n)$-matrix								
	12	13	14	15	16	17	18	19	20
3 46 13	.2148	.2168	.2188	.2209	.2229	.2250	.2271	.2291	.2312
14	.2627	.2669	.2710	.2752	.2793	.2834	.2874	.2914	.2954
15	.3225	.3300	.3373	.3445	.3515	.3583	.3650	.3715	.3779
16	.3991	.4102	.4206	.4307	.4402	.4493	.4580	.4663	.4743
17	.4890	.5020	.5139	.5250	.5351	.5445	.5532	.5612	.5686
18	.5812	.5934	.6042	.6137	.6223	.6299	.6368	.6430	.6486
19	.6659	.6756	.6838	.6910	.6974	.7030	.7081	.7128	.7170
20	.7408	.7481	.7543	.7599	.7649	.7694	.7735	.7773	.7808
21	.8075	.8133	.8183	.8229	.8271	.8309	.8344	.8376	.8406
22	.8655	.8703	.8745	.8782	.8816	.8847	.8875	.8901	.8925
23	.9128	.9166	.9198	.9228	.9253	.9277	.9298	.9316	.9334
24	.9479	.9507	.9530	.9551	.9569	.9585	.9599	.9612	.9623
25	.9714	.9733	.9748	.9761	.9773	.9783	.9792	.9800	.9807
26	.9856	.9867	.9877	.9885	.9891	.9897	.9902	.9907	.9911
27	.9933	.9940	.9945	.9949	.9953	.9956	.9959	.9961	.9963
28	.9971	.9975	.9977	.9980	.9981	.9983	.9984	.9985	.9986
29	.9989	.9990	.9991	.9992	.9993	.9994	.9994	.9995	.9995
30	.9996	.9996	.9997	.9997	.9998	.9998	.9998	.9998	.9999
31	.9999	.9999	.9999	.9999	.9999	.9999	.9999	1.0000	1.0000
32	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
33	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
34	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
35	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
3 47 0	*.00000000*	*.00000000*	*.00000000*	*.00000000*	*.00000000*	*.00000000*	*.00000000*	*.00000000*	*.00000000*
1	.0002	.0002	.0002	.0003	.0003	.0003	.0003	.0004	.0004
2	.0003	.0003	.0004	.0004	.0005	.0005	.0005	.0006	.0006
3	.0006	.0006	.0007	.0008	.0009	.0009	.0010	.0011	.0011
4	.0007	.0008	.0009	.0009	.0010	.0011	.0011	.0012	.0013
5	.0015	.0016	.0016	.0017	.0018	.0019	.0019	.0020	.0021
6	.0043	.0043	.0044	.0045	.0045	.0046	.0047	.0048	.0048
7	.0113	.0114	.0115	.0115	.0116	.0117	.0117	.0118	.0119
8	.0254	.0255	.0255	.0256	.0257	.0258	.0258	.0259	.0260
9	.0492	.0494	.0496	.0498	.0500	.0502	.0504	.0506	.0508
10	.0804	.0806	.0809	.0811	.0813	.0816	.0818	.0821	.0823
11	.1174	.1177	.1181	.1184	.1188	.1191	.1195	.1199	.1202
12	.1568	.1575	.1581	.1588	.1594	.1601	.1608	.1615	.1621
13	.1980	.1995	.2010	.2025	.2040	.2055	.2070	.2086	.2101
14	.2422	.2454	.2485	.2517	.2549	.2580	.2612	.2643	.2674
15	.2952	.3012	.3071	.3129	.3186	.3243	.3298	.3353	.3406
16	.3632	.3727	.3819	.3908	.3994	.4077	.4156	.4233	.4307
17	.4465	.4589	.4705	.4814	.4915	.5010	.5099	.5183	.5261
18	.5373	.5501	.5616	.5720	.5814	.5899	.5976	.6046	.6111
19	.6243	.6351	.6445	.6527	.6599	.6663	.6721	.6773	.6820
20	.7021	.7103	.7173	.7234	.7289	.7338	.7382	.7423	.7461
21	.7715	.7779	.7834	.7883	.7928	.7969	.8007	.8041	.8074
22	.8336	.8388	.8434	.8475	.8513	.8547	.8579	.8608	.8634
23	.8867	.8910	.8947	.8980	.9010	.9037	.9061	.9084	.9104
24	.9286	.9318	.9346	.9371	.9393	.9413	.9430	.9446	.9460
25	.9585	.9608	.9627	.9644	.9659	.9672	.9684	.9694	.9703
26	.9779	.9794	.9806	.9817	.9826	.9834	.9841	.9847	.9852
27	.9892	.9900	.9908	.9914	.9919	.9923	.9927	.9931	.9934
28	.9951	.9956	.9960	.9963	.9966	.9968	.9970	.9972	.9973
29	.9980	.9982	.9984	.9985	.9987	.9988	.9989	.9990	.9990
30	.9992	.9993	.9994	.9995	.9995	.9996	.9996	.9997	.9997
31	.9997	.9998	.9998	.9998	.9998	.9999	.9999	.9999	.9999
32	.9999	.9999	.9999	.9999	1.0000	1.0000	1.0000	1.0000	1.0000

Table 33: Sheet Z10. Exact (in roman), approximate *(in italics)* and interpolated **(boldfaced)** probabilities of perfect ($i = 0$) and i-imperfect column triplets ($k = 3$) in Bernoulli ($m \times n$)-matrices. The rows with all 1's are omitted

k m i	Width n of Bernoulli $(m \times n)$-matrix								
	12	13	14	15	16	17	18	19	20
3 47 33	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
34	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
35	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
3 48 0	*.00000000*	*.00000000*	*.00000000*	*.00000000*	*.00000000*	*.00000000*	*.00000000*	*.00000000*	*.00000000*
1	**.0002**	**.0002**	**.0002**	**.0003**	**.0003**	**.0003**	**.0003**	**.0004**	**.0004**
2	**.0003**	**.0003**	**.0004**	**.0004**	**.0005**	**.0005**	**.0005**	**.0006**	**.0006**
3	**.0006**	**.0006**	**.0007**	**.0008**	**.0008**	**.0009**	**.0010**	**.0011**	**.0011**
4	**.0007**	**.0007**	**.0008**	**.0009**	**.0009**	**.0010**	**.0011**	**.0011**	**.0012**
5	**.0012**	**.0013**	**.0013**	**.0014**	**.0015**	**.0015**	**.0016**	**.0017**	**.0017**
6	**.0032**	**.0033**	**.0033**	**.0034**	**.0035**	**.0036**	**.0036**	**.0037**	**.0038**
7	**.0087**	**.0087**	**.0088**	**.0089**	**.0089**	**.0090**	**.0091**	**.0092**	**.0092**
8	**.0202**	**.0202**	**.0203**	**.0204**	**.0205**	**.0205**	**.0206**	**.0207**	**.0208**
9	**.0408**	**.0410**	**.0412**	**.0414**	**.0415**	**.0417**	**.0419**	**.0421**	**.0423**
10	**.0690**	**.0692**	**.0694**	**.0696**	**.0698**	**.0701**	**.0703**	**.0705**	**.0707**
11	**.1038**	**.1041**	**.1044**	**.1047**	**.1050**	**.1053**	**.1056**	**.1059**	**.1062**
12	**.1421**	**.1426**	**.1431**	**.1436**	**.1441**	**.1446**	**.1451**	**.1456**	**.1461**
13	**.1823**	**.1834**	**.1845**	**.1857**	**.1868**	**.1880**	**.1891**	**.1903**	**.1915**
14	**.2234**	**.2257**	**.2280**	**.2304**	**.2327**	**.2351**	**.2374**	**.2397**	**.2421**
15	**.2712**	**.2758**	**.2804**	**.2850**	**.2896**	**.2940**	**.2985**	**.3029**	**.3072**
16	**.3313**	**.3393**	**.3471**	**.3547**	**.3621**	**.3692**	**.3762**	**.3830**	**.3897**
17	**.4072**	**.4185**	**.4293**	**.4395**	**.4491**	**.4583**	**.4671**	**.4753**	**.4832**
18	**.4944**	**.5073**	**.5191**	**.5300**	**.5399**	**.5490**	**.5574**	**.5652**	**.5723**
19	**.5823**	**.5942**	**.6046**	**.6138**	**.6220**	**.6293**	**.6359**	**.6418**	**.6472**
20	**.6629**	**.6722**	**.6801**	**.6870**	**.6931**	**.6986**	**.7035**	**.7079**	**.7120**
21	**.7347**	**.7418**	**.7478**	**.7532**	**.7580**	**.7624**	**.7664**	**.7701**	**.7736**
22	**.7997**	**.8054**	**.8103**	**.8148**	**.8189**	**.8226**	**.8261**	**.8292**	**.8322**
23	**.8574**	**.8621**	**.8662**	**.8699**	**.8733**	**.8764**	**.8792**	**.8817**	**.8841**
24	**.9054**	**.9092**	**.9124**	**.9153**	**.9179**	**.9202**	**.9223**	**.9242**	**.9260**
25	**.9420**	**.9448**	**.9471**	**.9492**	**.9511**	**.9527**	**.9542**	**.9555**	**.9566**
26	**.9673**	**.9691**	**.9707**	**.9721**	**.9733**	**.9744**	**.9753**	**.9761**	**.9768**
27	**.9830**	**.9842**	**.9852**	**.9860**	**.9867**	**.9873**	**.9879**	**.9884**	**.9888**
28	**.9919**	**.9926**	**.9931**	**.9936**	**.9940**	**.9943**	**.9946**	**.9949**	**.9951**
29	**.9964**	**.9968**	**.9971**	**.9973**	**.9975**	**.9977**	**.9978**	**.9980**	**.9981**
30	**.9986**	**.9987**	**.9989**	**.9990**	**.9991**	**.9991**	**.9992**	**.9993**	**.9993**
31	**.9995**	**.9995**	**.9996**	**.9996**	**.9997**	**.9997**	**.9997**	**.9998**	**.9998**
32	**.9998**	**.9998**	**.9999**	**.9999**	**.9999**	**.9999**	**.9999**	**.9999**	**.9999**
33	**.9999**	**.9999**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
34	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
35	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
36	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**

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